Universal Intrinsic Spin Hall Effect

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We describe a new effect in semiconductor spintronics that leads to dissipationless spin currents in paramagnetic spin-orbit coupled systems. We argue that in a high-mobility two-dimensional electron system with substantial Rashba spin-orbit coupling, a spin current that flows perpendicular to the charge current is intrinsic. In the usual case where both spin-orbit split bands are occupied, the intrinsic spin-Hall conductivity has a universal value for zero quasiparticle spectral broadening.

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The science of devices whose operation is based in part on manipulation of the electronic spin degree of freedom, spintronics, has emerged [1] as an active subfield of condensed matter physics because of its potential impact on information technology and because of the challenging basic questions that it poses. Many spintronic concepts involve ferromagnets, in which spins are easier to manipulate because they behave collectively. Spintronic magnetoresistive [2–5] sensors based on the properties of ferromagnetic metals, for example, have reinvented the hard-disk industry over the past several years. Spintronics in semiconductors is richer scientifically than spintronics in metals because doping, gating, and heterojunction formation can be used to engineer key material properties and because of the intimate relationship in semiconductors between optical and transport properties. Practical spintronics in semiconductors has appeared, however, to be contingent on either injection of spin-polarized carriers [6–12] from ferromagnetic metals combined with long spin lifetimes [13] or on room-temperature semiconductor ferromagnetism [14]. In this Letter we explain a new effect [15] that might suggest a new direction for semiconductor spintronics research.

In the following paragraphs we argue that in high-mobility two-dimensional electron systems (2DES) that have substantial Rashba [16] spin-orbit coupling, spin currents always accompany charge currents. The Hamiltonian of a 2DES with Rashba spin-orbit coupling is given by [16]

$$H = \frac{p^2}{2m} - \frac{\lambda}{\hbar}\vec{\sigma}\cdot(\hat{z}\times\vec{p}), \qquad (1)$$

where λ is the Rashba coupling constant, $\vec{\sigma}$ is the Pauli matrices, m is the electron effective mass, and $\hat{z}$ is the unit vector perpendicular to the 2DES plane. The Rashba coupling strength in a 2DES can be modified by as much as 50% by a gate field [17]. Recent observations of a spin-galvanic effect [18,19] and a spin-orbit coupling induced metal-insulator transition in these systems [20] illustrate the potential importance of this tunable interaction in semiconductor spintronics [21]. The spin current we discuss is polarized in the direction perpendicular to the two-dimensional plane and flows in the planar direction that is perpendicular to the charge current direction. It is therefore a spin Hall effect, but unlike the effect conceived by Hirsch [22], it is purely intrinsic and does not rely on anisotropic scattering by impurities. Remarkably, in the usual case when both spin-orbit split Rashba bands are occupied, the spin Hall conductivity has a universal value independent of both the 2DES density and the Rashba coupling strength.

The basic physics of this effect is illustrated schematically in Fig. 1. In a translationally invariant 2DES, electronic eigenstates have definite momentum and, because of spin-orbit coupling, a momentum-dependent effective magnetic field that causes the spins (red or dark gray arrows) to align perpendicularly to the momenta (green or light gray arrows), as illustrated in Fig. 1(a). In the presence of an electric field, which we take to be in the $\hat{x}$ direction as shown in Fig. 1(b), electrons are accelerated and drift through momentum space at the rate $\dot{\vec{p}} = -eE\hat{x}$. Our spin Hall effect arises from the time dependence of the effective magnetic field experienced by the spin because of its motion in momentum space. For the Rashba Hamiltonian case of interest here, the effect can be understood most simply by considering the Bloch equation of a spin-1/2 particle, as we explain in the following paragraph. More generally the effect arises from nonresonant interband contributions to the Kubo-formula expression for the spin Hall conductivity that survive in the static limit.

The dynamics of an electron spin in the presence of time-dependent Zeeman coupling is described by the Bloch equation:

$$\frac{\hbar d\hat{n}}{dt} = \hat{n}\times\vec{\Delta}(t) + \alpha\frac{\hbar d\hat{n}}{dt}\times\hat{n}, \qquad (2)$$

where $\hat{n}$ is direction of the spin and α is a damping parameter that we assume is small. For the application we have in mind the $\vec{p}$ dependent Zeeman coupling term in the spin Hamiltonian is $-\vec{s}\cdot\vec{\Delta}/\hbar$, where $\vec{\Delta} = 2\lambda/\hbar(\hat{z}\times\vec{p})$. For a Rashba effective magnetic field with

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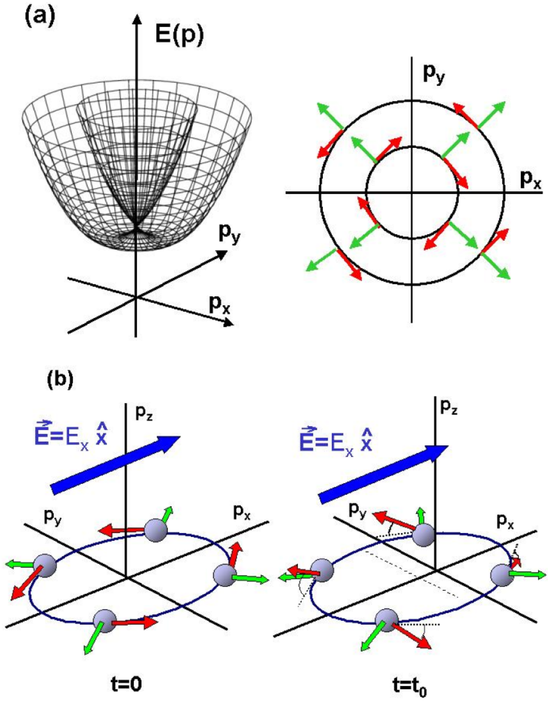

FIG. 1 (color online). (a) The 2D electronic eigenstates in a Rashba spin-orbit coupled system are labeled by momentum (green or light gray arrows). For each momentum the two eigenspinors point in the azimuthal direction (red or dark gray arrows). (b) In the presence of an electric field the Fermi surface (circle) is displaced an amount $|eE_x t_0/\hbar|$ at time t_0 (shorter than typical scattering times). While moving in momentum space, electrons experience an effective torque which tilts the spins up for $p_y > 0$ and down for $p_y < 0$, creating a spin current in the y direction.

magnitude Δ_1 that initially points in the $\hat{x}_1$ direction, then tilts (arbitrarily slowly) slightly toward $\hat{x}_2$, where $\hat{x}_1$ and $\hat{x}_2$ are orthogonal in-plane directions, it follows from the linear response limit of Eq. (2) that

$$\frac{\hbar dn_2}{dt} = n_z\Delta_1 + \alpha dn_z/dt,$$
$$\frac{\hbar dn_z}{dt} = -\Delta_1 n_2 - \alpha dn_2/dt + \Delta_2, \tag{3}$$

where $\Delta_2 = \vec{\Delta}\cdot\hat{x}_2$. Solving these inhomogeneous coupled equations using a Green function technique, it follows that to leading order in the slow-time dependences $n_2(t) = \Delta_2(t)/\Delta_1$, i.e., the $\hat{x}_2$ component of the spin rotates to follow the direction of the field, and that

$$n_z(t) = \frac{1}{\Delta_1^2}\frac{\hbar d\Delta_2}{dt}. \tag{4}$$

Our intrinsic spin Hall effect follows from Eq. (4). When a Bloch electron moves through momentum space, its spin orientation changes to follow the momentum-dependent effective field and also acquires a momentum-dependent $\hat{z}$ component. We now show that in the case of Rashba spin-orbit coupling, this effect leads to an intrinsic spin-Hall conductivity that has a universal value in the limit of zero quasiparticle spectral broadening.

For a given momentum $\vec{p}$, the spinor originally points in the azimuthal direction. An electric field in the $\hat{x}$ direction ($\dot{p}_x = -eE_x$) changes the y component of the $\vec{p}$-dependent effective field. Applying the adiabatic spin dynamics expressions explained above, identifying the azimuthal direction in momentum space with $\hat{x}_1$ and the radial direction with $\hat{x}_2$ we find that the z component of the spin direction for an electron in a state with momentum $\vec{p}$ is

$$n_{z,\vec{p}} = \frac{-e\hbar^2 p_y E_x}{2\lambda p^3}. \tag{5}$$

(Linear response theory applies for $eE_x r_s \ll \Delta_1$, where r_s is the interparticle spacing in the 2DES.) Summing over all occupied states the linear response of the $\hat{z}$ spin-polarization component vanishes because of the odd dependence of n_z on p_y, as illustrated in Fig. 1, but the spin current in the $\hat{y}$ direction is finite.

The Rashba Hamiltonian has two eigenstates for each momentum with eigenvalues $E_\pm = p^2/2m \mp \Delta_1/2$; the discussion above applies for the lower energy (labeled $+$ for majority spin Rashba band) eigenstate while the higher energy (labeled $-$) eigenstate has the opposite value of $n_{z,\vec{p}}$. Since Δ_1 is normally much smaller than the Fermi energy [17], only the annulus of momentum space that is occupied by just the lower energy band contributes to the spin current. In this case we find that the spin current in the $\hat{y}$ direction is [23]

$$j_{s,y} = \int_{\text{annulus}} \frac{d^2\vec{p}}{(2\pi\hbar)^2}\frac{\hbar n_{z,\vec{p}}}{2}\frac{p_y}{m} = \frac{-eE_x}{16\pi\lambda m}(p_{F+} - p_{F-}), \tag{6}$$

where p_{F+} and p_{F-} are the Fermi momenta of the majority and minority spin Rashba bands. We find that when both bands are occupied, i.e., when $n_{2\text{D}} > m^2\lambda^2/\pi\hbar^4 \equiv n_{2\text{D}}^*$, $p_{F+} - p_{F-} = 2m\lambda/\hbar$ and then the spin Hall (sH) conductivity is

$$\sigma_{\text{sH}} \equiv -\frac{j_{s,y}}{E_x} = \frac{e}{8\pi}, \tag{7}$$

independent of both the Rashba coupling strength and of the 2DES density. For $n_{2\text{D}} < n_{2\text{D}}^*$ the upper Rashba band is depopulated. In this limit p_{F-} and p_{F+} are the interior and exterior Fermi radii of the lowest Rashba split band, and σ_{sH} vanishes linearly with the 2DES density:

$$\sigma_{\text{sH}} = \frac{e}{8\pi}\frac{n_{2\text{D}}}{n_{2\text{D}}^*}. \tag{8}$$

The intrinsic spin Hall conductivity of the Rashba model can also be evaluated by using transport theories that are valid for systems with multiple spin-orbit spit bands, either the linear-response-theory Kubo-formula approach [24] or a generalized Boltzmann equation approach that accounts for anomalous contributions to wave-packet dynamics [25]. In the Kubo formalism approach, our universal intrinsic spin Hall effect comes from the static $\omega = 0$ limit of the nondissipative reactive term in the expression for the spin-current response to an electric field [26]:

$$\sigma_{xy}^{\mathrm{sH}}(\omega) = \frac{e\hbar}{V} \sum_{\mathbf{k}, n \neq n'} (f_{n',k} - f_{n,k}) \times \frac{\mathrm{Im}[\langle n'k|\hat{j}^{z}_{\mathrm{spin}\,x}|nk\rangle\langle nk|v_y|n'k\rangle]}{(E_{nk} - E_{n'k})(E_{nk} - E_{n'k} - \hbar\omega - i\eta)}, \tag{9}$$

where n, n' are band indices, $\vec{j}^{z}_{\mathrm{spin}} = \frac{\hbar}{4}\{\sigma_z, \vec{v}\}$ is the spin-current operator, ω and η are set to zero in the dc clean limit, and the velocity operators at each $\vec{p}$ are given [7] by

$$\begin{aligned} \hbar v_x &= \hbar \partial H(\vec{p})/\partial p_x = \hbar p_x/m - \lambda\sigma_y, \\ \hbar v_y &= \hbar \partial H(\vec{p})/\partial p_y = \hbar p_y/m + \lambda\sigma_x. \end{aligned} \tag{10}$$

For the Rashba model $n, n' = \pm$ and the eigenspinors are spin coherent states given explicitly by

$$|\mp, p\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \pm i e^{-i\phi} \\ 1 \end{pmatrix}, \tag{11}$$

where $\phi = \arctan p_x/p_y$. The energy denominators in Eq. (9) in this case are given by the Rashba splitting, and the velocity matrix elements can be evaluated using Eqs. (10) and (11). The integral over momentum space can be evaluated easily because the energy denominators are independent of orientation and the intrinsic spin Hall conductivity expressions in Eqs. (7) and (8) are recovered. This principal result of the Letter is summarized in Fig. 2 where σ_{sH} is plotted as a function of carrier density and Rashba coupling strength in the zero quasiparticle spectral broadening limit. The Kubo-formula analysis also makes it clear that, unlike the universal Hall conductivity value on a 2DES quantum Hall plateau, the universality of the intrinsic spin Hall effect is not robust against disorder and will be reduced whenever the disorder broadening is larger than the spin-orbit coupling splitting [27–29].

The intrinsic character of our spin Hall effect, compared to the extrinsic character of the effect discussed originally by Hirsch [22], is analogous to the intrinsic character that we have recently proposed for the anomalous Hall effect in some ferromagnets and strongly polarized paramagnets [27,30–36]. In both cases the skew-scattering contributions to the Hall conductivities can become important [37,38], when the overall electron scattering rate is small and the steady state distribution function of the current-carrying state is strongly disturbed compared to the equilibrium one. In the Kubo-formula approach these contributions to the spin-Hall conductivity appear as dissipative contributions from disorder scattering of Fermi-energy quasiparticles. A general and quantitative analysis of the disorder-potential-dependent interplay between intrinsic and skew-scattering contributions to anomalous Hall and spin-Hall effects is a subject of current research that is beyond the scope of this Letter.

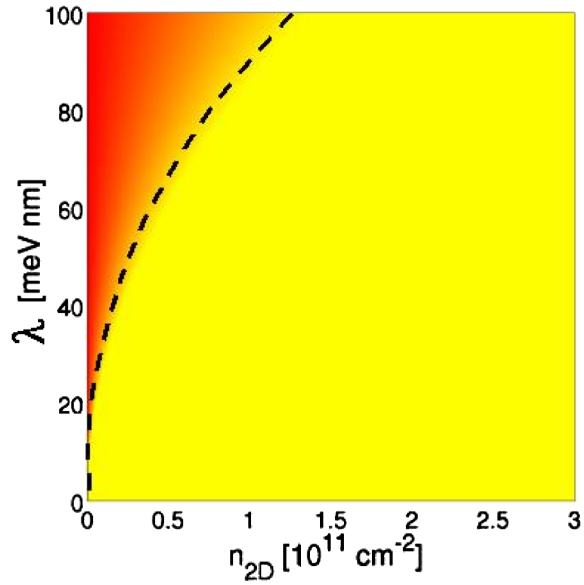

FIG. 2 (color online). Plot of σ_{sH} vs the two-dimensional electron density n_{2D} and the Rashba coupling constant λ. The scale is yellow/light gray $= e/8\pi$ and dark red/dark gray $= 0$. The dashed line indicates the boundary between the universal region (constant σ_{sH}) and the linear density dependence region. Most of the current experimental samples are deep in the universal value region. We expect the intrinsic spin Hall contribution to be present only when the Rashba splitting is larger than the disorder broadening of the quasiparticle energy levels, i.e., when $\lambda p_F \tau/\hbar^2 > 1$.

Several schemes have been proposed for measuring the spin Hall effect in metals [22,39,40], and these can be generalized to the semiconductor case. In semiconductors the close relationship between optical properties [6,7] and spin polarizations opens up new possibilities for detecting nonequilibrium spins accumulated near contacts or near the sample perimeter by spin currents. Spatially resolved Faraday or Kerr effects should be able to detect spin accumulations induced by the spin currents we have evaluated. As in the case of the ferromagnetic semiconductor anomalous Hall effect, the origin of the intrinsic spin Hall-type effect is strong spin-orbit coupling. A sizable intrinsic spin Hall effect will occur in any paramagnetic material with strong spin-orbit coupling, including hole-doped bulk semiconductors [15], although the universal value we obtain here is a unique property of Rashba systems.

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