Pumping in an interacting quantum wire

R. Citro
Dipartimento di Fisica "E. R. Caianiello" and Unità I.N.F.M. di Salerno, Università di Salerno, Via S. Allende, 84081 Baronissi (Sa), Italy

N. Andrei
Center for Materials Theory, Serin Physics Laboratory, Rutgers University, Piscataway, New Jersey 08854-8019, USA

Q. Niu
Department of Physics, The University of Texas, Austin, Texas 78712, USA

(Received 22 May 2003; published 14 October 2003)

We study charge and spin pumping in an interacting one-dimensional wire. We show that a spatially periodic potential modulated in space and time acts as a quantum pump, inducing a dc−current component at zero bias. The current generated by the pump is strongly affected by the interactions. It has a power-law dependence on the frequency or temperature, with the exponent determined by the interaction in the wire, while the coupling to the pump affects the amplitudes only. We also show that pure spinpumping can be achieved without the presence of a magnetic field.

DOI: 10.1103/PhysRevB.68.165312 PACS number(s): 73.63.Nm, 73.63.Fg, 71.10.Pm, 72.25.Pn

I. INTRODUCTION

An adiabatic quantum pump is a device that generates a dc current (at zero bias) by a periodic slow variation of some system characteristic, the variation being slow enough so that the system remains close to its ground state throughout the pumping cycle. The physics of pumping has attracted considerable interest in the last two decades: In his original work Thouless[1] studied the integrated particle current on a finite torus produced by a slow variation of the potential and showed that the integral of the current over a period can vary continuously, but must have an integer value in a clean infinite periodic system with full bands. The robustness of the quantization in the latter system with respect to the influence of disorder, many-body interactions, and system size was shown in Refs. 2 and 3, and spectacular precision of quantization of the pumped current has also been achieved in experiment.[4] Since then, interest in this phenomenon has shifted to theoretical[5–8] and experimental[9,10] investigations of adiabatic pumping through open quantum dots where realization of the periodic time-dependent potential can be achieved by modulating gate voltages applied to the structure. In this regime, the pumped current is generally not quantized,[11,12] and interesting questions are raised on the nature of dissipation associated with the pumping.[13–16] Recently, theoretical studies of quantum pumping have extended to systems with exotic leads, such as superconductor wires[17,18] and Luttinger liquid quantum wires.[19,20] A single-wall carbon nanotube represents an ideal realization of such an interacting quantum wire, and parametric pumping can be achieved by applying gate voltages on the sides[21] or surface acoustic wave propagating along the wire.[22]

In this paper, we report our results on quantum pumping through an interacting one-dimensional wire in the adiabatic regime. The pump we propose consists of a spatially periodic potential V extending from $-L/2$ to $+L/2$ and oscillating wave like with frequency ω_0 and momentum q_0, acting on an interacting clean quantum wire of infinite length; see Fig. 1. We shall show that dc spin and charge currents are induced.

The low-energy properties of the quantum wire are described by a Luttinger liquid, the fixed-point Hamiltonian of the wire, and we carry out the pumping at low temperatures and small ω_0, staying this way in the neighborhood of the fixed point. In this regime, the charge is not quantized as expected, and the results reflect the intrinsic properties of the Luttinger liquid. An anomalous response will be observed since the external periodic potential couples to electrons while the quasiparticles of the interacting systems are Luttinger-like bosons. We will also address the issue of a pure spin pumping through an interacting quantum wire.

The paper is organized as follows. In Sec. II we introduce our physical setup and the Hamiltonian that describes the pump in a one-dimensional (1D) wire, making use of the Luttinger liquid description. In Sec. III we introduce the nonequilibrium Keldysh formalism appropriate to calculate the charge and spin current in the wire. After that we discuss the results for the current at zero and finite temperature. Finally we draw the conclusions in Sec. IV, discussing further perspectives of our analysis and the implications for experimental realization of a device.

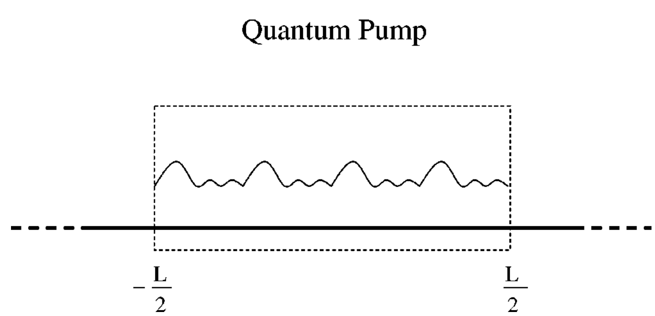

FIG. 1. Quantum wire in presence of a periodic potential extending from $-L/2$ to $L/2$ and oscillating with frequency ω_0.

II. HAMILTONIAN OF A PUMPED 1D WIRE

There are several experimental realizations of one-dimensional systems, among which are nanotubes, quantum wires, and organic conductors, such as Bechgaard salts. These systems are described by interacting one-dimensional Hamiltonians, generally of the form

$$H_T = H_0 + H_{el,el}, \tag{1}$$

$$H_0 = \sum \epsilon_k c^\dagger_{k\sigma} c_{k\sigma}, \tag{2}$$

$$H_{el,el} = \sum_{k\in BZ} U^{\sigma_1\sigma_2\sigma_3\sigma_4}_{(k_1,k_2,k_3,k_4)} c^\dagger_{k_1\sigma_1} c^\dagger_{k_2\sigma_2} c_{k_3\sigma_3} c_{k_4\sigma_4}, \tag{3}$$

where $c^\dagger_{k\sigma}$ is an electron creation operator with momentum k and spin component σ, $c^\dagger_{j\sigma}$ (its Fourier transform) creates the electron at lattice site $x_j = ja$, and U is an arbitrary electron-electron interaction.

If we wish to study the low-energy physics of such a model, as in the case of *adiabatic* pumping, it suffices to consider it close to its fixed point—typically the Luttinger liquid—to which it flows under the action of the renormalization group (RG).[23] The low-energy dynamics takes place close to the Fermi points $\pm k_F$ and is expressed in terms of the fermionic low-energy fields $\psi_{\alpha\sigma}(x)$ describing the right-moving modes ($\alpha = R$) with spin σ around k_F and the left-moving modes ($\alpha = L$) describing the physics around $-k_F$. The Luttinger Hamiltonian is

$$H_{LL} = -iv_F \int dx[\Psi^\dagger_{R\sigma}(x)\partial_x\Psi_{R\sigma}(x) - \Psi^\dagger_{L\sigma}(x)\partial_x\Psi_{L\sigma}(x)] + g\int dx\rho(x)^2, \tag{4}$$

where v_F is the Fermi velocity, g measures the strength of interactions ($g>0$ for repulsive interactions), and $\rho = \rho_R + \rho_L$ is the sum of the left- and right-moving electron densities $\rho_{\alpha,\sigma} = \Psi^\dagger_{\alpha\sigma}\Psi_{\alpha\sigma}$, with $\rho_\alpha = \Sigma_\sigma \rho_{\alpha,\sigma}$. Note that the number of left and right movers is conserved by the Luttinger Hamiltonian.

Consider now the wire in the presence of an external periodic potential. We add then to Eq. (1) the term

$$H^{latt} = \sum_j V_{\sigma,\sigma'}(x_j) c^\dagger_{j,\sigma} c_{j,\sigma'}, \tag{5}$$

where $V_{\sigma,\sigma'}(x_j)$ is a periodic external potential of x_j (with period l) acting on a section of length L of the wire. A possible way to realize the periodic potential is to embed a section L of the long quantum wire in a semiconductor heterostructure with a meander line on top (or bottom) of the sandwich, generating a spatial periodic electric field oscillating in time with a fixed frequency ω_0 at the interface. The interfacial electric field would be such that the effective potential experienced by the Luttinger-bosonic-like quasiparticles will result in a sinusoidal potential modulated in space and time. (In such a system magnetized contacts could be used to preferentially inject and detect specific spin orientation.[24,25])

Also the periodic potential will flow under the action of the renormalization group, and in the low-energy limit it will be represented by a sum over umklapp operators H^U_{n,m,n_s} transferring n electrons and n_s units of spin from right to left Fermi points (and vice versa), while absorbing from the lattice m units of lattice momentum $G = 2\pi/l$ (Ref. 26). The umklapp operators to which Eq. (5) flows under the RG describe high-energy processes which are irrelevant (in the RG sense) at low energies when we consider systems close to a Luttinger fixed point. However, we shall examine the system at small but finite energy scales at which the RG flow stops and the umklapp terms make the main contribution to pumping.

Leading umklapp terms are of the form

$$H^U_{0,m,0} \approx g^U_{0,m,0}\int dx[e^{i\Delta k_{0,m}x}(\rho_R+\rho_L)^2 + \text{H.c.}],$$

$$H^U_{1,m,0} \approx g^U_{1,m,0}\sum_\sigma \int dx[e^{i\Delta k_{1,m}x}\Psi^\dagger_{R\sigma}(x)\Psi_{L\sigma}(x)\rho_{-\sigma} + \text{H.c.}],$$

$$H^U_{1,m,1} \approx g^U_{1,m,1}\int dx[e^{i\Delta k_{1,m}x}\Psi^\dagger_{R\downarrow}(x)\Psi_{L\uparrow}(x)\rho + \text{H.c.}],$$

$$H^U_{2,m,0} \approx g^U_{2,m,0}\int dx[e^{i\Delta k_{2,m}x}\Psi^\dagger_{R\uparrow}(x)\Psi^\dagger_{R\downarrow}(x)\Psi_{L\downarrow}(x)\Psi_{L\uparrow}(x) + \text{H.c.}], \tag{6}$$

with the couplings g_{n,m,n_s} determined from the microscopic Hamiltonian (including the driving potential $V_{\sigma,\sigma'}$) via a full RG analysis. The quantity $\Delta k_{n,m} = n2k_F - mG$ in the exponential of H_{n,m,m_s} is the momentum transfer associated with the umklapp process n,m. Note that a commensurability between the electron density and the imposed periodicity implies $\Delta k_{n,m} = 0$ for some n,m. At commensurate filling some umklapp operator may become relevant. This is the case with $H^U_{2,1,0}$ at half filling for any value of the coupling $g^U_{2,1,0}$. This would also be the case with other commensurate fillings, but with a finite critical value of the coupling. When any of the umklapp operators is relevant the low-energy behavior is no longer given by the Luttinger liquid. We shall assume in what follows that we are away from half filling and, when considering other commensurate filling, that the coupling is below its critical value.

Also boundary terms may be generated under the RG process. The periodic potential acts on a section of the wire and we assumed sharp edges at $\pm L/2$; hence terms of the form

$$H^{boundary} = V_0[\psi^\dagger_{R\sigma}(L/2)\psi_{L\sigma}(L/2) + \psi^\dagger_{R\sigma}(-L/2)\psi_{L\sigma}(-L/2) + \text{H.c.}] \tag{7}$$

will appear. Such terms were shown by Kane and Fisher to be relevant in the low-energy limit.[27]

We now allow the external periodic potential to oscillate with frequency ω_0 and propagate with some momenta $\{q\}, q \approx q_0 + \delta q$, with $\delta q \ll q_0$,

$$V(x) \to V(t,x) = \sum_q A_q \cos(\omega_0 t - qx) V(x). \tag{8}$$

Again, close to the Luttinger fixed point, the potential renormalizes to a sum of umklapp terms with time- (and phase-) dependent coupling constants:

$$g^U_{n,m,n_s}(t) = g^U_{n,m,n_s} e^{i(\omega_0 t - \varphi_{n,m})}. \tag{9}$$

The momenta $\{q\}$ in the driving potential break the mirror symmetry of the oscillating potential and are reflected in the effective low-energy Hamiltonian by the umklapp phases $\varphi_{n,m}$. For very weak periodic potential one expects $\varphi_{n,m} \approx nq_0/\omega_0$. When mirror symmetry is present $\varphi_{n,m}=0$ (and we shall see that no current is induced). Together with the periodic potential also the boundary terms will oscillate and we have, for the leading term,

$$H^{boundary}(t) = V_0[e^{i\omega_0 t}\psi^\dagger_{R\sigma}(L/2)\psi_{L\sigma}(L/2) + e^{i(\omega_0 t - \varphi)}\psi^\dagger_{R\sigma}(-L/2)\psi_{L\sigma}(-L/2) + \text{H.c.}],$$

where φ is the temporal phase shift between the two edges.

The low-energy effective Hamiltonian

$$H_{eff}(t) = H_{LL} + H^{pump}(t), \tag{10}$$

$$H^{pump}(t) = H^{bulk}(t) + H^{boundary}(t), \tag{11}$$

$$H^{bulk}(t) = \sum_{m,n,n_s} H^U_{n,m,n_s}(t), \tag{12}$$

describes the time evolution of the system close to the fixed point and is valid therefore (over a cycle) when all energy scales such as ω_0, T are small. We shall show that the oscillating potential acts as a quantum pump, inducing spin and charge dc currents. We shall find that both the bulk term $\Sigma_{m,n,n_s} H^U_{n,m,n_s}(t)$ and the boundary term $H^{boundary}(t)$ induce charge and spin currents. The bulk contribution dominates in the large pump limit, i.e., for $L \to \infty$, holding ω_0 fixed but small. In the other limit $\omega_0 \to 0$, holding L large but fixed, the boundary contribution dominates.

We wish to study the effect of the oscillating terms on the current operators,

$$I_c(x) = \sum_\sigma [\psi^\dagger_{R\sigma}(x)\psi_{R\sigma}(x) - \psi^\dagger_{L\sigma}(x)\psi_{L\sigma}(x)], \tag{13}$$

$$I_s(x) = \sum_{\sigma,\sigma'} [\psi^\dagger_{R\sigma}\tau^z_{\sigma\sigma'}(x)\psi_{R\sigma'}(x) - \psi^\dagger_{L\sigma}\tau^z_{\sigma\sigma'}(x)\psi_{L\sigma'}(x)]. \tag{14}$$

To do so it is convenient to rewrite the problem in terms of bosonic fields ϕ_σ, Π_σ: Defining the chiral components $\phi_{R\sigma}, \phi_{L\sigma} = \frac{1}{2}[\phi_\sigma \pm \int^x \Pi_\sigma(x')dx']$, the fermionic fields are given by

$$\psi_{R,\sigma}(x,t) = \frac{1}{\sqrt{2\pi a}} e^{i\phi_{R\sigma}},$$

$$\psi_{L,\sigma}(x,t) = \frac{1}{\sqrt{2\pi a}} e^{-i\phi_{L\sigma}}, \tag{15}$$

where a is a spatial cutoff (essentially the *electron* lattice spacing, to be distinguished from l). Rewriting the interacting Hamiltonian, Eq. (4), by means of the bosonic fields, it can be brought into a quadratic form by a Bogliubov rotation.[28,29] It is convenient to introduce the combinations $\phi_c = (\phi_\uparrow + \phi_\downarrow)/\sqrt{2}$ and $\phi_s = (\phi_\uparrow - \phi_\downarrow)/\sqrt{2}$, the spin and charge bosonic fields, in terms of which

$$H_{LL} = \frac{1}{2\pi} \sum_{\nu=c,s} v_\nu \int dx \left(K_\nu \Pi^2_\nu + \frac{1}{K_\nu}(\partial_x \phi_\nu)^2 \right), \tag{16}$$

where the momenta Π_ν are conjugate to ϕ_ν, $v_{c,s}$ are the charge and spin velocities, and K_ν are the Luttinger parameters, $v_c/K_c = v_F + g/\pi$ and $v_s/K_s = v_F - g/\pi$. The bosonic version of the umklapp terms is

$$H^U_{n,m,n_s}(t) = \frac{g^U_{n,m,n_s}}{(2\pi a)^n} \int dx \{ e^{i(\omega_0 t - \varphi_{n,m})} e^{i\Delta k_{n,m} x} e^{i\sqrt{2}(n\phi_c + n_s\phi_s)} + \text{H.c.}\}, \tag{17}$$

while the local boundary term is

$$H^{boundary}(t) = \frac{V}{(2\pi a)} \{ e^{i\omega_0 t} e^{i\sqrt{2}[\phi_c(L/2,t) + \phi_s(L/2,t)]} + e^{i(\omega_0 t - \varphi)} e^{i\sqrt{2}[\phi_c(-L/2,t) + \phi_s(-L/2,t)]} + \text{H.c.}\}. \tag{18}$$

In terms of bosonic fields the charge current and spin current are given by

$$I_c(x,t) = \frac{e\sqrt{2}}{\pi} \partial_t \phi_c(x,t),$$

$$I_s(x,t) = \frac{\hbar\sqrt{2}}{\pi} \partial_t \phi_s(x,t), \tag{19}$$

where e denotes the electric charge. In the following, we shall consider the oscillating lattice as a perturbation around the Luttinger liquid fixed point and compute the current perturbatively. This is a controlled expansion in the low-energy limit as noted before. As we will show, the boundary term, though relevant with respect to the Luttinger liquid, as $\omega_0 \to 0$, will lead to a subdominant contribution in the large pump limit.

III. NONEQUILIBRIUM TRANSPORT FORMALISM

In the system described above we consider an external source pumping energy into it; therefore the general formalism of this nonequilibrium situation is given by the Keldysh technique.[30] Our purpose is to calculate the charge and spin currents generated by the pumping. They are given by

$$\langle I_{c,s}(x,t)\rangle = \left\langle T_C\left\{ I_{c,s}(x,t)\exp\left(-i\oint dt_1 H^{pump}(t_1)\right)\right\}\right\rangle, \tag{20}$$

where T_C is the time ordering operator along the Keldysh contour. Expressing T_C in terms of the ordering (antiordering) operator T_K along the upper (lower) Keldysh branches, we adopt the convention[31] that the indices $\eta,\eta_{1,2}=\pm$ identify the upper (lower) branch of the Keldysh contour.

We shall begin by studying the bulk contribution of the pump. We then expand in the irrelevant umklapp operators around the Luttinger liquid fixed point. Expanding the exponential to first order we obtain

$$\langle I_{c,s}(x,t)\rangle^{(1)} = -i\sum_{\eta\eta_1}\eta_1\left\langle T_K\left\{ I_{c,s}(x,t^{\eta})\int dt_1 H^{bulk}(t_1^{\eta_1})\right\}\right\rangle. \tag{21}$$

Starting from the expression (17) of the Hamiltonian in terms of the bosonic fields and using the identity $\lim_{\gamma\to 0}(i\gamma)^{-1}\partial_t \exp[i\sqrt{2}\gamma\phi_c] = \sqrt{2}\partial_t\phi_c$, in order to cast the time-ordered averages into correlators of exponentials only, we have

$$\begin{aligned}\langle I_c(x,t)\rangle^{(1)} &= -i\frac{e}{\pi}\sum_{n,m,n_s}\frac{g^U_{n,m,n_s}}{(2\pi a)^n}\sum_{\epsilon=\pm}\sum_{\eta\eta_1}\eta_1\int dt_1\int_{-L/2}^{L/2}dx_1 e^{i\epsilon(\omega_0 t_1-\varphi_{n,m})}\\ &\quad\times e^{i\epsilon\Delta k_{n,m}x_1}\lim_{\gamma\to 0}(i\gamma)^{-1}\partial_t\langle T_K\{e^{i\gamma\sqrt{2}\phi_c(x,t^{\eta})}e^{i\sqrt{2}\epsilon[n\phi_c(x_1,t_1^{\eta_1})+n_s\phi_s(x_1,t_1^{\eta_1})]}\}\rangle\\ &= \frac{2e}{\pi}\sum_{n,m,n_s}\left(\frac{L}{a}\right)^{-n^2K_c/2-n_s^2K_s/2}\frac{g^U_{n,m,n_s}}{(2\pi a)^n}\sum_{\eta\eta_1}\eta_1\int dt_1\\ &\quad\times\int_{-L/2}^{L/2}dx_1\sin(\omega_0 t_1-\varphi_{n,m}+\Delta k_{n,m}x_1)\partial_t G^{\phi_c\phi_c}_{\eta\eta_1}(x-x_1,t-t_1),\end{aligned} \tag{22}$$

where we have introduced $\epsilon=\pm$ for the Hermitian conjugates, and the bosonic Keldysh Green's function is

$$\begin{aligned}G^{\phi_c\phi_c}_{\eta\eta_1}&(x-x_1,t-t_1)\\ &= \langle T_K[\sqrt{2}\phi_c(x,t^{\eta})\sqrt{2}\phi_c(x_1,t_1^{\eta_1})]\rangle\\ &= -\frac{K_c}{2}\sum_{\alpha=\pm}\ln\{a+ih_{\eta\eta_1}(t-t_1)[v_c(t-t_1)\\ &\quad -\alpha(x-x_1)]\}\end{aligned} \tag{23}$$

with $\alpha=\pm$ for R/L movers, respectively, and $h_{\pm\pm}(t)=\pm\,\mathrm{sgn}(t), h_{\pm\mp}(t)=\mp 1$. The nontrivial L dependence is arising from the correlator of the exponential for a finite-size system.

Using the definition of the Keldysh Green's function matrix elements and the symmetry property $G(x,\tau)=G(x,|\tau|)$, only the terms with $\eta=-\eta_1$ can be retained; thus

$$\begin{aligned}\langle I_c(x,t)\rangle^{(1)} &= 2\frac{e}{\pi}\sum_{n,m,n_s}\left(\frac{L}{a}\right)^{-n^2K_c/2-n_s^2K_s/2} n\frac{g^U_{n,m,n_s}}{(2\pi a)^n}\\ &\quad\times\sum_{\eta}\eta\int dt_1\int_{-L/2}^{L/2}dx_1\sin(\omega_0 t_1-\varphi_{n,m}\\ &\quad+\Delta k_{n,m}x_1)\partial_t G^{\phi_c\phi_c}_{\eta-\eta}(x-x_1,t-t_1).\end{aligned} \tag{24}$$

A further change of variables leads to a final form of the first-order contribution to the charge current,

$$\begin{aligned}\langle I_c(x,t)\rangle^{(1)} &\propto \sum_{n,m,n_s}\left(\frac{L}{a}\right)^{-n^2K_c/2-n_s^2K_s/2} n\frac{g^U_{n,m,n_s}}{(2\pi a)^n}\\ &\quad\times\sin(\omega_0 t-\varphi_{n,m}+\Delta k_{n,m}x),\end{aligned} \tag{25}$$

and the spin current,

$$\begin{aligned}\langle I_s(x,t)\rangle^{(1)} &\propto \sum_{n,m,n_s}\left(\frac{L}{a}\right)^{-n^2K_c/2-n_s^2K_s/2} n_s\frac{g^U_{n,m,n_s}}{(2\pi a)^n}\\ &\quad\times\sin(\omega_0 t-\varphi_{n,m}+\Delta k_{n,m}x).\end{aligned} \tag{26}$$

However, as these terms oscillate in space and in time no pumping takes place to first order. As we will show, to have a dc current at least two umklapp operators with a nonzero phase difference are required, in accordance with the general idea of pumping.

To second order we have

$$\begin{aligned}\langle I_{c,s}(x,t)\rangle^{(2)} &= -\frac{1}{2}\sum_{\eta\eta_1\eta_2}\eta_1\eta_2\left\langle T_K\left\{ I_{c,s}(x,t^{\eta})\int dt_1\right.\right.\\ &\quad\left.\left.\times\int dt_2 H^{bulk}(t_1^{\eta_1})H^{bulk}(t_2^{\eta_2})\right\}\right\rangle.\end{aligned} \tag{27}$$

By using the bosonic expression of H^{bulk} we find

$$\langle I_c(x,t)\rangle^{(2)}=-\frac{e}{2\pi}\sum_{n,m,n_s}\sum_{n',m',n_s'}\frac{g^U_{n,m,n_s}}{(2\pi a)^n}\frac{g^U_{n',m',n_s'}}{(2\pi a)^{n'}}\sum_{\epsilon_{1,2}=\pm}\sum_{\eta\eta_1\eta_2}\eta_1\eta_2\int dt_1\int dt_2\int_{-L/2}^{L/2}dx_1\int_{-L/2}^{L/2}dx_2$$
$$\times e^{i\epsilon_1(\omega_0 t_1-\varphi_{n,m})}e^{i\epsilon_2(\omega_0 t_2-\varphi_{n',m'})}e^{i\epsilon_1\Delta k_{n,m}x_1}e^{i\epsilon_2\Delta k_{n',m'}x_2}\lim_{\gamma\to 0}(i\gamma)^{-1}\partial_t\langle T_K(e^{i\gamma\sqrt{2}\phi_c(x,t^\eta)}$$
$$\times e^{i\epsilon_1\sqrt{2}[n\phi_c(x_1,t_1^{\eta_1})+n_s\phi_s(x_1,t_1^{\eta_1})]}e^{i\epsilon_2\sqrt{2}[n'\phi_c(x_2,t_2^{\eta_2})+n_s'\phi_s(x_2,t_2^{\eta_2})]})\rangle. \quad (28)$$

A dc contribution to the current arises only from the term with $\epsilon_1=-\epsilon_2$ and a nonzero phase difference. We proceed to calculate it:

$$\langle I_c(x,t)\rangle^{(2)}_{dc}=-\frac{e}{\pi}\sum_{n,m,n_s}\sum_{n',m',n_s'}\left(\frac{L}{a}\right)^{-(n^2+n'^2)K_c/2-(n_s^2+n_s'^2)K_s/2}\frac{g^U_{n,m,n_s}}{(2\pi a)^n}\frac{g^U_{n',m',n_s'}}{(2\pi a)^{n'}}$$
$$\times\sum_{\eta\eta_1\eta_2}\eta_1\eta_2\int dt_1\int dt_2\int_{-L/2}^{L/2}dx_1\int_{-L/2}^{L/2}dx_2\sin[\omega_0(t_1-t_2)-\Delta\varphi^{n',m'}_{n,m}+\Delta k_{n,m}x_1-\Delta k_{n',m'}x_2]$$
$$\times e^{-nn'G^{\phi_c\phi_c}_{\eta_1\eta_2}(x_1-x_2,t_1-t_2)}e^{-n_s n_s' G^{\phi_s\phi_s}_{\eta_1\eta_2}(x_1-x_2,t_1-t_2)}[n\partial_t G^{\phi_c\phi_c}_{\eta\eta_1}(x-x_1,t-t_1)-n'\partial_t G^{\phi_c\phi_c}_{\eta\eta_2}(x-x_2,t-t_2)], \quad (29)$$

where $\Delta\varphi^{n',m'}_{n,m}=(\varphi_{n,m}-\varphi_{n',m'})$ is the phase difference and the Keldysh spin bosonic Green function is given by

$$G^{\phi_s\phi_s}_{\eta\eta_1}(x-x_1,t-t_1)=\langle T_K[\sqrt{2}\phi_s(x,t^\eta)\sqrt{2}\phi_s(x_1,t^{\eta_1})]\rangle=-\frac{K_s}{2}\sum_{\alpha=\pm}\ln\{a+ih_{\eta\eta_1}(t-t_1)[v_s(t-t_1)-\alpha(x-x_1)]\}. \quad (30)$$

An expression similar to Eq. (29) will hold for the spin current, except that in this case the derivative of the spin bosonic Green's function will appear, multiplied by n_s (the spin umklapp quantum numbers), instead of charge umklapp quantum numbers n.

The calculation of the contribution to the current from the boundary terms is carried in an analogous way by considering $H^{boundary}$ in Eqs. (21) and (27) instead of H^{bulk}.

A. Zero-temperature pumping

1. Bulk current

Evaluating the integral (29) for the charge current and the corresponding one for the spin current (for details see the Appendix), we find that the leading-order contribution to the dc currents at zero temperature is

$$I_c^{dcbulk}(\omega_0)=eK_c v_c\sum_{n,m,n_s}\sum_{n',m',n_s'}(n-n')\left(\frac{L}{a}\right)^{-(n-n')^2K_c/2-(n_s-n_s')^2K_s/2}\mathcal{A}^{n',m',n_s'}_{n,m,n_s}I^{n'm'n_s'}_{nmn_s}(\omega_0,[\Delta k_+]^{n',m'}_{n,m})\frac{\sin([\Delta k_-]^{n',m'}_{n,m})\frac{L}{2}}{[\Delta k_-]^{n',m'}_{n,m}},$$

$$I_s^{dcbulk}(\omega_0)=\hbar K_s v_s\sum_{n,m,n_s}\sum_{n',m',n_s'}(n_s-n_s')\left(\frac{L}{a}\right)^{-(n-n')^2K_c/2-(n_s-n_s')^2K_s/2}\mathcal{A}^{n',m',n_s'}_{n,m,n_s}I^{n'm'n_s'}_{nmn_s}(\omega_0,[\Delta k_+]^{n',m'}_{n,m})\frac{\sin([\Delta k_-]^{n',m'}_{n,m})\frac{L}{2}}{[\Delta k_-]^{n',m'}_{n,m}}, \quad (31)$$

where $[\Delta k_\pm]^{n',m'}_{n,m}=(\Delta k_{n,m}\pm\Delta k_{n',m'})/2$ and

$$\mathcal{A}^{n',m',n_s'}_{n,m,n_s}=\frac{g^U_{n,m,n_s}}{(2\pi a)^n}\frac{g^U_{n',m',n_s'}}{(2\pi a)^{n'}}\sin\Delta\varphi^{n',m'}_{n,m} \quad (32)$$

is the area enclosed in a pumping cycle by the periodic parameters $g^U_{n,m,n_s}(t)$ and $g^U_{n',m',n_s'}(t)$. The expression $I^{n'm'n_s'}_{nmn_s}$ for $v_c=v_s$ is given by (the case $v_s\neq v_c$ is treated in the Appendix)

$$I^{n'm'n_s'}_{nmn_s}(\omega_0,[\Delta k_+]^{n',m'}_{n,m}) = \mathrm{sgn}(\omega_0)\left(\frac{a}{2v}\right)^{2K^{nn'}_{n_sn_s'}}\Gamma^{-2}(K^{nn'}_{n_sn_s'}) \times(\omega_0^2-v^2[\Delta k_+^2]^{n',m'}_{n,m})^{K^{nn'}_{n_sn_s'}-1} \times\theta(|\omega_0|-|v[\Delta k_+]^{n',m'}_{n,m}|), \tag{33}$$

where $K^{nn'}_{n_sn_s'}=nn'K_c/2+n_sn_s'K_s/2$, K_c and K_s are the Luttinger parameters defined earlier, and the function $\mathrm{sgn}(\omega_0)$ is defined as $\mathrm{sgn}(\omega_0)=0$ for $\omega_0=0$ in addition to the usual definition $\mathrm{sgn}(\omega_0)=\pm1$ for ω_0 positive and negative.

The nontrivial dependence of the current on the size of the pump L arises technically from the exponential of the Keldysh correlators evaluated on the finite size of the pump with the usual "charge neutrality" violated, $n\neq n'$, $n_s\neq n_s'$. This violation is a manifestation of the nonequilibrium process taking place during the pumping with "charges" in the upper part of the Keldysh contour not canceling the charges in lower part. Thus, the pumping can be viewed as the action of the potential on the section L of the wire creating charge unbalance and resulting in a net current in one direction.[32]

2. Discussion

We now discuss the physical characteristics of our results. First, the nonlinear dependence on the size of the pumping region strongly suppresses for large L/a terms with large $|n-n'|$ or $|n_s-n_s'|$. Therefore, the leading contribution to the charge current comes from terms with $n_s=n_s'$ and $n-n'=\pm1$, and the leading contribution to the spin current comes from terms with $n=n'$ and $n_s-n_s'=\pm1$. Second, depending on the lattice having only charge umklapp terms (i.e., $n,n'\neq0$ but $n_s,n_s'=0$) or only spin umklapp terms ($n,n'=0$ but $n_s,n_s'\neq0$), a pure charge or pure spin current will be induced. This spin pumping takes place without spin-orbit coupling and without magnetic field or spontaneous symmetry breaking, unlike the mechanisms in Refs. 19 and 33. This is possible only due to interactions. Third, the charge and spin pumped per cycle are not quantized but depend linearly on the area $\mathcal{A}^{n',m',n_s'}_{n,m,n_s}$ enclosed by the interaction. Note that at least two umklapp terms are needed to have a nonzero dc current. This accords with the observation that at least two umklapp terms are required to represent a lattice[26] and also in agreement with the picture that the electron pump is induced by the out-of-phase variation of any pair of independent parameters. The current would vanish if under the RG a single umklapp term is induced, even if associated with several phases. In case of mirror symmetry we have $\varphi_{n,m},\varphi_{n',m'}=0$, resulting in a zero dc current. Thus the breaking of mirror symmetry is a necessary condition for the pumping. Most importantly, the response of the non-Fermi-liquid (Luttinger) quasiparticles to a fermionic coupling produces anomalous frequency dependence in the pumped current. Consider first the commensurate case where $\Delta k_+=0$. Equation (33) reduces to a power law in frequency dependence with an exponent $2(K^{nn'}_{n_sn_s'}-1)$. In the noninteracting limit $K_c=1,K_s=1$, the lowest value of the exponent will correspond to $K^{12}_{11}=3/2$, giving the expected linear ω_0 behavior at commensurability. In this case we get charge and spin pumping with a frequency-independent pumping conductance

$$\mathcal{G}_{c,s}=\frac{e^2}{h}\frac{2\pi}{\omega_0}I^{dc}_{c,s},$$

similar to Refs. 5 and 19. With interaction, the frequency dependence of the current is generally nonlinear with an exponent depending on the strength of the Luttinger interaction. For $K^{nn'}_{n_sn_s'}>1$, the current goes to zero smoothly in the zero-frequency limit, connecting to the expected result of no current when the lattice does not oscillate. In the range $K^{nn'}_{n_sn_s'}<1$, the Luttinger fixed point would become unstable and a new charge-density-wave (CDW) or spin-density-wave (SDW) ground state forms, where our considerations do not apply. This RG argument manifests itself as a "dynamic Stoner instability" with $I(\omega_0)$ diverging as $\omega_0\to0$ in this case. Note, however, that the stable regime includes both the superlinear and sublinear behaviors in frequency dependences of the current. Such nontrivial power laws are never seen for conventional pumps. In the incommensurate case, the current vanishes in the frequency window $|\omega_0|<|v[\Delta k_+]^{n',m'}_{n,m}|$. This reflects the physical requirement that sufficient (photon) energy must be supplied from the pumping source in order to make the transition. The nontrivial power law appears again immediately beyond the frequency threshold.

3. Boundary current

We still need to examine the boundary contribution. Carrying out the calculation along the lines described above we find that the mixed bulk-boundary contribution to the dc current vanishes while the pure boundary interference yields (cf. Ref. 19)

$$I^{dcboundary}_c=V_0^2\left(\frac{L}{a}\right)^{-K_c-K_s}|\omega_0|^{K_c+K_s-1}\,\mathrm{sgn}(\omega_0)\sin\varphi. \tag{34}$$

We then conclude that for $L\to\infty$ (holding ω_0 fixed so that no further renormalization of V_0 and g^U_{nm} takes place) the bulk contribution will dominate due to umklapp terms with $|n-n'|=1$ and $\omega_0>|v[\Delta k_+]^{n'}_n|$. The irrelevant terms acting over a large distance win over the relevant terms from the edges.

B. Finite-temperature pumping

Our considerations are easy to extend to small but finite temperature (which leave the system in the vicinity of the Luttinger fixed point). We start by considering the contribution from the bulk first. Using the finite-temperature expression for the correlation functions of the boson operators[34,35] the expression for $I_{nmn_s}^{n'm'n'_s}(\omega_0,[\Delta k_+]_{n,m}^{n',m'})$ in Eq. (33) will read

$$I_{nmn_s}^{n'm'n'_s,T}(\omega_0,[\Delta k_+]_{n,m}^{n',m'}) = \left(\frac{2\pi a T}{v}\right)^{2K_{n_s n'_s}^{nn'}-2}\sin(\pi K_{n_s n'_s}^{nn'}) \times B\left(-\frac{i}{2\pi}s_+ + \frac{K_{n_s n'_s}^{nn'}}{2};1-K_{n_s n'_s}^{nn'}\right) \times B\left(-\frac{i}{2\pi}s_- + \frac{K_{n_s n'_s}^{nn'}}{2};1-K_{n_s n'_s}^{nn'}\right)\sinh\left(\frac{\omega_0}{\pi T}\right), \tag{35}$$

where $s_\pm=(\omega_0\pm[v\Delta k_+]_{n,m}^{n',m'})/2T$; $B(x,y)=\Gamma(x)\Gamma(y)/\Gamma(x+y)$ is the Euler beta function.

When we consider incommensurate fillings $[\Delta k_+]_{n,m}^{n',m'}\neq 0$, assuming $T\ll v[\Delta k_+]_{n,m}^{n',m'}$, two interesting regimes occur depending on whether $T\ll\omega_0$, or $T\gg\omega_0$. In the first case, we get

$$I_{nmn_s}^{n'm'n'_s,T}(\omega_0,[\Delta k_+]_{n,m}^{n',m'}) \simeq \mathrm{sgn}(\omega_0)\sin(\pi K_{n_s n'_s}^{nn'})\Gamma^2(1-K_{n_s n'_s}^{nn'})\left(\frac{a}{2v}\right)^{2K_{n_s n'_s}^{nn'}-2} \times(\omega_0^2-v^2[\Delta k_+^2]_{n,m}^{n',m'})^{K_{n_s n'_s}^{nn'}-1} \times\theta(|\omega_0|-|v[\Delta k_+]_{n,m}^{n',m'}|), \tag{36}$$

coinciding in the limit with the result at $T=0$. For $T\gg\omega_0$ and ω_0 not too small compared to $v[\Delta k_+]_{n,m}^{n',m'}$ we find

$$I_{nmn_s}^{n'm'n'_s,T}(\omega_0,[\Delta k_+^2]_{n,m}^{n',m'}) \simeq \sin^2(\pi K_{n_s n'_s}^{nn'})\Gamma^2(1-K_{n_s n'_s}^{nn'})\left(\frac{a}{2v}\right)^{2K_{n_s n'_s}^{nn'}-2} \times(\omega_0^2-v^2[\Delta k_+^2]_{n,m}^{n',m'})^{K_{n_s n'_s}^{nn'}-1} \times e^{-\frac{v[\Delta k_+]_{n,m}^{n',m'}}{2T}}\sinh\left(\frac{\omega_0}{\pi T}\right), \tag{37}$$

where the exponential factor describes the suppression of processes between the initial and final states of energy $v|[\Delta k_+]_{n,m}^{n',m'}|/2$ involving momentum transfer $[\Delta k_+]_{n,m}^{n',m'}$. When $\omega_0\to 0$ at low temperature the exponential factor in Eq. (37) prevails and the processes with the smallest $[\Delta k_+]_{n,m}^{n',m'}$ are favored and the current is suppressed.

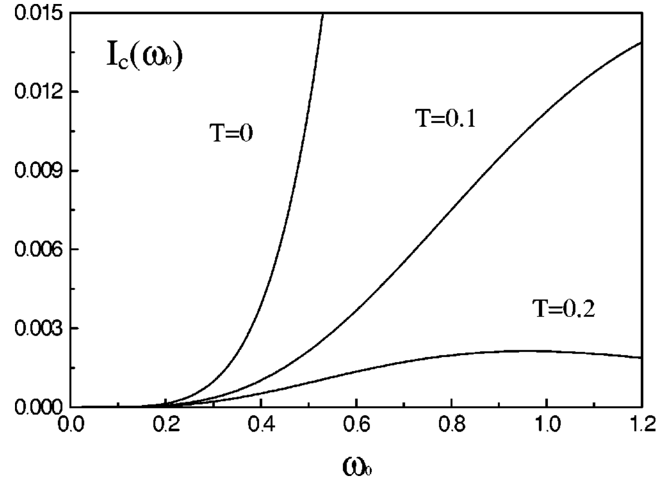

FIG. 2. The low-frequency behavior of the charge current $I_c(\omega_0)$ at $T=0$ and $T=0.1,0.2$, having taken into account umklapp terms $g_{2,0},g_{2,1},g_{2,2},g_{3,1},g_{3,2},g_{3,3},g_{4,1},g_{4,2},g_{4,3}$. We have chosen $g_{n,m}=1$, $K_c=0.7$, and $K_s=1$; ω_0 and T are measured in units of $v\Delta k_{20}$ and $I(\omega_0)$ in units of $ev_F/2$.

At a typical commensurate point $[\Delta k_+]_{n_0,m_0}^{n_1,m_1}\sim 0$ and temperature not too low, we have to balance algebraic and exponential suppression in Eq. (37). In the limit $\omega_0\ll T$, the dominant contribution to the dc current will be given by

$$I_c^{dcbulk}\sim eK_c v_c n_0 \mathcal{A}_{n_0,m_0,n_{0s}}^{n_1,m_1,n_{1s}}\left(\frac{\sin\left(\frac{G}{2n_0}L\right)}{\frac{G}{2n_0}}\right) \times\cos(\pi K_{n_{0s}n_{1s}}^{n_0 n_1})^2 B^2(K_{n_{0s}n_{1s}}^{n_0 n_1}/2,1-K_{n_{0s}n_{1s}}^{n_0 n_1}) \times\left(\frac{2\pi a T}{v}\right)^{2K_{n_{0s}n_{1s}}^{n_0 n_1}-2}\frac{\omega_0}{T}\mathrm{sgn}(\omega_0). \tag{38}$$

In the noninteracting limit the lowest value of the exponent corresponds to $K_{11}^{12}=3/2$ and one recovers again the usual Fermi liquid behavior $I\simeq\max(T,\omega_0)$ for the noninteracting gas.[35] With interactions present, the current behaves as a power law of the temperature with an exponent depending on the interactions, indicating a strong renormalization of the scattering process due to various fluctuations of a one-dimensional electron gas. A similar expression will hold also for the spin current with a coefficient $K_s v_s n_{0s}$ instead of $K_c v_c n_0$.

Figure 2 shows the low-frequency behavior of the charge current at zero and finite temperature, taking into account few umklapp terms.

When considering the bulk-boundary contribution to the dc current the same argument as in Sec. III A holds. For $T\ll\omega_0$ we recover the zero-temperature expression (34), and for $\omega_0 \ll T$ we do have

$$I_c^{dcboundary} \propto (L/a)^{-K_c-K_s}|T|^{K_c+K_s-1}\,\mathrm{sgn}(\omega_0)\sin\varphi,$$

so that none of the previous conclusions is invalidated when $L\to\infty$, taking ω_0 or T fixed.

IV. CONCLUSIONS

We have introduced and studied charge and spin parametric pumps for an interacting quantum wire. We have demonstrated that the pump, consisting of a periodic potential oscillating in space and in time over a size L of a long clean wire, induces dc spin and charge currents. At finite and fixed frequency, the leading contribution to the current arises from the interference of two out-of-phase umklapp operators, in agreement with the picture of a phase coherent quantum transport, while edges contribution dominates at large but fixed size of the pump in the small frequency limit. We have shown that the pumped current is strongly affected by the interaction in the wire, displaying a nonuniversal behavior that depends on the filling and the interaction itself. We have also discussed how to realize a pure spin pumping in the wire as an alternative picture to the existing coherent spin transport methods, without assuming any magnetic field present. We have finally addressed the question of the charge and spin transported into a cycle across the section of the wire. We have shown that the charge and spin are not quantized even if the adiabatic conditions are satisfied.

It would be interesting to address further questions regarding the thermal current pumped into the system, dissipation, and noise. However, the most interesting question would concern the experimental detection of our proposed pumping effect.

ACKNOWLEDGMENTS

We thank E. Andrei, E. Orignac, E. Shimshoni, and in particular A. Rosch for useful and enlightening discussions and L. Ioffe and L. Glazman for helpful comments on the manuscript. Q.N. acknowledges support from DOE Grant DE-FG03-02ER45958, Welch Foundation, and Texas Advanced Research Program.

APPENDIX: EVALUATION OF THE CURRENT INTEGRAL TO SECOND ORDER

1. Calculation at $T=0$

Due to the symmetry properties of the Green's function, only the terms with $\epsilon_1=-\epsilon_2$ contribute in the current integral (28). Changing variables it can be rewritten as Eq. (29):

$$\langle I_c(x,t)\rangle^{(2)} = -\frac{ie}{\pi}\sum_{n,m}\sum_{n',m'}\frac{g^U_{n,m}}{(2\pi a)^n}\frac{g^U_{n',m'}}{(2\pi a)^{n'}}\sum_{\eta\eta_1}\int_{-L/2}^{L/2}dx_1\int_{-L/2}^{L/2}dx_2\int_{-\infty}^{\infty}dT\,\sin(\omega_0 T-\Delta\varphi+\Delta k_{n,m}x_1-\Delta k_{n',m'}x_2)$$
$$\times e^{-nn'G^{\phi_c\phi_c}_{\eta_1-\eta_1}(x_1-x_2,T)}e^{-n_s n_s' G^{\phi_s\phi_s}_{\eta_1-\eta_1}(x_1-x_2,T)}\int_{-\infty}^{\infty}dT'[n\partial_T G^{\phi_c\phi_c}_{\eta\eta_1}(x-x_1,T')$$
$$-n'\partial_T G^{\phi_c\phi_c}_{\eta-\eta}(x-x_2,T')], \tag{A1}$$

where $\Delta\varphi=\Delta\varphi^{n',m'}_{n,m}$.

Another variable change $x_1,x_2\to x=(x_1-x_2),x'=(x_1+x_2)$, so that we must evaluates the integrals over (x,T), x', and T', which we denote by J_1, J_2, and J_3, respectively. The integral over x' is simply

$$J_2=2\int_0^{L/2}dx'\cos\left(\frac{\Delta k_{n,m}-\Delta k_{n',m'}}{2}x'\right).$$

The integral over (x,T) is

$$J_1=\sum_{\eta_1}\int_{-\infty}^{\infty}dT\int_{-L/2}^{L/2}dx\frac{\sin\left(\omega_0 T-\Delta\varphi+\frac{\Delta k_{n,m}+\Delta k_{n',m'}}{2}x\right)}{\prod_{\alpha=\pm}[a+ih_{\eta_1-\eta_1}(v_cT-\alpha x)]^{K_c^{nn'}}[a+ih_{\eta_1-\eta_1}(v_sT-\alpha x)]^{K_s^{n_sn_s'}}}, \tag{A2}$$

where $K_c^{nn'}=nn'K_c/2$ and $K_s^{n_sn_s'}=n_sn_s'K_s/2$.

We first consider the case when $v_s=v_c$ and make the variable change $s=(vT-x)/v$, $s'=(vT+x)/v$, so that Eq. (A2), becomes

$$\left(\frac{v}{2}\right)\sum_{\eta_1}\int_{-\infty}^{\infty}ds\int_{-\infty}^{\infty}ds'\frac{\sin[(\omega_0-v\Delta k_+)s+(\omega_0+v\Delta k_+)s']+\Delta\varphi}{[a+2ih_{\eta_1-\eta_1}vs]^{K^{nn'}_{n_sn_s'}}[a+2ih_{\eta_1-\eta_1}vs']^{K^{nn'}_{n_sn_s'}}}, \tag{A3}$$

where $K^{nn'}_{n_s n'_s}=K^{nn'}_c+K^{n_s n'_s}_s$, $(\Delta k_{n,m}+\Delta k_{n',m'})/2=\Delta k_+$, indicated as $[\Delta k_+]^{n',m'}_{n,m}$ in the main text.

Use the integrals 3.382.6/7 from Ref. 36,

$$\int_{-\infty}^{\infty}(\beta-ix)^{-\mu}e^{-ipx}dx=2\pi\frac{e^{-\beta p}(p)^{\mu-1}}{\Gamma(\mu)}\theta(p),\qquad \int_{-\infty}^{\infty}(\beta+ix)^{-\mu}e^{-ipx}dx=2\pi\frac{e^{-\beta p}(-p)^{\mu-1}}{\Gamma(\mu)}\theta(-p), \tag{A4}$$

we find the final result of the main text, Eq. (33).

When $v_s\neq v_c$ and $v_s<v_c$, we change variables,[37] $s=(v_sT+x)/(v_c+v_s)$ and $s'=(v_cT-x)/(v_c+v_s)$, permitting us to rewrite the integral (A2) as

$$J_1=\sum_{\eta_1}\frac{(v_s-v_c)}{(v_s+v_c)}\int ds\frac{e^{i\Delta\varphi}e^{-i\Omega s}}{[a+ih_{\eta_1-\eta_1}(v_c+v_s)s]^{K^{nn'}_s}}F(s)-(s\rightarrow -s;s'\rightarrow -s'), \tag{A5}$$

where

$$F(s)=\int ds'\frac{e^{-i\Omega' s'}}{[a+ih_{\eta_1-\eta_1}(v_c+v_s)s']^{K^{nn'}_c}\{a+ih_{\eta_1-\eta_1}[2v_cs+(v_c-v_s)s']\}^{K^{nn'}_c}\{a+ih_{\eta_1-\eta_1}[2v_ss'+(v_s-v_c)s]\}^{K^{n_sn'_s}_s}}$$

and

$$\Omega=(\omega_0+v_c\Delta k_+),\qquad \Omega'=(\omega_0-v_s\Delta k_+).$$

We expect singularities in J_1 near $\omega_0=\pm v_s\Delta k_+$ and $\omega_0=\pm v_c\Delta k_+$. Near $\omega_0=v_s\Delta k_+$, $\Omega'\simeq 0$ and $\Omega=(v_c-v_s)\Delta k_+$. The integral in s is dominated by $s<1/\Omega$ where $\Omega\simeq(v_c-v_s)\Delta k_+$, whereas that in s' is dominated by very large values. Power counting does imply that

$$I(\omega_0)\sim\Theta(\omega_0-v_s\Delta k_+)(\omega_0-v_s\Delta k_+)^{K^{nn'}_c+K^{n_sn'_s}_s-1}. \tag{A6}$$

Near $\omega_0=-v_c\Delta k_+$, the integrand in s' is dominated by $s'<1/\Omega'$, where $\Omega'\simeq(v_c-v_s)\Delta k_+$, and that in s by very large values. By power counting we obtain the singular form of I:

$$I(\omega_0)\sim\Theta(-\omega_0-v_c\Delta k_+)(-\omega_0-v_c\Delta k_+)^{K^{n_sn'_s}_s+K^{nn'}_c-1}. \tag{A7}$$

The role of v_c and v_s will be exchanged if $v_c<v_s$.

In other ranges of ω_0, the current may be written in terms of a single integration as shown in Ref. 38. We use integrals 3.384.7/8 from Gradshteyn to perform first the integral over s:

$$\int_{-\infty}^{\infty}(\beta-ix)^{-\mu}(\gamma-ix)^{-\nu}e^{-ip}dx=2\pi\frac{e^{-\beta p}(p)^{\mu+\nu-1}}{\Gamma(\mu+\nu)}\Phi(\mu;\mu+\nu;(\beta-\gamma)p)\theta(p),$$

$$\int_{-\infty}^{\infty}(\beta+ix)^{-\mu}(\gamma+ix)^{-\nu}e^{-ip}dx=-2\pi\frac{e^{\beta p}(-p)^{\mu+\nu-1}}{\Gamma(\mu+\nu)}\Phi(\mu;\mu+\nu;(\beta-\gamma)p)\theta(-p), \tag{A8}$$

where Φ is the degenerate hypergeometric function, and in our case $\beta=(a-ih_{\eta_1-\eta_1}s')/(v_c-v_s)$ and $\gamma=(a+ih_{\eta_1-\eta_1}s')/(v_c+v_s)$. Next, one employs the integral representation of the hypergeometric function:

$$\Phi(a,b,z)=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}\int_0^1 dse^{-zs}(1-z)^{b-a-1}s^{a-1}. \tag{A9}$$

The resulting integral over s' may be written in terms of the Γ functions by using 3.382.7. In the last step we use the integral 3.197.3 to recast the current in terms of hypergeometric functions.

In the region $v_s\Delta k\leqslant\omega_0\leqslant v_c\Delta k$ we obtain the following result of the integral $I^{n'm'n'_s}_{nmn_s}(\omega_0)$:

$$I^{n'm'n'_s}_{nmn_s}(\omega_0)=\frac{2\pi a^K}{\Gamma(K)}\frac{(v_c+v_s)^{1-K^{n_sn'_s}_s}}{(2v_c)^{K^{nn'}_c+1}}(\omega_0-v_s\Delta k_+)^{K^{nn'}_{n_sn'_s}-1}(\omega_0+v_c\Delta k_+)^{K^{nn'}_{n_sn'_s}-1}F\left(1,K^{n_sn'_s}_s,K^{nn'}_{n_sn'_s};\frac{(v_c+v_s)}{2v_c}\frac{\omega_0-v_s\Delta k_+}{\omega_0+v_c\Delta k_+}\right),$$

where $K^{nn'}_{n_s n_s'} = (K_c^{nn'} + K_s^{n_s n_s'})$.

To have the current in its final form we must evaluate the integral over T' involving $\partial_{T'} G$:

$$J_3^c = \sum_{\eta_1} \int_{-\infty}^{\infty} dT' [n \partial_{T'} G^{\phi_c \phi_c}_{\eta \eta_1}(x-x_1,T') - n' \partial_{T'} G^{\phi_c \phi_c}_{\eta - \eta_1}(x-x_2,T')] \tag{A10}$$

for the charge current, where

$$\sum_{\eta_1} \int_{-\infty}^{\infty} dT' \partial_{T'} G^{\phi_c \phi_c}_{\eta \eta_1}(x-x_1,T') = -i \frac{K_c v_c}{2} \sum_{\alpha} \sum_{\eta_1} \int_{-\infty}^{\infty} dT' \left[\frac{h_{\eta \eta_1}}{[a + i h_{\eta \eta_1} \alpha (x-x_1)] + i h_{\eta \eta_1} v_c T'} \right]$$

$$= \frac{K_c v_c}{2} \sum_{\alpha} \sum_{\eta_1} \lim_{T_m \to \infty} \ln\{a + i h_{\eta \eta_1}[\alpha (x-x_1) + v_c T']\}\Big|_{-T_m}^{T_m}$$

$$= \frac{K_c v_c}{2} \sum_{\alpha} \sum_{\eta_1} \lim_{T_m \to \infty} \left\{ \frac{1}{2} \ln\{a^2 + h^2_{\eta \eta_1}[\alpha (x-x_1) + v_c T']^2\}_{-T_m}^{T_m} \right.$$

$$\left. + i \tan^{-1}\left(h_{\eta \eta_1} \frac{(\alpha (x-x_1) + v_c T')}{a} \right)\Bigg|_{-T_m}^{T_m} \right\}$$

$$= i K_c v_c \pi, \tag{A11}$$

and similarly for $\int_{-\infty}^{\infty} dT' \partial_T C^{\phi_c \phi_c}_{\eta - \eta_1}(x-x_2,T')$ with $\eta_1 \to -\eta_1$.

We can perform the same type of calculation for the spin bosonic Green's function. Thus we finally have

$$J_3^c = i K_c v_c (n-n') \pi, \tag{A12}$$

$$J_3^s = i K_s v_s (n_s - n_s') \pi. \tag{A13}$$

The result shows that when $n_s', n_s = 0$ we have a pure charge current $I_s = 0$, while if $n, n' = 0$ we have a pure spin current.

2. Finite temperature

For simplicity we make the calculation in the case $v_c = v_s$. Using the finite-temperature expression for the bosonic Green's function, we must evaluate the integral:

$$J_1 = \sum_{\eta_1} \left[\frac{\pi a T}{v} \right]^{2K^{nn'}_{n_s n_s'}} \int_{-\infty}^{\infty} dT \int dx \frac{\sin(\omega_0 T + \Delta k_+ x)}{\sinh \pi T \left[h_{\eta_1 - \eta_1} \left(\frac{vT - x}{v} \right) + ia \right]^{K^{nn'}_{n_s n_s'}} \sinh \pi T \left[h_{\eta_1 - \eta_1} \left(\frac{vT + x}{v} \right) - ia \right]^{K^{nn'}_{n_s n_s'}}}. \tag{A14}$$

We first perform the variable change $s = vT - x$ and $s' = vT + x$ and afterwards we use the integral

$$\int_{-\infty}^{\infty} ds |[\sinh(\pi T s)]|^{-K^{nn'}_{n_s n_s'}} e^{-isz} = \frac{2^{K^{nn'}_{n_s n_s'} - 2}}{\pi T} B\left(\frac{K^{nn'}_{n_s n_s'}}{2} - \frac{iz}{2\pi}, 1 - K^{nn'}_{n_s n_s'} \right) \cosh\left(\frac{z}{2T} \right) \left[1 + \tanh\left(\frac{z}{2T} \right) \right], \tag{A15}$$

which permits us to write $I(\omega_0)$ in the final form shown in the text. The integral (A10) gives the same result at finite temperature and similarly for the spin part.

[1] D. Thouless, Phys. Rev. B **27**, 6083 (1983).
[2] Q. Niu, Phys. Rev. Lett. **64**, 1812 (1990).
[3] Q. Niu and D. Thouless, J. Phys. A **17**, 2453 (1984).
[4] J. Shilton, V.I. Talyanskii, M. Pepper, D.A. Ritchie, J.E.F. Frost, C.J.B. Ford, C.G. Smith, and G.A.C. Jones, J. Phys.: Condens. Matter **8**, L531 (1996).
[5] P. Brouwer, Phys. Rev. B **58**, 10 135 (1998).
[6] I. Aleiner and A. Andreev, Phys. Rev. Lett. **81**, 1286 (1998).
[7] M. Polianski and P. Brouwer, Phys. Rev. B **58**, 10 135 (1998).
[8] F. Zhou, B. Spivak, and B. Altshuler, Phys. Rev. Lett. **82**, 608 (1999).
[9] M. Switkes, C. Marcus, K. Campman, and A.C. Gossard, Science

283, 1905 (1999).
[10] B. Altshuler and L. Glazman, Science **283**, 1864 (1999).
[11] Y. Levinson, O. Entin-Wohlman, and P. Wolfe, Physica A **302**, 335 (2001).
[12] O. Entin-Wohlman and A. Aharony, Phys. Rev. B **66**, 035329 (2002).
[13] T. Shutenko, I. Aleiner, and B. Altshuler, Phys. Rev. B **61**, 10 366 (2000).
[14] M. Moskalets and M. Buttiker, Phys. Rev. B **66**, 035306 (2002).
[15] M. Polianski, M. Vavilov, and P. Brouwer, Phys. Rev. B **65**, 245314 (2002).
[16] J. Avron, A. Elgart, G. Graf, and L. Sadun, Phys. Rev. Lett. **87**, 236601 (2001).
[17] J. Wang and B. Wang, Phys. Rev. B **65**, 153311 (2002).
[18] B. Wang and J. Wang, Phys. Rev. B **66**, 201305 (2002).
[19] P. Sharma and C. Chamon, Phys. Rev. Lett. **87**, 096401 (2001).
[20] P. Sharma and C. Chamon, cond-mat/0209291 (unpublished).
[21] Y. Wei, J. Wang, H. Guo, and C. Roland, Phys. Rev. B **64**, 115321 (2001).
[22] V. Talyanskii, D.S. Novikov, B. Simons, and L. Levitov, Phys. Rev. Lett. **87**, 276802 (2001).
[23] F.D.M. Haldane, J. Phys. C **14**, 2585 (1981).
[24] S. Datta and B. Das, Appl. Phys. Lett. **56**, 665 (1990).
[25] F. Mireles and G. Kirczenow, Phys. Rev. B **64**, 024426 (2001).
[26] A. Rosch and N. Andrei, Phys. Rev. Lett. **85**, 1092 (2000).
[27] C. Kane and M. Fisher, Phys. Rev. B **46**, 15 233 (1992).
[28] J. Sólyom, Adv. Phys. **28**, 209 (1979).
[29] V. J. Emery, in *Highly Conducting One-Dimensional Solids*, edited by J. T. Devreese *et al.* (Plenum, New York, 1979), p. 327.
[30] L. Keldysh, Sov. Phys. JETP **20**, 1018 (1965).
[31] A. Crepieux, R. Guyon, P. Devillard, and T. Martin, cond-mat/0209291 (unpublished).
[32] C. Chamon, D. Freed, and X. Wen, Phys. Rev. B **51**, 2363 (1995).
[33] M. Governale, F. Taddei, and R. Fazio, cond-mat/0211211 (unpublished).
[34] H. Schulz, Phys. Rev. B **34**, 6372 (1986).
[35] T. Giamarchi, Phys. Rev. B **44**, 2905 (1991).
[36] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series and Products* (Academic Press, London, 1965).
[37] A. Gogolin, A. Nersesyan, and A. Tsvelik, in *Bosonization and Strongly Correlated Systems* (Cambridge University Press, London, 1998), p. 268.
[38] V. Meden and K. Schönhammer, Phys. Rev. B **46**, 15 753 (1992).