Charge Hall effect driven by spin-dependent chemical potential gradients and Onsager relations in mesoscopic systems

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We study theoretically the spin-Hall effect as well as its reciprocal phenomenon (a transverse charge current driven by a spin-dependent chemical potential gradient) in electron and hole finite size mesoscopic systems. The Landauer-Buttiker-Keldysh formalism is used to model samples with mobilities and Rashba coupling strengths which are experimentally accessible and to demonstrate the appearance of measurable charge currents induced by the spin-dependent chemical potential gradient in the reciprocal spin-Hall effect. We also demonstrate that within the mesoscopic coherent transport regime the Onsager relations are fulfilled for the disorder averaged conductances for electron and hole mesoscopic systems.

Introduction. In the very active field of semiconductor based spintronics the control of spin can be achieved by the manipulation of the strength of spin-orbit (SO) interactions in paramagnetic systems. Within this context, the newly proposed intrinsic spin-Hall effect (SHE) in p-doped semiconductors by Murakami *et al.*[1] and in a two-dimensional electron system (2DES) by Sinova *et al.* [2] offers new possibilities for spin current manipulation and generation in high mobility paramagnetic semiconductor systems. In contrast to the earlier proposed extrinsic spin-Hall effect [3, 4, 5], which is associated with scattering from impurities, the intrinsic spin-Hall arises purely from host semiconductor band structure and represents a spin-current response generated perpendicular to the driving electric field.

Recently, the spin Hall effect was experimentally observed by Kato *et al.* [6] in n-doped GaAs using the Kerr effect and by Wunderlich *et al.* [7] in the p-n junction light-emitting diodes based on two dimensional hole gas (2DHG)(Al,Ga)As. Although, the experiment by Wunderlich *et al.* seems to be in the regime where the intrinsic effect in 2DHG is dominant, the main theoretical focus has been concentrated so far on 2DEG with Rashba SO interactions, where Rashba term is linear with k [2, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21].The influence of disorder on infinite 2DEG is still unclear(for a recent review see [21]). For δ-function impurities the analytical calculations of vertex corrections in the ladder approximation seem to cancel the intrinsic spin-Hall effect in a weak scattering regime [12, 14]. However,these calculations have been challenged recently [20]. Furthermore, the numerical calculations based on Kubo formula using continuum model in momentum space [13] and discrete model in the real space [22] show finite spin-conductivity in a weak scattering regime which goes to zero in the thermodynamic limit for electron systems [22, 23]. In contrast, for the infinite 2DHG the vertex corrections vanish [24, 25]. Moreover, the numerical calculations based on Kubo formula using continuum model in mo-

mentum space show finite value of SHE in a weak scattering regime which goes to constant in the thermodynamic limit [23].

The calculations within the Landauer-Buttiker (LB) formalism on finite size systems [17, 18, 19, 26] model a sample of micro/nanosize attached to contacts. The calculations on electron mesoscopic systems show that spin Hall conductance is a fraction of $e/8\pi$ in a weak scattering regime [17, 18, 19]. Moreover, a mesoscopic spin-Hall conductance is robust against the disorder [17, 18, 19]. Very recently Wu and Zhou considered the Luttinger model [27], showing as expected that SHE can be much larger in hole systems in comparison with the electron ones. Although the experimental measurement by Wunderlich at al [7] concern 2DHG systems with broken inversion symmetry, the pure cubic Rashba term was not considered in detail within the LB formalism so far.

The observation of spin-Hall effect through transport measurements is one of the urgent experimental challenges facing this spin-transport physics. Recently, an H-probe structure has been proposed to measure the effect where the spin-Hall effect could be measured indirectly by detecting charge voltages induced by the reciprocal spin-Hall effect (RSHE) [19]. This RSHE, where transverse charge current is driven by spin dependent chemical potential, was proposed in a context of extrinsic spin-Hall effect by Hirsch [4] and formulated in a semiclassical approach by Zhang and Niu [28]. Also, the Onsager relation between the spin-Hall conductivity and reciprocal charge-Hall conductivity was established within a wave packet model through a redefinition of the spin-current including spin-torque terms in the bulk [28]. We show here that within the mesoscopic regime, and more specifically within the Landauer-Buttiker-Keldysh formalism, the Onsager relations are satisfied within the models studied for the disorder averaged conductances. Because the conductances are formulated with respect to the leads which have no spin-orbit coupling, it is not necessary nor consequential to introduce the spin current

redefinition in our problem [28].

In this paper we compare the magnitude of the SHE as well as the RSHE in finite electron and hole mesoscopic systems within the LB formalism. We show that the conductances associated with both effects are significantly larger in the hole systems. Furthermore, we analyze the possible experimental setup to measure the RSHE. We show that the charge current driven by a spin-dependent chemical gradient is on the order of hundred nano-ampers for typical voltages in hole systems and should be experimentally measurable.

Model Hamiltonian for hole system and LB treatment of the spin-Hall effect and its reciprocal correspondent. The observation of the spin-Hall effect and its reciprocal phenomenon in transport is the next experimental challenge in the subfield of spintronics using spin-orbit interactions to manipulate the spin.

The continuum effective mass model for 2DHG in a narrow inversion asymmetrical well is given by [29]: $\hat{H} = \frac{\hat{p}^2}{2m^*} + i\frac{\lambda}{2\hbar^3}(p_-^3\hat{\sigma}_+ - p_+^3\hat{\sigma}_-) + H_{dis}$, where H_{dis} describes disorder. We use the tight-binding approximation [30] to model the disordered conductor within the LB formalism. Within this approximation the continuum effective mass envelope function Hamiltonian becomes:

$$
\begin{aligned}
H =\ & \sum_{j,\sigma}\epsilon_j c_{j,\sigma}^\dagger c_{j,\sigma} - t\sum_{j,\vec{\delta},\sigma} c_{j+\vec{\delta},\sigma}^\dagger c_{j,\sigma} \\
& + t_{SO-k^3}[\sum_j c_{j,\uparrow}^\dagger c_{j+2a_x,\downarrow} - c_{j,\uparrow}^\dagger c_{j-2a_x,\downarrow} \\
& + i\sum_j c_{j,\uparrow}^\dagger c_{j+2a_y,\downarrow} - c_{j,\uparrow}^\dagger c_{j-2a_y,\downarrow} \\
& + 3(1-i)\sum_j c_{j,\uparrow}^\dagger c_{j-a_x+a_y,\downarrow} - c_{j,\uparrow}^\dagger c_{j+a_x-a_y,\downarrow} \\
& + 3(1+i)\sum_j c_{j,\uparrow}^\dagger c_{j-a_x-a_y,\downarrow} - c_{j,\uparrow}^\dagger c_{j+a_x+a_y,\downarrow} \\
& + 4i\sum_j c_{j,\uparrow}^\dagger c_{j+a_y,\downarrow} - c_{j,\uparrow}^\dagger c_{j-a_y,\downarrow} \\
& + 4\sum_j c_{j,\uparrow}^\dagger c_{j+a_x,\downarrow} - c_{j,\uparrow}^\dagger c_{j-a_x,\downarrow} + h.c.],
\end{aligned}
\tag{1}
$$

where $t = \hbar^2/2m^*a_0^2$ and $t_{SO-k^3} = -\lambda/2a_0^3$, a_0 is the mesh lattice spacing, and $\vec{\delta} = \pm a_0\hat{x}, \pm a_0\hat{y}$. The first term represents a quenched disorder potential and disorder is introduced by randomly selecting the on-site energy ϵ_j in the range [-W/2,W/2]. The continuum effective mass model for 2DES and its tight-binding correspondent can be found elsewhere [18, 19].

Within the leads the SO coupling is zero and therefore each lead should be considered as having two independent spin-channels. Moreover, in leads without the SO coupling, the spin current is measured in a medium where spin is conserved removing the ambiguity of spin-current definition. For SHE, leads constitute reservoirs of electrons at chemical potential $\mu_1, \ldots, \mu_N$,
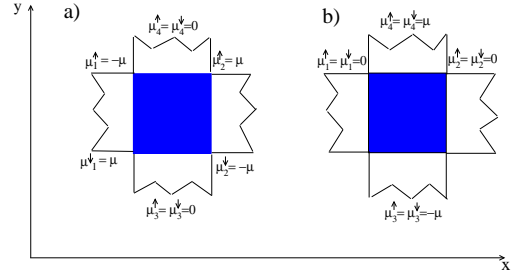

FIG. 1: The schematic picture of setup to measure (a)charge Hall effect driven by spin dependent chemical potential and (b)spin-Hall effect.
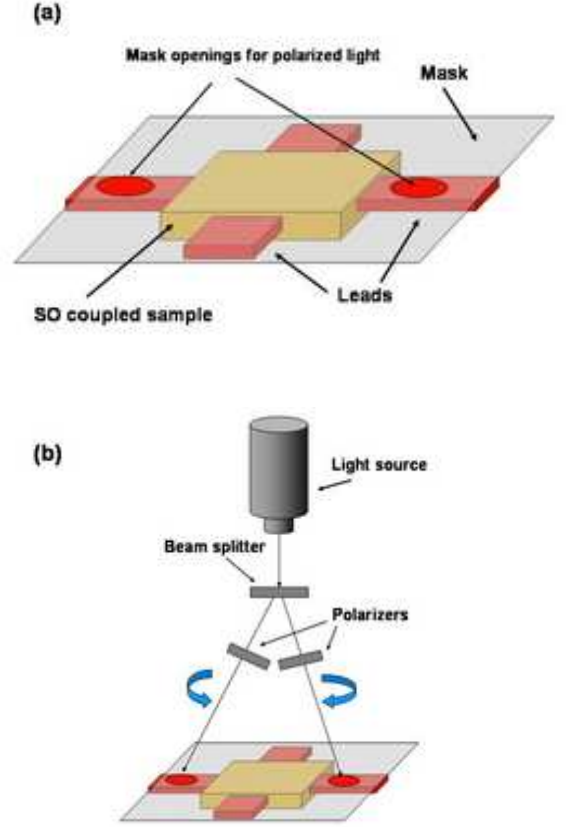

FIG. 2: Proposal of experimental setup to measure the reciprocal spin-Hall effect. a) Mask (light-gray) covers a sample and leads except for two holes in the longitudinal leads where circularly polarized light shines. (b) Full schematics of the experimental set-up.

where N is the number of leads that we consider to be four (see Fig. 1b). For RSHE, the chemical potential is spin dependent in leads 1 and 2 allowing the generation of spin-force in the x direction (see Fig. 1a). In the low temperature limit $k_BT \ll E_F$ and for low bias-voltage, the *particle* current going through a particular channel is given within the LB formalism by [30] $I_{p,\sigma} = (e/h)\sum_{q\sigma'} T_{p,\sigma;q,\sigma'}[V_p - V_q]$, where p labels the

lead and $T_{p,\sigma;q,\sigma'}$ is the transmission coefficient at the Fermi energy E_F between the (p,σ) channel and the (q,σ') channel. This transmission coefficient is obtained by $T_{p,\sigma;q,\sigma'} = \text{Tr}[\Gamma_{p,\sigma} G^R \Gamma_{q,\sigma'} G^A]$ where $\Gamma_{p,\sigma}$ is given by $\Gamma_{p,\sigma}(i,j) = i[\Sigma^R_{p,\sigma}(i,j) - \Sigma^A_{p,\sigma}(i,j)]$. The retarded and advance Green's function of the sample $G^{R/A}$ with the leads taken into account through the self energy $\Sigma^{R/A}_{p,\sigma}(i,j)$ has a form $G^{R/A}(i,j) = [E\delta_{i,j} - H_{i,j} - \sum_{p,\sigma} \Sigma^{R/A}_{p,\sigma}(i,j)]^{-1}$. Here the position representation of the matrices $\Gamma_{p,\sigma}$, G^R, $H_{i,j}$, and Σ^R are in the subspace of the sample. Within the above formalism the spin current through each channel is given by $I^s_{p,\sigma} = (e/4\pi) \sum_{q\sigma'} T_{p,\sigma;q,\sigma'}[V_{p,\sigma} - V_{q,\sigma'}]$. The spin force driven charge-Hall conductance, G^{yx}_{CS}, is defined as the ratio of charge current in the y- direction induced by the spin-dependent chemical potential along the x- axis to this spin-dependent chemical potential difference (see Fig 1a).

$$G^{yx}_{CS} = \frac{(I^s_{3\uparrow} + I^s_{3\downarrow})}{V^\uparrow_2 - V^\uparrow_1}, \qquad (2)$$

where $V^\uparrow_i = \mu^\uparrow_i/e$. The spin-Hall conductance, G^{xy}_{SC}, is defined as the ratio of spin-current in the x-direction induced by charge voltage difference in y direction to this voltage difference

$$G^{xy}_{SC} = \frac{(I^s_{1\uparrow} - I^s_{1\downarrow})}{V_4 - V_3}, \qquad (3)$$

where $V_i = \mu_i/e$, and the labels are indicated in Fig. 1b. G^{yx}_{SC} and G^{xy}_{CS} are defined by analogy. We set the absolute value of voltage for spin and spin force driven charge-Hall effects as $V = \mu/e = 2.5\text{mV}$.
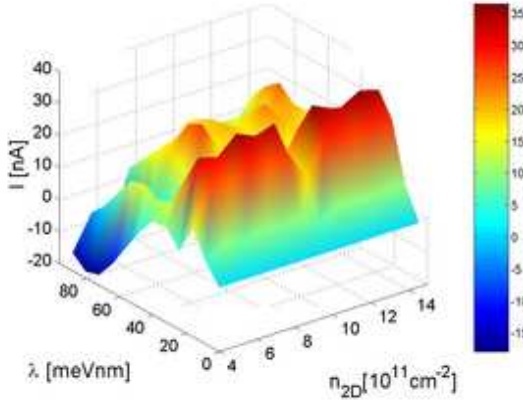

FIG. 3: The spin-force driven charge current as a function of electron concentration and Rashba coupling ,λ, for mesoscopic square sample 100nm by 100nm and μ =250,000 cm^2/Vs.

Results and discussion. In order to address the key issue of experimental observation of spin driven charge-Hall effect as well as to establish the Onsager relation

between spin-Hall effect and it reciprocal correspondent we choose realistic parameters for our calculations which model currently attainable systems. We consider an effective mass of $m^* = 0.05m_e$ for electron systems and $m^* = 0.5m_e$ for hole ones. The disorder strength $W = 0.09$ meV corresponds to the mobility of 250,000 cm^2/Vs, which is typical for a semiconductor like (In,Ga)As. We take the Rashba parameter λ in the range from 0 to 100 meV nm, easily obtained in experiments [31, 32], and we choose the electron concentration n_{2D} in a range between $3 \cdot 10^{11}$cm^{-2} and $1.3 \cdot 10^{12}$ cm^{-2}. The Fermi energy is obtained from the chosen carrier concentration assuming an infinite 2D gas. In the detection of spin
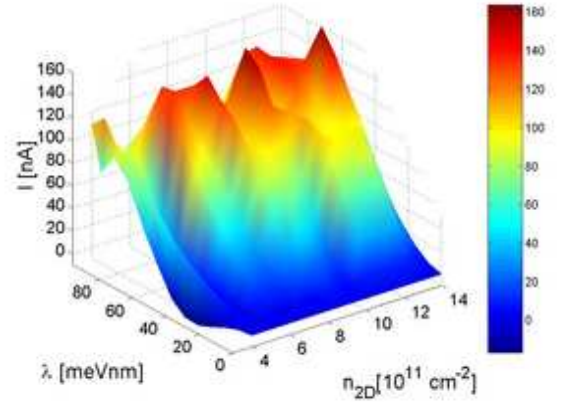

FIG. 4: The spin-force driven charge current as a function of hole concentration and Rashba coupling ,λ, for mesoscopic square sample 100nm by 100nm and μ =250,000 cm^2/Vs.

force driven charge-Hall effect the first task is to generate a spin force which can be realized by spin dependent chemical potential in the leads. The ferromagnetic leads are not the good candidates because although magnetization exists in ferromagnetic leads, the chemical potential is the same for both spin directions. Here, we propose the optical method of spin-dependent chemical potential generation by shining the beam of circularly polarized light on the leads (see Fig. 2). The right-circularly polarized beam shines on the right lead (lead 2 in Fig.1a) and the left circularly polarized beam on the left one (lead 1 in Fig.1a). The sample as well as the transverse leads should be covered by mask, preventing the light absorption anywhere except the small part of longitudinal leads as shown in Fig.2a. Choosing semiconductor leads for this setup e.g. GaAs, will cause the opposite spin polarizations in left and right leads through optical selection rules. Using the beam splitter should produce the same light intensity in each leads providing simultaneously the spin-dependent chemical potential between leads 1 and 2 and the total charge current across a sample equal zero. Having produced the spin-dependent chemi-

cal potential in the leads, we perform calculations using non-equilibrium Green function method presented in previous section. Fig.3 and Fig.4 present the charge current $I_3 = I_{3\uparrow}^s + I_{3\downarrow}^s$ (see sample configuration Fig. 1a) as a function of SO coupling λ and electron or hole concentrations, respectively. The charge current for electron systems show oscillations with respect to the electron density and SO coupling. The period of current oscillations depends on system size, however its maximal value seems to be around 40nA for systems sizes achievable in our calculations. For hole mesoscopic systems (see Fig.4), the charge current also oscillates with λ and for system sizes on the order of 100 nanometeres is on the order of hundred nano-ampers in a wide range of densities starting from $6 \cdot 10^{11} \text{cm}^{-2}$. Hence the charge current is much larger in hole systems and shall be detectable in experiments. Fig.5 presents the spin-Hall conductance G_{SC}^{yx} as
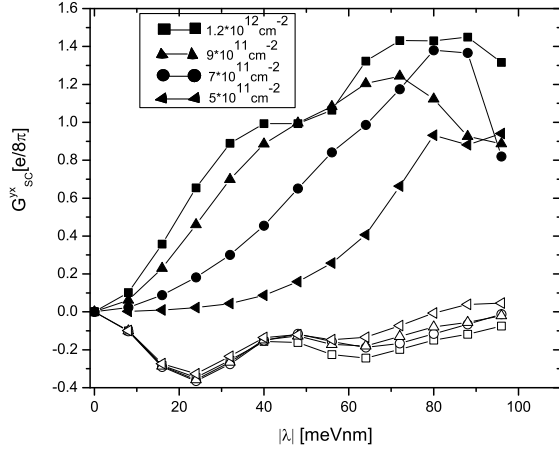

FIG. 5: The spin-Hall conductance as a function of SO coupling for 100nm by 100nm square system for different concentration of holes (close symbols) and electrons (open symbols).

a function of λ for electron and hole systems. One can see that spin-Hall conductance (Fig.5) and spin-force driven charge conductances (see Figs.3 and 4) behave similarly. Spin-Hall conductances for electron and hole systems oscillate with SO coupling. Moreover, G_{SC}^{yx} is several times larger for hole systems in comparison with electron ones, which is associated with much larger effective mass in a case of hole systems. Let us emphasize that for electron and hole systems with the same effective masses the spin-Hall effect can be larger for electron systems. Moreover, in mesoscopic systems where Fermi energy as well as multichannel effects are important the straightforward renormalization of effective mass of electron and hole systems to compare the spin-Hall conductance suggested by [27] does not have to be correct. Our calculations in mesoscopic systems are in agreement with the

linear response Kubo calculation for 2DEG and 2DHG which show that spin-conductance is much more larger for hole systems [2, 29]. The sign of spin-Hall and charge Hall conductances in mesoscopic systems depends on the Fermi energy.
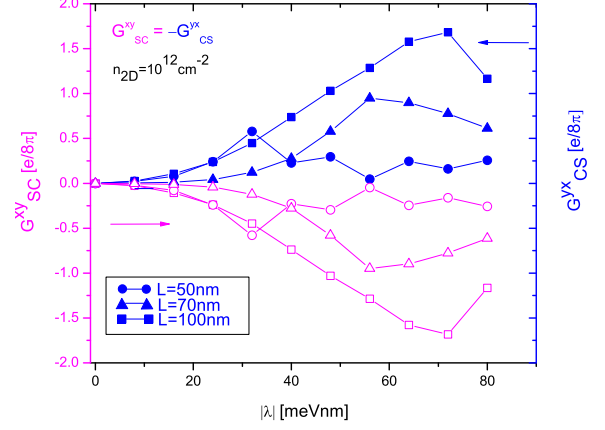

FIG. 6: Onsager relation between the spin-Hall conductance G_{SC}^{xy} (open symbols) and spin-force driven charge-Hall conductance G_{CS}^{yx} (close symbols) for different L by L hole systems.

The Onsager relations express the symmetry between the transport coefficients describing reciprocal processes in systems with a linear dependence of response to the driving forces. Within the models studied, considering disorder averaged conductances, the relations can be derived by utilizing the time reversal symmetry and the inversion symmetry on the x-y plane which relate $T_{\sigma n;\sigma m} = T_{\bar{\sigma} m;\bar{\alpha} n}$ and $T_{\sigma n;\alpha m} = T_{\sigma m;\alpha n}$ respectively, where σ, α and n, m are the spin and lead labels and a bar represents the opposite direction. Within the SHE $V_1^\sigma = V_2^\sigma = 0$ and $V_3^\sigma = -V_4^\sigma = V_0/2$ following the labels of Fig. 1. Within the RSHE $V_3^\sigma = V_4^\sigma = 0$ and $V_1^\sigma = V_2^{\bar{\sigma}} = s(\sigma)V_0/2$ where $s(\uparrow) = +1$ and $s(\downarrow) = -1$. Within the LB formalism and these boundary conditions we obtain for the spin-current associated with the SHE $I_1^{spin} \equiv I_1^\uparrow - I_1^\downarrow = \frac{V_0}{2} \sum_{\sigma,\alpha} s(\sigma)(T_{\sigma 1;\alpha 3} - T_{\sigma 1;\alpha 4})$ and for the charge current associated with the RSHE $I_3^c \equiv I_3^\uparrow + I_3^\downarrow = \frac{V_0}{2} \sum_{\sigma,\alpha} s(\alpha)(T_{\sigma 3;\alpha 1} - T_{\sigma 3;\alpha 2})$. The above symmetries imply that $T_{\sigma 1;\alpha 4} = T_{\sigma 2;\alpha 3} = T_{\bar{\sigma} 3;\bar{\alpha} 2}$. This then yields $I_1^{spin} = -I_3^c$ which implies the Onsager relation

$$G_{SC}^{xy} = -G_{CS}^{yx} \qquad (4)$$

This is verified numerically in Fig.6 which presents the disorder averaged spin-Hall conductance and charge Hall conductance for hole systems of different sizes. For a specific disorder realization this relation does not hold and is only approximate depending on the strength of

the fluctuations induced by disorder. This relation between G_{SC} and G_{CS} is consistent with predictions of semiclassical wave-packet theory, where standard definition of spin-current was modified by a spin-torque term [28, 33]. However, as seen from the above derivation and noted in the introduction, our finding of an Onsager relations in mesoscopic coherent systems do not involve such spin-torque term since all spin-currents are defined in the non-spin-orbit coupled leads.

Summary. We have analyzed the spin Hall effect as well as its reciprocal effect in mesoscopic hole and electron systems. We have shown that the spin-Hall as well as the spin-dependent chemical potential gradient driven charge-Hall conductances are several times larger for hole systems. Further we have proposed the experimental setup to detect the transverse charge current driven by the spin-dependent chemical potential gradient through transport measurements. We have shown that this charge current is of the order of hundred nano-ampers in hole systems and should be detectable. Also, we have established a direct relation between the disorder average spin-Hall conductances and their reciprocal in mesoscopic systems.

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