Charge-Hall effect driven by spin force: reciprocal of the spin-Hall effect

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A new kind of charge-Hall effect is shown. Unlike in the usual Hall effect, the driving force in the longitudinal direction is a spin force, which may originate from the gradient of a Zeeman field or a spin-dependent chemical potential. The transverse force is provided by a Berry curvature in a mixed position-momentum space. We can establish an Onsager relation between this effect and the spin-Hall effect provided the spin current in the latter is modified by a torque dipole contribution. This remarkable relation leads to new ways for experimental detection of spin accumulation predicted by the spin Hall effect.

PACS numbers: 72.10.Bg, 72.20.Dp, 73.63.Hs

The generation of an electric current in the transverse direction of an electric field is known as a Hall effect. The transverse force is usually provided by the Lorentz force from a magnetic field, but can also arise from spin-orbit interactions in magnetic materials[1] or Magnus forces on vortices in superconductors[2]. Apart from such ordinary and anomalous Hall effects, other variants, such as the thermal Hall effect[3], can manifest when the longitudinal electric field is replaced by other type of electromotive forces like a temperature gradient. Likewise, a longitudinal electric field can also cause non-electrical response in the transverse direction, such as in the spin Hall effect where a transverse spin current may be generated[4, 5, 6].

Recently, extensive theoretical work[7, 8, 9, 10] has established the importance of intrinsic origin of the anomalous Hall effect in ferromagnets. Unlike the traditional mechanisms of skew scattering[11], Berry phase in the Bloch bands due to spin-orbit coupling can lead to a Hall current depending only on the equilibrium part of the carrier distribution function. In a similar fashion, intrinsic spin-Hall effect has been proposed for paramagnetic materials[12, 13], which has generated much interest recently[14, 15, 16, 17, 18, 19, 20, 21, 22, 23] in association with electrical manipulation of spins for spintronics applications[24, 25, 26].

In this Letter, we propose a novel charge Hall effect driven by a spin force along the longitudinal direction. This spin force can be provided by the gradient of a Zeeman field or a spin dependent chemical potential. In the intrinsic regime considered here, the transverse force is provided by a Berry curvature in the mixed position-momentum space, also stemming from spin-orbit coupling in the band structure. As shown below, there is not a longitudinal electric current accompanying the spin-force in this intrinsic regime, so the spin force should not be interpreted as a variant electromotive force. Therefore, this effect is essentially different from the conventional thermal Hall effect, which is known to be just a usual Hall effect driven by a variant electromotive force.

Another goal of this Letter is to construct a "reciprocal" version of the intrinsic spin-Hall effect[12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23] which is at present being in active investigation. Since a longitudinal charge force (electric field) may drive a spin-Hall current through the spin-orbit interaction, one naturally expects that a spin force can also induce a charge-Hall current. To our surprise, we discovered that an exact Onsager relation between these intrinsic Hall effects cannot readily be established; only when the spin current is modified by including a torque dipole term, is the spin-Hall conductivity in response to an electric field found to correspond to our charge-Hall conductivity in response to a spin force. Interestingly, it is this modified spin current that is responsible for spin accumulation at a sample boundary as shown recently based on the spin continuity equation in a semiclassical theory[14]. Our finding on the Onsager relation further points to the importance of this modified spin current. Spin accumulation due to spin Hall effect can thus be tested through the Onsager relation by a measurement of our charge Hall effect in response to a spin force.

To construct the theory, we consider the conduction electrons in semiconductor quantum wells. In this two-dimensional electron gas (2DEG) system, spin-orbit interaction arises from the quantum well asymmetry in the growth (z) direction and has a standard Rashba form[27] $V_{so} = (\alpha/\hbar)(\sigma \times \mathbf{k}) \cdot \hat{\mathbf{z}}$, where σ is the Pauli matrix vector and $\mathbf{k}$ is the 2D wave vector in the x-y plane. The Rashba coefficient, α, can be tuned over a wide range by a vertical electric field. Taking into account the kinetic energy and a Zeeman field normal to the heterostructure, we can write the $\mathbf{k} \cdot \mathbf{p}$ Hamiltonian as

$$H = \gamma k^2 + V_{so} - \mu \sigma_z, \tag{1}$$

where $\gamma = \hbar^2/2m^*$ with m^* being the band effective mass, and μ is the Zeeman field (in unit of energy), which is assumed to be weak in this paper. The orbital effect of the magnetic field can be taken into account in our semiclassical formalism, but will be neglected in this work for simplicity. To introduce a spin force, we consider a non-uniform Zeeman field as in the Stern-Gerlach experiment for separating spin-up and spin-down electrons. We thus assume $\mu = \mu_0 + \mu_1 x$, where μ_0 describes the average strength of the Zeeman field while $F_s = (\frac{2}{\hbar})\partial\mu/\partial x = \frac{2}{\hbar}\mu_1$

gives the spin force in the x (longitudinal) direction. The Zeeman field can be created by exchange interaction with the moments of doped magnetic ions, which can be polarized by a weak magnetic field (with negligible Lorentz force) at low temperatures. An inhomogeneous Zeeman field can be produced either by a magnetic field gradient or a temperature gradient. Alternatively, one may consider using the spin-dependent chemical potential gradient present near an interface with a ferromagnetic material.

To access transport properties, we adopt the formalism of semiclassical wavepacket dynamics[28], which is a powerful tool for studying the influence of slowly varying perturbations such as the spin force term in Eq.(1) on the dynamics of Bloch electrons. Consider a wave packet centered at $\mathbf{r}_c$ and with a narrow distribution around the mean wave vector $\mathbf{k}$. Then the semiclassical equations of motion for $\mathbf{r}_c$ and $\mathbf{k}$ are [28]

$$\dot{\mathbf{r}}_c = \frac{1}{\hbar}\frac{\partial \varepsilon_n}{\partial \mathbf{k}} + \frac{1}{\hbar}\frac{\partial \Delta\varepsilon_n}{\partial \mathbf{k}} - \mathbf{\Omega}_n^{\mathbf{kr}}\cdot\dot{\mathbf{r}}_c - \mathbf{\Omega}_n^{\mathbf{kk}}\cdot\dot{\mathbf{k}}, \tag{2a}$$

$$\dot{\mathbf{k}} = -\frac{1}{\hbar}\frac{\partial \varepsilon_n}{\partial \mathbf{r}_c} - \frac{1}{\hbar}\frac{\partial \Delta\varepsilon_n}{\partial \mathbf{r}_c} + \mathbf{\Omega}_n^{\mathbf{rr}}\cdot\dot{\mathbf{r}}_c + \mathbf{\Omega}_n^{\mathbf{rk}}\cdot\dot{\mathbf{k}}, \tag{2b}$$

where n is the band index, and ε_n is the local band energy obtained by diagonalizing the Hamiltonian (1) with the position operator $\mathbf{r}$ identified with the wavepacket center $\mathbf{r}_c$. The result is $\varepsilon_{1,2}(\mathbf{k},\mathbf{r}_c) = \gamma k^2 \mp \sqrt{\mu_c^2+\alpha^2k^2}$ with $\mu_c = \mu_0 + \mu_1 x_c$. In the second term in Eqs.(2), $\Delta\varepsilon_n$ is the energy correction in the gradient of the Zeeman field. For the Hamiltonian (1) we get $\Delta\varepsilon_1(\mathbf{k},\mathbf{r}_c) = \Delta\varepsilon_2(\mathbf{k},\mathbf{r}_c) = \alpha^2 k_y\mu_1/2(\mu_c^2+\alpha^2k^2)$. Finally, the last two terms in Eqs.(2) denote Berry-curvature corrections to the orbital motion of the electron. They are defined by

$$\left(\mathbf{\Omega}_n^{\mathbf{kr}}\right)_{\alpha\beta} \equiv \Omega_n^{k_\alpha x_\beta} = i\left[\langle\frac{\partial u_n}{\partial k_\alpha}|\frac{\partial u_n}{\partial x_\beta}\rangle - \langle\frac{\partial u_n}{\partial x_\beta}|\frac{\partial u_n}{\partial k_\alpha}\rangle\right], \tag{3}$$

where u_n is the eigenstate of the nth band. The other Berry curvature tensors are defined similarly. One can see that the equations of motion involve Berry curvatures between every pair of parameters in combined real and momentum spaces. Recent studies[7, 8, 9] of the anomalous Hall effect in ferromagnetic materials have revealed the fundamental role of the Berry curvature $\mathbf{\Omega}^{\mathbf{kk}}$ in the k space. As we will see below, it is the Berry curvature in the mixed position-momentum space, $\mathbf{\Omega}^{\mathbf{kr}}$, which has not attracted attention in previous work, that determines the charge-Hall effect driven by a spin force.

In most cases, calculation of the Berry curvatures is an involved work. For real materials with complicated crystal and ferromagnetic structures, first-principle calculations are needed. Fortunately, in the present two-band case, the Berry curvatures can be obtained analytically. For the bottom band with energy ε_1, the non-zero components of the Berry curvatures are

$$\Omega^{k_x,k_y} = -\frac{\alpha^2\mu_c}{2\Delta_c^3}, \Omega^{k_x,x} = \frac{\alpha^2k_y\mu_1}{2\Delta_c^3}, \Omega^{k_y,x} = -\frac{\alpha^2k_x\mu_1}{2\Delta_c^3}, \tag{4}$$

where $\Delta_c = \sqrt{\mu_c^2+\alpha^2k^2}$. The Berry curvatures for the top band are different from the above by a sign. Considering these non-zero Berry curvatures, and keeping terms up to first order in spin force, we get $\dot{k}_x = -\partial\varepsilon_n/\hbar\partial x_c$ and $\dot{k}_y = 0$. The orbital motion in the nth band is given by

$$\dot{x}_c = (1-\Omega_n^{k_x,x})v_n^x + \frac{\partial\Delta\varepsilon_n}{\hbar\partial k_x}, \tag{5a}$$

$$\dot{y}_c = v_n^y + \frac{\partial\Delta\varepsilon_n}{\hbar\partial k_y} - \Omega_n^{k_y,x}v_n^x + \Omega_n^{k_y,k_x}\frac{\partial\varepsilon_n}{\hbar\partial x_c}, \tag{5b}$$

where $v_n^\alpha = (1/\hbar)\partial\varepsilon_n/\partial k_\alpha$ is band group velocity. The longitudinal and transverse currents are obtained by averaging $\dot{x}_c$ and $\dot{y}_c$ with the distribution function, respectively. The full distribution function consists of an equilibrium part and a nonequilibrium part (maintained by a competition between k_x-drift and scattering relaxation). As for the anomalous Hall effect and spin Hall effect, there is an intrinsic contribution from the equilibrium part of the distribution. The extrinsic contribution from the non-equilibrium part will be small if scatterings are strong enough to maintain the distribution near to equilibrium and weak enough (compared to the Rashba splitting in energy scale) so that interband couplings are small. In this work we will focus attention to the intrinsic regime in order to make contact with similar work for the spin Hall effect. Using the equilibrium distribution, we find that the longitudinal current vanishes. However, the intrinsic Hall current in the y direction is not zero, and is given by, using Eq.(5b),

$$\begin{aligned} J_y^c &= e\sum_{n=1}^{2}\int\frac{d^2k}{(2\pi)^2}f_n\Omega_n^{k_y,x}v_n^x \\ &= -F_s\int\frac{d^2k}{(2\pi)^2}\frac{e\hbar\alpha^2k_x}{4\Delta^3}(f_1v_1^x - f_2v_2^x) \\ &\equiv \sigma_{yx}^{cs}F_s, \end{aligned} \tag{6}$$

where f_n is the equilibrium Fermi-Dirac distribution function, and $\Delta = (\mu_0^2+\alpha^2k^2)^{1/2}$. We note that the contributions from the second and last terms in Eq.(5b) cancel each other under the k integral, thus only the Berry curvature $\Omega^{k_y,x}$ contributes to the charge-Hall current.

Equation (6) is a central result in this paper. Figure 1 summarizes the behavior of the charge-Hall conductivity. Panel (a) is a (colored) contour plot of this quantity in the parameter plane of electron density and Rashba coefficient. The (blue) region below the dashed line corresponds to the case of single band occupation, where the charge-Hall conductivity is small. At high electron densities, σ_{yx}^{cs} remains large within a wide range of α. Panel

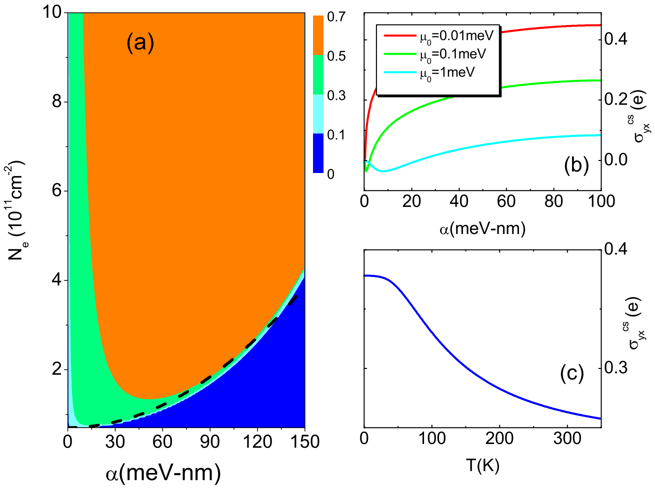

FIG. 1: Charge-Hall conductivity (in unit of e) in GaAs quantum well system as a function of (a) Rashba coefficient α and the electron density N_e (contour plot) for $T = 0$ and $\mu_0 = 0.01$meV, (b) Rashba coefficient for different values of μ_0, and (c) temperature.

(b) shows the dependence on the Rashba coefficient for a set of Zeeman energies. In these panels, the temperature is set to zero. Panel (c) shows that the charge-Hall effect is robust against thermal broadening, a property stemming from the intrinsic nature of σ_{yx}^{cs}. For estimate, we consider a GaAs quantum well sample with electron density $N_e \sim 5 \times 10^{11}\text{cm}^{-2}$ and a typical Rashba coefficient of $\alpha \sim 20$meV-nm. A spin force corresponding to $\mu_1 \sim 1\text{meV}/\mu\text{m}$ (one millivolt of Zeeman energy change over a micron) can yield a charge-Hall current on the order of 1mA/cm, implying that the predicted effect is observable.

Now we turn our attention to a basic and enlighten question: is there an Onsager relation between our spin force-driven charge-Hall effect and the electric field-driven spin-Hall effect? In the presence of an electric field along the y direction, the Hamiltonian is now

$$H = \gamma k^2 + V_{so} - \mu_0 \sigma_z + eEy, \tag{7}$$

where we have removed the spin force but kept the uniform part of the Zeeman field. Based on the spin current operator, defined as the symmetric product of the velocity and spin operators, $\frac{1}{2}(s_z \dot{x} + \dot{x} s_z)$, one can obtain a spin current in the transverse (x) direction either from the Kubo formula or the semiclassical wavepacket formalism. In the intrinsic regime, this spin current for the Rashaba model is given by

$$J_x^s = -E \int \frac{d^2k}{(2\pi)^2} \frac{e\hbar^2\alpha^2 k_x^2}{4m^*\Delta^3}(f_1 - f_2). \tag{8}$$

At zero temperature and $\mu_0 = 0$, this yields a universal spin Hall conductivity [13, 16, 17, 18, 20, 21, 22], $\sigma_{xy}^{sc} = -\sigma_{yx}^{sc} = -e/8\pi$, which does not depend on the Rashba coefficient. Unfortunately, this universality is absent in the spin force-driven charge-Hall conductivity σ_{yx}^{cs} [see Eq.(6) and Fig.1]. In fact, the expression for σ_{yx}^{cs} in Eq.(6) can be reduced to the expression for σ_{xy}^{sc} in Eq.(8) *only* when the band group velocity $v_{1,2}^x = \hbar k_x/m^* \mp \alpha^2/\hbar\Delta^2$ is replaced by its first term $\hbar k_x/m^*$; the neglected term will cause a finite difference for non-zero Rashba coefficient. Thus contrary to our intuitive expectation, a full Onsager relation between these two kinds of Hall conductivities fails to be established.

To understand the origin of this violation of the Onsager relation and to search for a remedy, we notice that the spin-current operator is not really an observable in conjugation with the spin force. To illustrate this point, let us put the charge force (the electric field) and the spin force in an equal footing by including both of them in the perturbative Hamiltonian

$$\delta H = -F_1 d_1 - F_2 d_2, \tag{9}$$

where $F_1 = E$ and $F_2 = F_s$ are the generalized forces applied on the charge and spin degrees of freedom, whereas $d_1 = -ey$ and $d_2 = s_z x$ are the corresponding displacement operators. The response currents are obtained as expectation values of the generalized velocity operators in the perturbed state

$$\dot{d}_1 = -e\dot{y}, \tag{10a}$$

$$\dot{d}_2 = s_z \dot{x} + \dot{s}_z x, \tag{10b}$$

where symmetrization between operators in Eq.(10b) is implied. It is clear now that the current conjugate to the spin force (F_2) not only involves the spin current given from $s_z\dot{x}$[12, 13], but also a torque dipole term $\dot{s}_z x$. We therefore introduce a modified spin current defined by

$$\tilde{J}_x^s = \sum_n \int \frac{d^2k}{(2\pi)^2} f_n \langle s_z \dot{x} + \dot{s}_z x \rangle_n, \tag{11}$$

where the bracket indicates quantum mechanical average over the wavepacket made out of the nth band. Due to the spin-orbit interaction, besides driving the wavepacket in the **k**-space, the electric field will also give rise to a nonadiabatic correction to the spin wavefunction. This correction has been taken into account when constructing our wavepacket for each band.

In terms of the wave packet center coordinate x_c and velocity $\dot{x}_c$, we can rewrite the above expression as [14]

$$\tilde{J}_x^s = \sum_n \int \frac{d^2k}{(2\pi)^2} f_n \left[\dot{x}_c \langle s_z \rangle_n + \frac{d}{dt}(p_x^{s_z})_n + x_c \frac{d}{dt}\langle s_z \rangle_n \right], \tag{12}$$

where $(p_x^{s_z})_n = \langle (x - x_c) s_z \rangle_n$ is the x-component of the spin dipole in the wave packet. To first order in the electric field, the time derivatives in the second and third terms can be replaced by $-eE(\partial/\hbar\partial k_y)$, and the quantities under the time derivatives can be evaluated at zero field as $\langle s_z\rangle_{1,2} = \pm\mu_0\hbar/2\Delta$, and $(p_x^{s_z})_{1,2} = -\hbar\alpha^2 k_y/4\Delta^2$. It is then clear from symmetry that the last term in the above equation vanishes after the k integral. The wave packet velocity has a first order field correction due to Berry phase, $\dot{x}_c = v_n^x + (e/\hbar)E\Omega_n^{k_x k_y}$, where $v_n^x = \partial\varepsilon_n/\hbar\partial k_x$ is the band group velocity. The Berry curvature tensor $\mathbf{\Omega}_n^{\mathbf{kk}}$ has been given in Eq.(4). Similarly, the wave packet spin also has a first order field correction, given by $E\langle s_z\rangle'_n$ with $\langle s_z\rangle'_{1,2} = \mp e\alpha^2 k_x/4\Delta^3$. After these considerations, the modified spin-Hall current can be written as

$$\tilde{J}_x^s = E\sum_n \int \frac{d^2k}{(2\pi)^2} f_n[v_n^x\langle s_z\rangle'_n + \frac{e}{\hbar}\Omega_n^{k_x k_y}\langle s_z\rangle_n - \frac{e}{\hbar}\frac{\partial}{\partial k_y}(p_x^{s_z})_n]. \tag{13}$$

Remarkably, we find that the contributions from the Berry curvature term and the spin dipole term completely cancel each other. Thus only the first term in Eq.(13) contributes to the modified spin current, yielding a spin Hall conductivity

$$\sigma_{xy}^{sc} = -\int \frac{d^2k}{(2\pi)^2}\frac{e\hbar\alpha^2 k_x}{4\Delta^3}(f_1 v_1^x - f_2 v_2^x). \tag{14}$$

A comparison between Eq.(14) and Eq.(6) gives the Onsager relation

$$\sigma_{yx}^{cs}(\mu_0) = \sigma_{xy}^{sc}(\mu_0). \tag{15}$$

It should be pointed out that these two kinds of Hall conductivities have time-reversal symmetry T, which is absent in the conventional Ohm's law. Since the spin force and the charge-Hall current are odd under T, thus they can be related via T-invariant σ_{yx}^{cs}. On the other hand, the spin current and the electric field are even under T; the resultant spin-Hall conductivity σ_{xy}^{sc} is also T-invariant. Due to this time-reversal symmetry, the general Onsager relation $\sigma_{\alpha\beta}(\mu_0) = \sigma_{\beta\alpha}(-\mu_0)$ reduces to Eq.(15) in the present context. We have also shown explicitly[29] that Eq.(15) remains to be true for the 4-band Luttinger model, and we believe that this is a generally valid relationship.

This remarkable Onsager relation is another central result in this paper. As a consequence, the spin Hall conductivity (for the modified spin current) should behave the same as we depicted in Fig. 1 for our charge Hall conductivity in response to a spin force. All previous discussions on spin Hall effect measurement are based on spin accumulation on a sample boundary due to a bulk spin current; Ref.[14] showed that it is the modified spin current that is relevant to such a measurement. The Onsager relation derived here provides another method for testing this modified spin Hall effect.

In conclusion, we have shown that a spin force can drive a novel charge-Hall current through the spin-orbit coupling. The predicted charge Hall effect has a remarkable Onsager relation with a spin-Hall effect driven by an electric field. This Onsager relation is only attainable when the spin current is corrected by a torque dipole density.

We thank D. Culcer, J. Sinova, and A.H. MacDonald for useful discussions. PZ was supported by NSF Grand No. DMR-0071893 and Welch Foundation in Texas. QN was supported by DE-FG03-02ER45958.

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