Proper Definition of Spin Current in Spin-Orbit Coupled Systems

Junren Shi
施均仁

Institute of Physics
Chinese Academy of Sciences

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Collaborators: Ping Zhang (九所)
Di Xiao, Qian Niu(UT-Austin)
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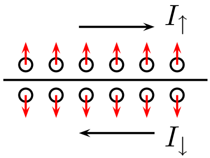
Outline

Concept of Spin Current

Key concept – Spin current:

- Spin transport
- Spin-based information exchange
- More general than the “spin polarized current”



Intuitive definition of spin current:

$$I_s = I_\uparrow - I_\downarrow$$

Pure spin current:

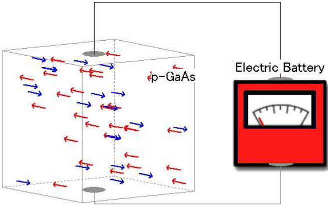
$$I_\uparrow = -I_\downarrow$$

$$I_c = I_\uparrow + I_\downarrow = 0$$

$$I_s = I_\uparrow - I_\downarrow = 2I_\uparrow$$

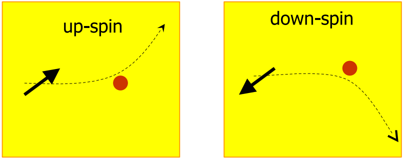
Generating Spin Current – Spin Hall Effect

- Generating pure spin current by applying electric field
- Present in non-magnetic semiconductors



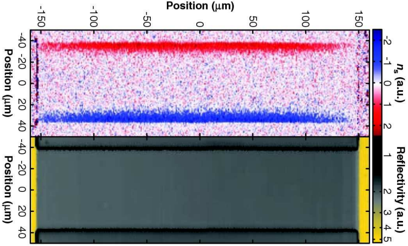
Mechanisms:

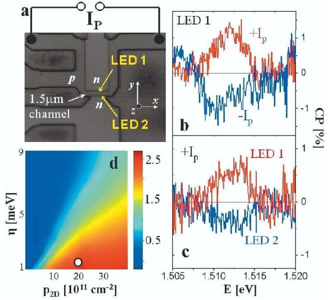
- Extrinsic mechanism – spin dependent skew scattering [Dyakonov and Perel 1972; Hirsch 1999; S. Zhang 2000]



- Intrinsic mechanism – spin dependent anomalous velocity (Berry phase in momentum space) [Murakami et al. 2003; Sinova et al. 2004 and many others]

Spin Accumulation Experiments





Y.K. Kato, R.C. Myers, A.C. Gossard, D.D. Awschalom, Science, **306**, 1910 (2004).

J. Wunderlich, B. Kaestner, J. Sinova, T. Jungwirth, Phys. Rev. Lett. **94**, 047204 (2005).

- Experiments – boundary spin accumulation
- Theory – bulk spin current

Determine the spin current from the spin accumulation?

Spin current and spin accumulation

Simplest theory:

$$\frac{\partial S}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{J}_s = -\frac{S}{\tau_s}$$

$$\int S d\boldsymbol{n} = \boldsymbol{J}_s \cdot \boldsymbol{n}\tau_s$$

However, spin–current/spin–accumulation relation is nontrivial:

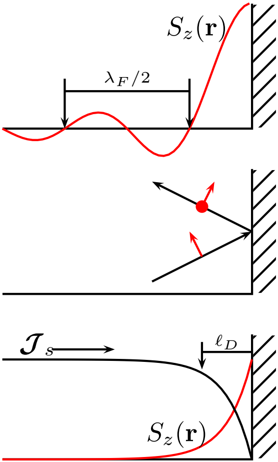
- The relation is valid only for the specific form of spin relaxation.
- Boundary contribution – bulk spin current is not the only source contributing to the boundary spin accumulation.

Spin accumulation may not be an appropriate way to determine the spin current.

Spin Accumulation: Boundary or Bulk Contribution

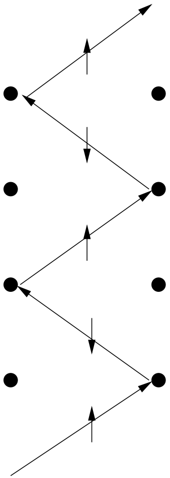
Origins of spin accumulation:

- Boundary effect:
  - Boundary induced spin density wave – similar to Friedel oscillation for charge density.
  - Boundary spin torque: scattering by boundary may induce spin flipping.
- Bulk contribution: spins are transported to the boundary region from the bulk – spin current.



Spin current is only relevant to the spin accumulation contributed by the bulk.

Can the Spin Current Really Describe the Spin Transport?



- Electron is localized along x-direction by the impurity scattering.
- Spin current is non-zero due to the spin-flip scattering
- The electron cannot contribute to the boundary spin accumulation even it carries nonzero spin current.

Issues:

- The spin current is NOT continuous.
- The spin current does not vanish even in a localized state.

The spin current cannot describe the spin transport when spin is not conserved!

Fundamentally Flawed Definition of Spin Current

Conventional definition of spin current:

$$\boldsymbol{J}_e = -e\boldsymbol{v} \longrightarrow \boldsymbol{J}_s = \hat{s}_z\boldsymbol{v}$$

However, this definition is fundamentally flawed:

- Not conserved in spin-orbit coupled systems

$$\frac{\partial S_z}{\partial t} + \nabla \cdot \boldsymbol{J}_s = \mathcal{T}_z \neq 0$$

- Not vanishing even in localized states – Rashba, 2003
- No conjugate force exists – Not a standard flow in the sense of the non-equilibrium statistical physics.

Current	Conjugate force	$\mathrm{d}Q/\mathrm{d}t$
$\boldsymbol{J}_e$	$\boldsymbol{E}$	$\boldsymbol{j}_e \cdot \boldsymbol{E}$
$\boldsymbol{J}_s$	?	?

Motivation

The conventional spin current:

- cannot be directly measured by any known procedure;
- cannot descibe true spin transport.

The proper definition of spin current must be:

- describing the true spin transport.
- measurable as a macroscopic current.

It must:

- conserve:

$$\frac{\partial S_z}{\partial t} + \nabla \cdot \boldsymbol{\mathcal{J}}_s = 0$$

- vanish in (Anderson) insulators
- be in conjugation to a force – spin force

A Conserved Spin Current

Continuity equation

$$\frac{\partial S_z}{\partial t} + \nabla \cdot \boldsymbol{J}_s = \mathcal{T}_z \equiv \langle \dot{s}_z \rangle$$

Assume zero spin generation in the bulk

$$\frac{1}{V} \int \mathrm{d}V \mathcal{T}_z(\boldsymbol{r}) = 0$$

Torque dipole density

$$\mathcal{T}_z(\mathbf{r}) = -\nabla \cdot \boldsymbol{P}_\tau(\boldsymbol{r})$$

Current conserved

$$\frac{\partial S_z}{\partial t} + \nabla \cdot (\boldsymbol{J}_s + \boldsymbol{P}_\tau) = 0$$

New definition:

$$\boldsymbol{\mathcal{J}}_s = \boldsymbol{J}_s + \boldsymbol{P}_\tau$$

$$\int \mathrm{d}V \boldsymbol{P}_\tau = \int \mathrm{d}V \langle \dot{s}_z \boldsymbol{r} \rangle$$

$$\boldsymbol{\mathcal{J}}_s = \langle s_z \dot{\boldsymbol{r}} \rangle + \langle \dot{s}_z \boldsymbol{r} \rangle = \left\langle \frac{\mathrm{d}(s_z \boldsymbol{r})}{\mathrm{d}t} \right\rangle$$

Spin Torque Dipole

- Definition of Spin Torque Dipole

$$\mathcal{T}_z(\mathbf{r}) = -\nabla \cdot \mathbf{P}_\tau(\mathbf{r})$$

- Macroscopic average:

$$\frac{1}{V}\int dV \mathbf{P}_\tau(\mathbf{r}) = \frac{1}{V}\int dV \mathbf{r}\mathcal{T}(\mathbf{r})$$

- Analogy to the charge dipole density:

$$\mathbf{P}(\mathbf{R}) = \frac{1}{V}\int_{V_\mathbf{R}} \mathbf{r}\rho(\mathbf{r})$$

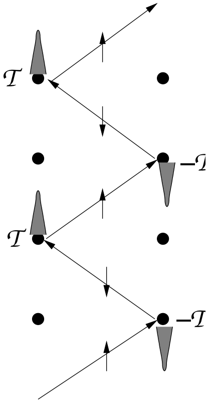
Effective Conserved Spin Current Operator

$$\int dV \mathbf{P}_\tau(\mathbf{r}) \simeq \int dV \mathbf{r}\mathcal{T}(\mathbf{r}) \equiv \int dV \mathrm{Re}\psi^*(\mathbf{r})\frac{1}{2}\left\{\hat{\mathbf{r}}, \frac{\mathrm{d}\hat{s}_z}{\mathrm{d}t}\right\}\psi(\mathbf{r})$$
$$\int dV \mathbf{J}_s(\mathbf{r}) \equiv \int dV \mathrm{Re}\psi^*(\mathbf{r})\frac{1}{2}\left\{\frac{\mathrm{d}\hat{\mathbf{r}}}{\mathrm{d}t}, \hat{s}_z\right\}\psi(\mathbf{r})$$
$$\int dV \boldsymbol{\mathcal{J}}_s(\mathbf{r}) = \int dV \mathrm{Re}\psi^*(\mathbf{r})\left[\frac{\mathrm{d}(\hat{\mathbf{r}}\hat{s}_z)}{\mathrm{d}t}\right]\psi(\mathbf{r})$$

$$\hat{\boldsymbol{\mathcal{J}}}_s = \frac{\mathrm{d}(\hat{\mathbf{r}}\hat{s}_z)}{\mathrm{d}t}$$

This is not microscopic definition of the spin current operator – it is an effective one defined in the macroscopic level.

Testing Case



- Spin torque density $\mathcal{T}_z$ is non-zero – spin flip process.
- Spin torque dipole:

$$P_\tau = -\mathcal{T} l$$

- Conserved spin current:

$$\boldsymbol{\mathcal{J}}_s = \boldsymbol{J}_s + \boldsymbol{P}_\tau = 0$$

Spin torque dipole deducts the local polarization contribution from the spin current. The resulting conserved spin current describes the true spin transport.

In Insulators

- Zero expectation value in any spatially localized states:

$$\langle \ell | \hat{\boldsymbol{\mathcal{J}}}_s | \ell \rangle \equiv \left\langle \ell \left| \frac{\mathrm{d}(\hat{\boldsymbol{r}}\hat{s}_z)}{\mathrm{d}t} \right| \ell \right\rangle = \frac{\mathcal{E}_\ell - \mathcal{E}_\ell}{i\hbar} \langle \ell | \hat{\boldsymbol{r}}\hat{s}_z | \ell \rangle = 0$$

- Zero spin transport coefficient in Anderson insulators:

$$\begin{aligned} \sigma^s &= -e\hbar \sum_{\ell \neq \ell'} f_\ell \frac{\mathrm{Im}\langle \ell | \mathrm{d}(\hat{\boldsymbol{r}}\hat{s}_z)/\mathrm{d}t | \ell' \rangle \langle \ell' | \hat{\boldsymbol{v}} | \ell \rangle}{(\epsilon_\ell - \epsilon_{\ell'})^2} \\ &= -e\hbar \sum_{\ell} f_\ell \langle \ell | [\hat{\boldsymbol{r}}\hat{s}_z, \, \hat{\boldsymbol{r}}] | \ell \rangle = 0 \end{aligned}$$

Conjugate Force for spin current

Origin of spin force:

- gradient of an inhomogeneous Zeeman field
- spin dependent chemical potential near ferromagnet-metal interface

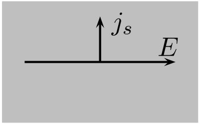
$$H = H_0 - \hat{s}_z \hat{\boldsymbol{r}} \cdot \boldsymbol{F}_s$$

$$\frac{\mathrm{d}Q}{\mathrm{d}t} \equiv \frac{\mathrm{d}H_0}{\mathrm{d}t} = \frac{\mathrm{d}(\hat{s}_z \hat{\boldsymbol{r}})}{\mathrm{d}t} \cdot \boldsymbol{F}_s \equiv \boldsymbol{\mathcal{J}}_s \cdot \boldsymbol{F}_s$$

The new definition conforms to the standard near-equilibrium transport theory.

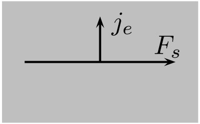
Onsager Relation for Charge/Spin Transport

Spin Hall effect



$$\mathcal{J}_s^x = \sigma_{xy}^{sc} E^y$$

Inverse spin Hall effect



$$j_e^y = \sigma_{yx}^{cs} F_s^x$$

Onsager Relation:

$$\sigma_{xy}^{sc} = -\sigma_{yx}^{cs}$$

The spin transport can be connected to the charge transport.

Onsager Relation – General theory

- A system under two driving forces:

$$H = H_0 - X_1 F_1 - X_2 F_2$$

- Transport coefficients defined by:

$$\begin{aligned} \langle \dot{X}_1 \rangle &= \sigma_{11} F_1 + \sigma_{12} F_2 \\ \langle \dot{X}_2 \rangle &= \sigma_{21} F_1 + \sigma_{22} F_2 \end{aligned}$$

- Onsager relation

$$\sigma_{12}(S) = s_1 s_2 \sigma_{21}(S^*)$$

Direct application:

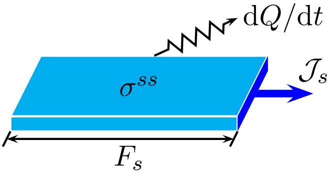
$$X_1 \to -e\hat{\boldsymbol{r}} \qquad\qquad X_2 \to \hat{s}_z \hat{\boldsymbol{r}}$$

Direct measurement of spin current

Thermodynamic method:

$$\mathcal{J}_s = \frac{1}{F_s}\frac{\mathrm{d}Q}{\mathrm{d}t}$$

Technique to measure the Zeeman field gradient is required.

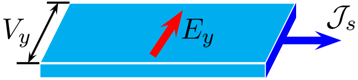


Electric method:

$$\mathcal{J}_s = \sigma^{sc}_{xy}\frac{V_y}{L_y}$$

σ^{sc}_{xy} can be determined from the inverse spin Hall effect:

$$\sigma^{sc}_{xy} = -\sigma^{cs}_{yx}$$



Linear Response Theory

To calculate the spin torque dipole:

- Calculate the spin torque response to external field at finite wave vector $\boldsymbol{q}$:

$$\mathcal{T}(\boldsymbol{q}) = \chi_\nu(\boldsymbol{q}) E_\nu(\boldsymbol{q})$$

- Spin torque dipole is related to the spin torque by:

$$\mathcal{T}(\boldsymbol{q}) = -i\boldsymbol{q} \cdot \boldsymbol{P}_\tau(\boldsymbol{q}) \equiv -iq_\mu \sigma^{\mathcal{T}}_{\mu\nu} E_\nu(\boldsymbol{q})$$

- Long wave limit $\boldsymbol{q} \to 0$:

$$\sigma^{\mathcal{T}}_{\mu\nu} = i \lim_{\boldsymbol{q}\to 0} \frac{1}{q_\mu} \chi_\nu(\boldsymbol{q}) = i\, \partial_\mu \chi_\nu(\boldsymbol{q})\big|_{\boldsymbol{q}\to 0}$$

Intrinsic Spin Hall Coefficients

Conventional values:

- Rashba model:

$$-\frac{e}{8\pi}$$

- Cubed-k Rashba model:

$$\frac{9e}{8\pi}$$

- Luttinger model:

$$-\frac{3e\gamma_1}{12\pi^2\gamma_2}(k_H-k_L)+\frac{e}{6\pi^2}k_H$$

$\gamma_2 \to 0$: $\frac{e}{6\pi^2}k$

New values:

- Rashba model:

$$\frac{e}{8\pi}$$

- Cubed-k Rashba model:

$$-\frac{9e}{8\pi}$$

- Luttinger model:

$$\frac{e(\gamma_2-\gamma_1)}{6\pi^2\gamma_2}(k_H-k_L)+\frac{e}{6\pi^2}k_H$$

$\gamma_2 \to 0$: $-\frac{e}{6\pi^2}k$

Disorder effect

(a) Rashba model

Impurity potential	Born approx.	Definition of spin current	
		$\langle J_s \rangle$	$\mathcal{J}_s$
$\delta(\mathbf{r})$	1st	0	0
	higher	0	0
$V_{\mathbf{p}-\mathbf{p}'}$	1st	0	0
	higher	0	Finite

(b) Cubic Rashba model

Impurity potential	Born approx.	Definition of spin current	
		$\langle J_s \rangle$	$\mathcal{J}_s$
$\delta(\mathbf{r})$	1st	Finite	0
	higher	Finite	0
$V_{\mathbf{p}-\mathbf{p}'}$	1st	Finite	0
	higher	Finite	Finite

Sugimoto, Onoda, Murakami and Nagaosa, cond-mat/0503475.

Conclusion

A proper definition of spin current is established:

- Conserved – Kirchhoff's law for spin current
- Vanishes in Anderson insulators – True transport current
- Measurable – Conjugate force exists

$$\mathcal{J}_s = \langle s_z \dot{\boldsymbol{r}} \rangle + \langle \dot{s}_z \boldsymbol{r} \rangle = \left\langle \frac{\mathrm{d}(s_z \boldsymbol{r})}{\mathrm{d}t} \right\rangle$$

Physical consequences: (somewhat disappointing)

- A few widely studied semiconductor models (Rashba and cubic Rashba) turn out to have NO intrinsic spin Hall effect.
- There is still NO known non-magnetic system that can generate a spin current in the presence of an electric field.

Thank You For Your Attention!