Screening, nonadiabaticity, and quantized acoustoelectric current

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Quantized single-electron transport driven by surface acoustic waves (SAW) through a pinched-off narrow constriction is studied theoretically. Long-range Coulomb interaction causes the tunneling coupling between the two-dimensional electron gas (2DEG) and the moving minimum of the SAW-induced potential to decay rapidly with time. The energy scale $\hbar/\tau$, associated with the characteristic time of this decay τ, controls both the width of the transition regions between the plateaus and the slope of the plateaus. This sets a limit for the accuracy of the quantization of acoustoelectric current at low temperature.

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Recent experiments[1,2] demonstrated that surface acoustic waves (SAW) induce charge transport through a narrow constriction, formed in GaAs heterostructure by a split-gate depletion technique. In the pinch-off regime[1] the acoustoelectric current, as function of the potential at the gate electrode, exhibits plateaus, where

$$I_{ae} = N_0 e f. \tag{1}$$

Here f is SAW frequency, e is the electron charge, and N_0 is an integer. The plateaus were demonstrated to be stable over the range of temperature, SAW power, and applied source-drain voltage. The remarkable accuracy of the quantization (of the order of 10^{-5}), and high frequency of operation[3]

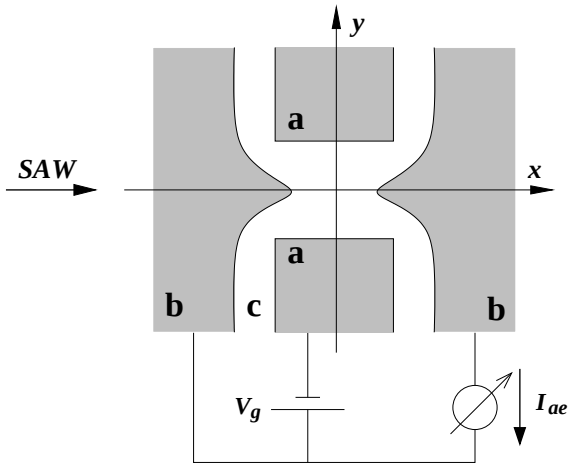

Fig. 1. SAW device: (a) split gates; (b) regions, occupied by two-dimensional electron gas; (c) depleted region. The arrow indicates direction of SAW propagation.

immediately suggest possible metrological applications of the effect as a tool for maintaining an independent current standard[4] . It is therefore important to the underlying physics of the effect, and, therefore, possible sources of errors.

Qualitatively, the effect is explained by a simple picture of moving quantum dots.[1] Electrons, trapped in the moving minima ('dots') of SAW-induced potential, are dragged through the potential barrier. The strong Coulomb repulsion prevents excess occupation of the dot. Increase of SAW power deepens the dots, more states in the dot become available for electrons to occupy, and new plateaus appear. The slope of the potential barrier can be lowered by changing the gate voltage, which has a similar effect.

In principle, quantization of the acoustoelectric current in one-dimensional channels is expected on the quite general theoretical ground.[5] However, the effect was not observed in the open channel regime.[2] The reason is that the mechanism of quantization[5] requires that the DC conductance for each instantaneous configuration of SAW-induced potential is zero. This can be achieved, if the channel is long (much longer than SAW wavelength λ_s), and the Fermi level lies in the gap of the spectrum. These conditions are difficult to realize experimentally.[2] In the alternative approach, put forward in,[1] the channel is biased beyond the pinch-off (see Fig. 1). This ensures that the DC conductance is zero. However, as shown below, the rapid change of SAW-induced potential near the entrance of the channel

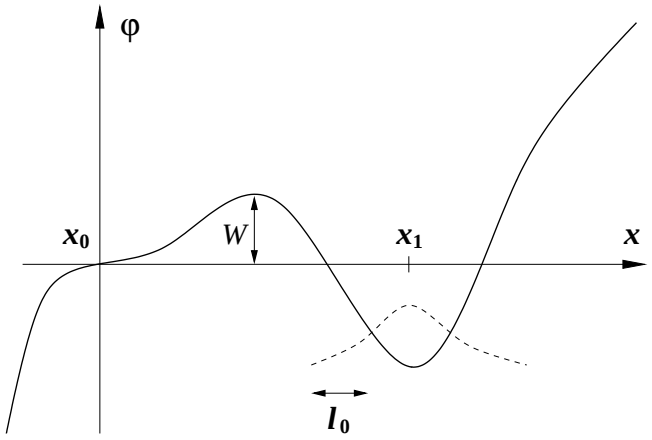

Fig. 2. The instantaneous electrostatic potential $\varphi(x, y)$ along the line $y = 0$. 2DEG is confined to the region $x < x_0$ (so that the line $\varphi = 0$ coincides with the Fermi level). The location x_1 of the SAW-induced potential minimum moves to the right with the sound velocity v_s. The dashed line shows the wave function of the trapped electron.

may lead to significant non-adiabatic corrections to (1).

Since the Fermi velocity v_s is much larger than the sound velocity v_s, the 2DEG is able to follow the changing in time SAW-induced potential. Therefore, the effective potential $\varphi(x, y)$, as seen by the electrons in the system, can be determined from a self-consistent solution of the instantaneous electrostatic problem, combined with the Thomas-Fermi-type relation between the density of the electron gas and φ. This problem is still extremely complex. However, important properties of the solution can be understood as follows. The crucial observation is that, since the screening length in 2DEG (~ 10 nm) is small compared to SAW wavelength ($\lambda_s \sim 1$ μm), the SAW-induced potential is screened almost completely in the regions, occupied by 2DEG (see Fig. 1,2). On the other hand, the screening in the depleted region is lacking. The SAW induce a potential minimum (the 'dot'), located near some point x_1 in the depleted region (see Fig. 2). As the system evolves (φ depends parametrically on time), x_1 moves to the right with the sound velocity v_s. At the same time, the position of the edge of 2DEG, x_0, is expected to oscillate with SAW frequency f. However, the amplitude of these oscillations is small (since the amplitude of the SAW-induced potential is definitely small, compared to V_g). Therefore, the width of the potential barrier, that separates 2DEG and the dot, grows linearly with time. As a result, the tunneling amplitude decays approximately exponentially with the

characteristic time $\tau \sim l_0/v_s$, where l_0 is the distance over which the localized wave function extends under the potential barrier (see Fig. 2). Since, evidently, $l_0 \ll \lambda_s$, the inequality

$$f\tau \ll 1 \tag{2}$$

holds. Due to this inequality, the time-dependence of all other parameters of the system can be neglected. For example, the change $\delta\varepsilon_0$ of the energy of the localized level ε_0 during the time τ can be estimated as

$$\delta\varepsilon_0 = \varepsilon_0\left(t+\tau\right) - \varepsilon_0\left(t\right) \sim \left(d\varepsilon_0/dt\right)\tau \sim \varepsilon_0 f\tau \ll \varepsilon_0,$$

and is negligibly small.

The rapid decay of the tunneling coupling results in nonadiabatic corrections to (1). These corrections are controlled by the energy scale $\hbar/\tau$.

To obtain an order of magnitude estimate of τ, we expand the potential $V\left(x\right)$, seen by the electrons, near the minimum x_1 (see Fig.2), we obtain $V\left(x\right) \approx A_s q_s^2 \left(x-x_1\right)^2/2$, where $q_s = 2\pi/\lambda_s$. The amplitude A_s is related to the single-particle level spacing in the dot Δ via

$$A_s q_s^2 = m^* \left(\Delta/\hbar\right)^2,$$

where m^* is an effective mass, to the 'size' r of the wave function of the localized electron via

$$A_s q_s^2 r^2 \sim \Delta,$$

and to the charging energy E_c via

$$E_c \sim e^2/\epsilon r.$$

l_0 can be estimated from the relation

$$\frac{\left(\hbar/l_0\right)^2}{2m^*} \sim W + \varepsilon_0,$$

where W is barrier's height, and ε_0 is the energy of the localized level. Assuming that $W + \varepsilon_0 \sim A_s$, this gives us 4 equations for 5 unknown quantities. Turning to the experiments, we note that the quantization disappears above the activation temperature $T^* \sim 10\,\mathrm{K}$, which we identify as the energy it takes to overcome the Coulomb barrier. Substituting $E_c \sim k_B T^* \sim 1\,\mathrm{meV}$, together with $m^* = 0.07\,m_0$ (m_0 is a free electron mass), $\epsilon \approx 13$, $\lambda_s = 10^{-4}\,\mathrm{cm}$ to the equations above, we obtain $A_s \sim 0.5\,\mathrm{meV}$, $r \sim 100\,\mathrm{nm}$, $\Delta \sim 0.1\,\mathrm{meV}$. Using $v_s = 3\times10^5\,\mathrm{cm/s}$, we finally arrive at $\tau \sim 10$ ps. Since the corresponding energy scale $\hbar/\tau \sim 0.1\,\mathrm{meV}$

is finite (although small), the nonadiabaticity may affect significantly the accuracy of the quantization at low temperature.

This can be understood from the following model[6] , in which the Coulomb repulsion in the dot is treated by introducing a single parameter - the charging energy E_c:

$$\begin{aligned} \mathcal{H} &= \sum_{k\sigma} \xi_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_{n\sigma} E_n d_{n\sigma}^\dagger d_{n\sigma} + E_c \left(N - \mathcal{N}_g\right)^2 \\ &\quad + V(t) \sum_{kn\sigma} \left(c_{k\sigma}^\dagger d_{n\sigma} + \text{H.c.} \right). \end{aligned} \tag{3}$$

Here ξ_k and E_n are the single-particle energy levels in the lead and in the dot correspondingly, E_c is charging energy, $N = \sum_{n\sigma} d_{n\sigma}^\dagger d_{n\sigma}$ is number of electrons in the dot, and $\mathcal{N}_g$ is a linear function of the gate voltage V_g. In writing the Hamiltonian (3), we have assumed that while the dot is in the vicinity of the left electrode, the tunneling to the right is negligible. We have also assumed that the electron gas is in thermodynamic equilibrium at all times (by virtue of the inequality $v_s \ll v_F$). According to the discussion above, we have taken into account the time-dependence of the tunneling coupling only,

$$V(t) = V_0 e^{-t/\tau}. \tag{4}$$

Our task is to calculate the occupation of the dot $N_0 = \langle N \rangle_{t=\infty}$ at $t \to \infty$, given that the system is in thermodynamic equilibrium at $t = -\infty$. The acoustoelectric current is related to N_0 via (1). The model (3) answers the minimal requirements: it produces the correct adiabatic limit ($\tau \to \infty$), reproducing the staircase-like dependence of I_{ae} on the gate voltage, while allowing explicitly for a non-equilibrium occupation of the dot.

The effect of the time-dependence of the tunnelling coupling depends on how close the system is to the Coulomb blockade degeneracy points (half-integer $\mathcal{N}_g$). Away from these points, when the inequality

$$2E_c \left| \mathcal{N}_g - n_0 - 1/2 \right| \gg \max\{T, 1/\tau\} \tag{5}$$

is satisfied, the time-dependence is too slow to cause the transitions between the charge states of the dot, and in this respect the adiabatic approximation is justified. Here n_0 is the integer part of $\mathcal{N}_g$ and we use $\hbar = k_B = 1$ for the remaining part of the text. The occupation of the dot is given by the standard Coulomb blockade expression, with the temperature replaced by the effective temperature $T_{eff} \sim \max\{T, 1/\tau\}$, and N_0 is integer, apart from the exponentially small corrections. Close to the degeneracy points, when the inequality (5) breaks down, the time-dependence of the tunneling coupling is fast enough to induce the transitions between the different charge

states of the dot. This immediately implies that the width of the transition region is given by T_{eff}.

To illustrate these conclusions, we consider the limit when $1/\tau \ll \Delta \ll E_c$, and neglect the spin of the electrons. This might seem to be an over-simplification, but the result of the calculations for the more general case is qualitatively the same. If we also restrict our attention to the region, which includes only one transition between the plateaus, $n_0 < \mathcal{N}_g < n_0 + 1$ is satisfied, we can take into account only two charge states of the dot, that with $N = n_0$, and that with $N = n_0+1$ electrons. Projected onto these states, the Hamiltonian is written in terms of the fermion operator $d = |n_0\rangle \langle n_0 + 1|$:

$$\mathcal{H} = \sum_k \xi_k c_k^\dagger c_k + E_0 d^\dagger d + V(t) \sum_k \left(c_k^\dagger d + \text{H.c.} \right), \tag{6}$$

where

$$E_0 = 2E_c \left(1/2 + n_0 - \mathcal{N}_g\right).$$

The simplified model (6) has an advantage of being exactly solvable for arbitrary $V(t)$. The solution can be obtained in various ways. One possibility is to use the equation-of-motion technique[7] to derive a first order differential equation for

$$\langle N(t) \rangle = n_0 + \left\langle d^\dagger(t) d(t) \right\rangle,$$

which results in the exact expression for N_0 in the form of the functional of the time-dependent level width $\Gamma(t) = 2\pi\nu V^2(t)$, where ν is density of states at the Fermi level. With $V(t)$ given by (4), the result is written in the compact form

$$N_0 - n_0 = \frac{\tau_0}{2\pi} \int_{-\infty}^{\infty} d\omega \frac{n_F(\omega)}{\cosh\left[\pi\left(\omega - E_0\right)\tau_0\right]}, \tag{7}$$

where n_F is the Fermi function, and $\tau_0 = \pi\tau/2$. At $T = 0$ it reduces to

$$N_0 - n_0 = \frac{2}{\pi} \tan^{-1}\left(e^{-E_0\tau_0}\right)$$

The result (7) is described very well by the Fermi function

$$N_0 - n_0 \approx \left(e^{E_0/T_{eff}} + 1\right)^{-1}, \; T_{eff} = \sqrt{T^2 + (c/\tau_0)^2}. \tag{8}$$

We found that $c = 0.88$ gives the best numerical fit.

Close to the middle of the plateau ($\mathcal{N}_g \to n_0$), Eqs. (1) and (8) give the following expression for the plateau's slope:

$$S = \frac{1}{I_0}\left(\frac{dI_{ae}}{d\mathcal{N}_g}\right)_{\mathcal{N}_g \to n_0} \approx \left(2E_c/T_{eff}\right) e^{-E_c/T_{eff}}, \tag{9}$$

where $I_0 = n_0 e f$ corresponds to the ideal quantization. Strictly speaking, the two-sate approximation is not sufficient to obtain the correct value of S precisely at the middle of the plateau: the state with $N = n_0 - 1$ makes exactly the same contribution, as that with $N = n_0 + 1$. This complication, however, does not affect significantly the validity of (9). The exact result is expected to differ from (9) by the numerical factor of the order of 1 only. This can be seen by considering the high temperature limit $T \gg 1/\tau$ where one easily obtains the relation between the two-state and three-state approximation results $S_3/S_2 = 2$.

To conclude, we argued in this paper that nonadiabatic effects may limit the accuracy of the quantization of the current, driven by surface acoustic waves. The time-dependence can be described effectively by a single parameter τ, the decay time of the tunneling coupling between 2DEG and moving minimum of SAW-induced potential. The corresponding energy scale $\hbar/\tau$ controls both the slope of the plateaus, and the width of the transition region between the plateaus.

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REFERENCES

1. J. M. Shilton et al., *J. Phys. Condens. Matter* **8**, L531 (1996); see also J. Cunnungham et al. in this issue, and references therein.
2. J. M. Shilton et al., *J. Phys. Condens. Matter* **8**, L337 (1996).
3. For $f \sim 3$ GHz, $N_0 = 1$, Eq.(1) gives $I_{ae} \sim 1$ nA , while other devices, such as single-electron pumps, are bound to operate at low frequencies of the order of 1 MHz, and, therefore, deliver quantized currents in the pA range.
4. K. Flensberg et al., *Int. J. Mod. Phys. B* **13**, 2651 (1999).
5. D. J. Thouless, *Phys. Rev. B* **27**, 6083 (1983); Q. Niu, *Phys. Rev. Lett.* **64**, 1812 (1990); see also B. L. Altshuler and L. I. Glazman, *Science* **283**, 1864 (1999) for a recent discussion.
6. Time-independent version of this model is widely used in the theory of the Coulomb blockade (see the review by L. I. Glazman in this issue and references therein).
7. A.-P. Jauho, N. Wingreen, and Y. Meir, *Phys. Rev. B* **50**, 5528 (1994).