

Nonadiabaticity and single electron transport driven by surface acoustic waves

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Single-electron transport driven by surface acoustic waves (SAW) through a narrow constriction, formed in two-dimensional electron gas, is studied theoretically. Due to long-range Coulomb interaction, the tunneling coupling between the electron gas and the moving minimum of the SAW-induced potential rapidly decays with time. As a result, nonadiabaticity sets a limit for the accuracy of the quantization of acoustoelectric current.

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Recently, a new type of single electron devices was introduced. In the experiments¹, surface acoustic waves (SAW) induce, via piezo-electric coupling, charge transport through a point contact in GaAs heterostructure. When the point contact is biased beyond the pinch-off, the acoustoelectric current develops plateaus, where

$$I = N_0 e f. \quad (1)$$

Here f is SAW frequency, and N_0 is an integer. The plateaus were shown to be stable over a range of temperature, gate voltages, SAW power, and source-drain voltages. Remarkably high accuracy of the quantization (1), and high frequency of operation ($f \sim 3$ GHz) immediately suggest a possibility of metrological applications of the effect².

However, deep understanding of these results is still lacking. Qualitatively, the effect is explained by a simple picture of moving quantum dots¹. Electrons, captured in the local potential minima ('dots'), created by SAW, are dragged through the potential barrier. The strong Coulomb repulsion prevents excess occupation of the dot. Increase of the SAW power deepens the dots, more states become available for the electrons to occupy, and new plateaus appear. By changing the gate voltage, the slope of the potential barrier can be lowered, which has a similar effect.

Interestingly, the quantization was not observed in the open channel regime³, although it should be expected on the quite general theoretical ground⁴. For the mechanism of the quantization, discussed in⁴, it is essential that the DC conductance for each instantaneous configuration of the SAW-induced potential is zero. In the open channel regime it would require the channel length to be much longer than SAW wavelength λ , which is difficult to realize. In the experiments¹ this problem is avoided, since in the pinch-off regime the DC conductance is zero. However, as explained below, the rapid change of SAW potential near the entrance to the channel creates a new trouble, leading to nonadiabatic corrections to (1).

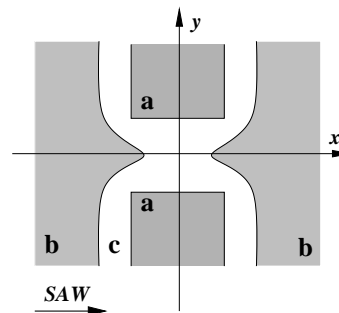


FIG. 1. Geometry of the system: (a) split gates; (b) regions, occupied by two-dimensional electron gas; (c) depleted region. The arrow indicates direction of SAW propagation.

Long-range Coulomb interaction plays crucial role in this phenomenon. The two-dimensional electron gas (2DEG) is depleted in the vicinity of the gates (see Fig.1). On the other hand, in the depleted region screening is lacking. The important parameters of the problem can be understood from the solution of the electrostatic model. In this approach, one assumes that, since the Fermi velocity v_F is large compared to the sound velocity v_s , the 2DEG is able to follow the changing in time SAW-induced potential. Therefore, it is sufficient to consider an instantaneous electrostatic problem, treating time as a parameter. Since the screening length in 2DEG (~ 10 nm) is much smaller, than $\lambda \sim 10^3$ nm, one can assume that the SAW-induced potential is completely screened in the 2DEG-occupied region. Therefore, one has to solve the Poisson equation subjected to complicated boundary conditions. In particular, the potential at the gates ($\varphi = V_g$), as well as the potential of 2DEG region ($\varphi = 0$), and density $\rho = \rho_0$ of the positive background charge in the depleted region are fixed. Furthermore, for simplicity, one can take an effect of SAW into account through a weak periodic modulation of ρ : $\rho \rightarrow \rho_0 + \delta\rho(x, t)$. The self-consistent solution would yield the location of the edge of 2DEG, potential in the depleted region $\varphi(\mathbf{r})$, and number density $n(\mathbf{r})$

of 2DEG. Then, using the Thomas-Fermi-type relation $U(\mathbf{r}) + (\pi\hbar^2/m^*)n(\mathbf{r}) = \epsilon_F$, one would be able to find an effective confining potential $U(\mathbf{r})$. However, the details of the full solution are not required, since the most important properties can be understood from the following simple arguments⁵.

Firstly, it is clear that the potential $\varphi(x, y)$ has a minimum in the gap between the gates (see Fig.1). Secondly, the very presence of plateaus shows that the charge states of the dot are separated by the finite energy gaps. Since both $N_0 = \text{odd}$ and $N_0 = \text{even}$ plateaus are observed the energy gap is associated with Coulomb repulsion rather than single-particle level spacing. This means that for any given x , $\varphi(x, y)$ has a sharp minimum near $y = 0$. The shape of this minimum is slowly changing with x , and this change is controlled by the geometry of the device. Note that the smoothness of the change of the confining potential in y -direction is supported by the fact that the same systems exhibit very nice pattern of conductance quantization in the open channel regime¹. Finally, the weak perturbation of the background charge density $|\delta\rho| \ll \rho_0$ should not affect significantly the position x_0 of the 2DEG edge.

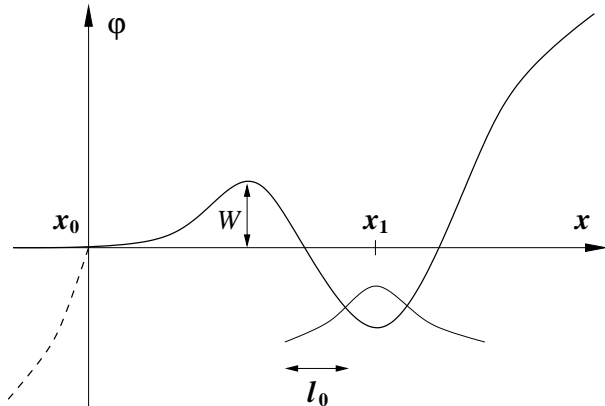


FIG. 2. The potential φ along the line $y = 0$. The dashed line shows the effective potential, that confines 2DEG to the region $x \leq x_0$. The SAW-induced potential minimum (x_1) moves to the right with the sound velocity v_s .

As function of time, $x_0(t)$ oscillates with SAW frequency f . However, the amplitude of the oscillations is small, compared to λ , therefore the velocity of the edge is negligible compared to v_s . On the other hand, the SAW-induced potential minimum (the dot) moves away from the edge with precisely the sound velocity v_s (see Fig.2). Therefore, the width of the potential barrier, separating 2DEG and the dot, increases linearly with time. This in turn means that the tunneling coupling between 2DEG and the level, localized in the dot, rapidly decreases, approximately exponentially. The characteristic time can be estimated as $\tau \sim l_0/v_s$, where l_0 is the distance over which the localized wave extends under the barrier. Since, evidently, $l_0 \ll \lambda$, the relation

$$f\tau \ll 1 \quad (2)$$

holds. Other parameters, such as the height of the barrier W , the energy of the localized level ϵ_0 , etc., change during the time, which is of the order of SAW period $1/f$. Thus, the time dependence of all these parameters during the time τ can be neglected.

Due to the rapid decrease of the tunneling coupling, the thermal equilibrium in the system can not be maintained, causing fluctuations of the occupation number of the dot. This results in nonadiabatic corrections to the quantized values of the acoustoelectric current. Whether these corrections have a significant impact on the accuracy of the quantization, depends on the value of the characteristic energy scale \hbar/τ , as compared to other energy scales in the problem. To obtain an order of magnitude estimate, we expand the potential near the minimum x_1 (see Fig.2), $V(x) \sim Aq^2(x - x_1)^2/2$, $q = 2\pi/\lambda$. The amplitude A is related to the single particle level spacing in the dot Δ via $Aq^2 = m^*(\Delta/\hbar)^2$ (m^* is effective electron mass), to the 'size' r of the localized wave function via $Aq^2r^2 \sim \Delta$, and to the charging energy E_c via $E_c \sim e^2/\epsilon r$. l_0 is estimated from WKB relation $(\hbar/l_0)^2 \sim 2m^*W$. Assuming that $W \sim A$, it gives us four equations for five unknown quantities. Additional relation follows from the experimental results. It was demonstrated¹, that the quantization disappears above the activation temperature $T^* \sim 10$ K, which we identify with the charging energy. Using typical parameters for the experiments¹, we find $\tau \sim 10$ ps⁶. All the parameters manage to pass minimal consistency requirements $r \ll \lambda$, $\Delta \ll A$, E_c , $f\tau \ll 1$. Since the corresponding energy scale $\hbar/\tau \sim 0.1$ meV, the nonadiabatic effects may have significant influence on (1) at low temperature.

This can be understood from the following model Hamiltonian:

$$H = H_{eg} + H_{dot} + H_T. \quad (3)$$

Here

$$H_{eg} = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} \quad (4)$$

describes electron gas in the lead,

$$H_{dot} = \sum_{n\sigma} E_n d_{n\sigma}^\dagger d_{n\sigma} + E_c (N - N_g)^2 \quad (5)$$

is the Hamiltonian of the dot ($N = \sum_{n\sigma} d_{n\sigma}^\dagger d_{n\sigma}$ is the total number of electrons in the dot), and

$$H_T = V(t) \sum_{kn\sigma} c_{k\sigma}^\dagger d_{n\sigma} + \text{H.c.}, \quad (6)$$

describes the tunneling coupling with time-dependent tunneling amplitude. We have included only one lead in the model, since tunneling coupling to the second lead is negligible. The electron gas in the lead is assumed to be in thermodynamic equilibrium at all times, by virtue of

the inequality $v_F \gg v_s$. As discussed above, we neglected time dependence of various parameters in (5), due to the separation of the time scales $f\tau \ll 1$. The last term in (5) describes the intra-dot Coulomb interaction. The parameter \mathcal{N}_g describes the effect of the gate voltage. Since the width of the plateaus is approximately independent on the plateau's number, it is a good approximation to assume that \mathcal{N} is a linear function of V_g . The most important ingredient of (6) is the time-dependent tunneling amplitude. We take it in the form

$$V(t) = V_0 e^{-t/\tau} \quad (7)$$

(other possible choices will be discussed below). The time-independent version of (3-6) is commonly used in the theory of the Coulomb blockade. Similar models have been also employed to study transfer of charge during the atom-surface scattering⁷, and nonadiabatic effects in charge pumping⁸.

Given that the system, described by (3-6), is in thermodynamic equilibrium at $t = -\infty$, our task is to calculate the occupation of the dot at $t \rightarrow \infty$, $N_0 = \langle N \rangle_{t=\infty}$. The acoustoelectric current is related to N_0 through (1).

Away from the Coulomb blockade degeneracy points (half-integer \mathcal{N}_g), when the inequality

$$2E_c |\mathcal{N}_g - n_0 - 1/2| \gg \max\{T, 1/\tau\} \quad (8)$$

is satisfied, the time-dependence is too slow to cause the transitions between different charge states of the dot. In (8), n_0 is integer part of \mathcal{N}_g ; units where $\hbar = k_B = 1$ are used throughout the rest of the paper. In this respect the evolution of the system is almost adiabatic, and the occupation of the dot N_0 is expected to coincide with equilibrium occupation, corresponding to the Hamiltonian (5), with T replaced by the effective temperature $T_{eff} \sim \max\{T, 1/\tau\}$. However, in the vicinity of the transition region between the plateaus, when (8) breaks down, the time-dependence mixes states with $N = n_0$ and $N = n_0 + 1$. This means that the width of the transition region is given by T_{eff} , and in the zero-temperature limit saturates to $1/\tau$. In this regime the nearly adiabatic picture fails. The width of the charge states due to tunneling $\Gamma(t)$ decreases with time. When $\Gamma(t) \lesssim 1/\tau$, the system can no longer follow the changing tunneling coupling. Effectively, it can be described within the sudden approximation, where $\Gamma(t)$ is replaced by the step-function, $\Gamma(t) \rightarrow \Gamma_s \theta(-t)$, $\Gamma_s \sim 1/\tau$. The occupation of the dot at $t \rightarrow \infty$ is therefore determined by (3-6) with *time-independent* tunneling amplitude, corresponding to the width Γ_s .

Due to the interaction term in (5), the model (3-6) is still difficult to solve analytically. To simplify the discussion, we limit our attention to the interval of \mathcal{N}_g , which includes only one transition region between the plateaus: $n_0 < \mathcal{N}_g < n_0 + 1$. Furthermore, we neglect the spin degeneracy, and consider the limit of the large single-particle level spacing in the dot, $\Delta \gg 1/\tau$. With these restrictions, at low temperature $T \ll \Delta$, only the

lowest energy configurations, corresponding to $N = n_0$ and $N = n_0 + 1$, are important. Since these states are non-degenerate, one can introduce the fermion operator $d = |n_0\rangle\langle n_0 + 1|$ to describe transitions between these states, and replace (5) by

$$H_{dot} = E_0 d^\dagger d, \quad E_0 = 2E_c (1/2 + n_0 - \mathcal{N}_g). \quad (9)$$

The advantage of the model (3),(6),(9) is that it is exactly solvable for arbitrary $V(t)$. Indeed, the occupation of the dot $n(t) = \langle d^\dagger(t)d(t) \rangle = N(t) - n_0$ satisfies equation of motion⁹

$$\frac{d}{dt} n(t) = -\Gamma(t)n(t) + \int d\varepsilon n_F(\varepsilon) A(\varepsilon, t), \quad (10)$$

$$A(\varepsilon, t) = -\frac{1}{\pi} \text{Im} \int dt' \sqrt{\Gamma(t)\Gamma(t')} e^{i\varepsilon(t-t')} G^R(t, t'). \quad (11)$$

Here $\Gamma(t) = 2\pi\nu V^2(t)$ is the width of the charge state, ν is density of states of conduction electrons at the Fermi level, $n_F(\varepsilon)$ is Fermi function and $G^R(t, t') = -i\theta(t-t') \langle \{d(t), d^\dagger(t')\} \rangle$ is the exact retarded Green function of the dot:

$$G^R(t, t') = -i\theta(t-t') e^{-i \int_{t'}^t dt_1 [E_0 - i\Gamma(t_1)/2]}.$$

Solution for N_0 follows by simple integration of (10-11). With $V(t)$ given by Eq. (7), the result is

$$N_0 - n_0 = \frac{\tau_0}{2\pi} \int_{-\infty}^{\infty} d\varepsilon \frac{n_F(\varepsilon)}{\cosh[(\varepsilon - E_0)\tau_0]}, \quad (12)$$

where $\tau_0 = \pi\tau/2$. At zero temperature, (12) reduces to

$$N_0 - n_0 = \frac{2}{\pi} \tan^{-1} [e^{-E_0\tau_0}]. \quad (13)$$

At finite temperature, (12) is described very well by the Fermi function

$$N_0 - n_0 \approx \left(e^{E_0/T_{eff}} + 1 \right)^{-1}, \quad (14)$$

with an effective temperature $T_{eff} = \sqrt{(c/\tau_0)^2 + T^2}$. We found that $c = 0.88$ gives the best numerical fit¹⁰.

For ($\mathcal{N}_g \rightarrow n_0$), Eqs. (1) and (14) give the following expression for the slope of the plateau:

$$S = \frac{1}{I_0} \left(\frac{dI}{d\mathcal{N}_g} \right)_{\mathcal{N}_g \rightarrow n_0} \approx (2E_c/T_{eff}) e^{-E_c/T_{eff}}. \quad (15)$$

Here $I_0 = n_0 e f$ corresponds to perfect quantization. Strictly speaking, to obtain the correct value of the slope precisely in the middle of the plateau, the two-state approximation is not sufficient: state with $N = n_0 - 1$ makes exactly the same contribution, as that with $N = n_0 + 1$. This complication, however, should not affect significantly the validity of (15): the exact result differs from

(15) by the factor of the order of 1 only¹¹. Due to the exponential factor in (15), S depends very strongly on the ratio E_c/T_{eff} . For example, for $E_c/T_{eff} = 10$, $S \sim 10^{-3}$, while for $E_c/T_{eff} = 20$, $S \sim 10^{-7}$.

According to the discussion above, in the transition region, the result can be obtained with the sudden approximation. Thus, we have

$$N_0 - n_0 \approx \int d\varepsilon n_F(\varepsilon) \frac{\Gamma_s/2\pi}{(\Gamma_s/2)^2 + (\varepsilon - E_0)^2},$$

or, for $T = 0$, $N_0 - n_0 \approx 1/2 - (1/\pi) \tan^{-1}(2E_0/\Gamma_s)$. This expression indeed coincides with (13) in the limit $E_0 \rightarrow 0$, if $\Gamma_s/2 = \tau_0^{-1}$. Note that the width of the transition region is determined by essentially the same T_{eff} , that enters (15).

The model (3-6) introduced above allows study of the nonadiabatic effects at the short time-scale. As the system evolves with time, the SAW-induced potential minimum moves uphill (see Fig.2), and may eventually cross the Fermi level. Due to the residual tunneling coupling in this regime, the leakage from the dot will introduce additional corrections to (1). These corrections, however, do not affect strongly the slope of the plateaus (15), and can be taken into account by multiplying (1) by the leakage factor $P_l \leq 1$: $I = P_l N_0 e f$, where P_l is expected to depend on system parameters, such as gate voltage and SAW power. Thus, the exact value of the quantized current $I_0 = P_l n_0 e f$ does not necessarily coincides with the transfer of precisely integer number of electrons per period n_0 , and the plateaus can move in parameter space.

In conclusion, we have shown that at low temperature, long-range Coulomb interactions may have dramatic effect on the accuracy of the quantization of the single-electron transport driven by surface acoustic waves through a narrow constriction, formed in two-dimensional electron gas. The effect of screening on the SAW-induced potential near the edge of 2DEG can be described by a single parameter - the time τ of the switching-off of the tunneling coupling between 2DEG and the moving quantum dot. As a result, both the slope of the plateaus and the width of the transition regions between the plateaus saturate at low temperature to the values, determined by the characteristic energy scale for nonadiabatic corrections \hbar/τ .

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 - ⁵ Due to the absence of translational invariance in y -direction (see Fig.1), the problem is untractable analytically even with additional simplifying assumptions, such as made in L. I. Glazman and I. A. Larkin, Semicond. Sci. Techn. **6**, 32 (1991).
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 - ⁹ A.-P. Jauho, N. Wingreen, and Y. Meir, Phys. Rev. B **50**, 5528 (1994).
 - ¹⁰ To study the universality of the choice (7), we use slightly different expression, $V(t) = V_0 (e^{t/2\tau} + 1)^{-1/2}$, which coincides with (7) for $t \gg \tau$. With this form of $V(t)$, the V_0 -independent result (12) is recovered, if $\Gamma_0 = 2\pi\nu V_0^2 \gg 1/\tau$. For a special choice of parameters, $\Gamma_0 = 1/\tau$, the result takes a form of (12), but with \cosh^2 instead of \cosh . This differs only little from (12).
 - ¹¹ For instance, in the high temperature limit $T \gg 1/\tau$, the results for the two-state and for the three-state approximations differ only by factor of 2: $S_3/S_2 = 2$.