Application of energy principle to multiparticle system

Geneneral formalism

- For a multiparticle system, the momentum of the system is by definition the momentum of the center of mass. This can be seen in the following way. For simplicity we will confine our discussions below to the nonrelativistic case. The total momentum of the system is given by

$$\mathbf{P}_{tot} = \Sigma_i \mathbf{p}_i. \tag{1}$$

On the other hand, the cm coordinate is defined by

$$\mathbf{r}_{cm} = \frac{\Sigma m_i \mathbf{r}_i}{M} \tag{2}$$

with $M = \Sigma m_i$. Its velocity

$$\mathbf{v}_{cm} = \frac{\Sigma m_i \mathbf{v}_i}{M} = \frac{P_{tot}}{M} \tag{3}$$

this leads to

$$\mathbf{P}_{cm} = M v_{cm} = P_{tot} \tag{4}$$

So one may think of the center of mass as a fictitious point plarticle (or a point ssytem) which has a mass M and a velocity $\mathbf{v}_{cm}$. Its momentum is Mv_{cm} and its kinetic energy is $\frac{1}{2}Mv_{cm}^2$.

- The momentum pricniple here leads to the equation of motion

$$\frac{\mathbf{P}_{tot}}{dt} = \frac{\mathbf{P}cm}{dt} = \mathbf{F}_{net} \tag{5}$$

Therefure so far the momentum content of the system is concerned,the motion of the whole system is descrfibed by the equation of motion of the cm point.

- Next consider the energy principle

$$\Delta E_{sys} = W. \tag{6}$$

When apply to the point system we have

$$\Delta E_{cm} = \Delta K_{cm} = W_{pt} \tag{7}$$

When applied to the whole system (the real system),

$$\Delta E_{real} = \Delta K_{cm} + \Delta K_{rel} = W_{real} \tag{8}$$

where the subscript "rel" denotes the motion relative to the center of mass. Thue

$$\Delta K_{rel} = \Delta K_{rot} + \Delta K_{vib} + \cdots = \Delta E_{int} + \tag{9}$$

The ΔK_{rel} is the sum of the kinetic energies of all motions relative to the cm point, the rotations, vitrations, etc. And if one is not interested in the relative motions it detail, one may refer to the macroscopic energy such as internal energy or the thermal energy or including over forms of energies. We see if we can identify the work done applied to the cm point and to the real system as a whole, we will be able to investigate the effect due to the relative motion with respect to cm.

Example 1: 9.P.33

Consider the setup shown in Figure 1 (due to technical difficulty it is not available, see lecture notes). A string is wrapped around a hokey puck which has a mass M and a radius R. The puck is placed on a horizontal ice surface. It is pulled by a force F starting from the rest at A. Consider the case where the pull of F has travelled by a distance d'. At the same time the center has displaced from A to B, which covers a distance d. Show that d'/d=1.5.

Solution. For the point-system, denote the kinetic energy at B by K_{cm} then

$$\Delta K_{cm} = (K_{cm} - 0) = W_{pt} = Fd. \tag{10}$$

For the real system, the total kinetic energy consists of K_{cm} and K_{rot}. So

$$\Delta K_{tot} = (K_{cm} - 0) + (K_{rot} - 0) = W_{real} = Fd'. \tag{11}$$

Dividing eq(11) by eq(10) leads to

$$\frac{d'}{d} = \frac{K_{cm} + K_{rot}}{K_{cm}}. \tag{12}$$

Earlier section result gives (see p355)

$$L_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{4}Mv_{cm}^2 = \frac{1}{2}K_{cm}. \tag{13}$$

where we assume the moment of inertia of the puck can be approximated by the moment of inertia a disk with mass M and radius R. Sibstotuting (13) into (12) gives

$$\frac{d'}{d} = \frac{K_{cm} + 0.5K_{cm}}{K_{cm}} = 1.5. \tag{14}$$

Example 2. 9.P.34.

- Consider the setup shown in Figure 2 in your lecture note. Given a chain of mass M and lengath L. Initiallly the chain is a pile at A. A force F is pulling at one end of the chain. When this end is pulled by a distance d' from A, find the change of the internal energy. Assume

$$\Delta E_{real} = \Delta K_{cm} + \Delta_{int}. \tag{15}$$

- Solution. For the cm point,

$$\Delta E_{cm} = \Delta K_{cm} = Fd \tag{16}$$

  For the real system,

$$\Delta E_{real} = \Delta K_{cm} + \Delta E_{int} = Fd' \tag{17}$$

  Substacting (17) from (16) gives

$$\Delta E_{int} = F(d' - d) = \frac{FL}{2}, \tag{18}$$

  where we have used d'=d+l/2, see figure. Digression: The temerature rasied can be estimated,assuming all the dissipataive energy converted into heat. This leads to

$$\Delta T = \frac{\Delta E_{int}}{CM} = \frac{FL}{2CM}, \tag{19}$$

  where C is the specific heat capacity of the chain It is interesting to ask to what extent this temperature increase can be verified by the experiment?