Entanglement Generation and Multiparticle Interferometry with Neutral Atoms

Artem M. Dudarev,[1,2] Roberto B. Diener,[1] Biao Wu,[1,3] Mark G. Raizen,[1,2] and Qian Niu[1]

[1]*Department of Physics, The University of Texas, Austin, Texas 78712-1081, USA*

[2]*Center for Nonlinear Dynamics, The University of Texas, Austin, Texas 78712-1081, USA*

[3]*Condensed Matter Sciences Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831-6032, USA*

(Received 11 December 2002; published 1 July 2003)

We study the preparation and manipulation of states involving a small number of interacting particles. By controlling the splitting and fusing of potential wells, we show how to interconvert Mott-insulator-like and trapped BEC-like states. We also discuss the generation of "Schrödinger cat" states by splitting a microtrap and taking into practical consideration the asymmetry between the resulting wells. These schemes can be used to perform multiparticle interferometry with neutral atoms, where interference effects can be observed only when all the participating particles are measured.

DOI: 10.1103/PhysRevLett.91.010402 PACS numbers: 03.75.Be, 03.65.Ud, 03.67.Mn, 03.75.Dg

Entanglement is at the root of Bell's theorem, which exposes the differences between quantum theory and a local classical theory based on elements of reality [1]. The predictions of quantum mechanics have been experimentally observed with entangled Einstein-Podolsky-Rosen (EPR) pairs [2,3] as well as Greenberger-Horne-Zeilinger triples [4]. A related consequence of entanglement is the possibility of multiparticle interferometry. Given a maximally entangled system of N particles (a "Schrödinger cat" state) a measurement of interference between different parts of the wave function corresponding to a single particle yields random results. It is only when performing a coincidence measurement on all N particles that an interference pattern is revealed [5]. Experimental confirmation of this result has been obtained using photonic EPR pairs [3,6] and internal states of four ions in the same trap [7], but no experiments have been performed using a larger number of particles. The latest generation of experiments with photons rely on parametric down-conversion, which has the technical disadvantage of an exponentially decreasing number of useful counts as N increases. Given that entanglement is the key ingredient in all quantum computation and quantum communication schemes, clean experimental studies of its consequences have become an active topic of research in the last decade.

In recent years several papers [8,9] have suggested the generation of entanglement between neutral atoms confined in traps by using their interaction in controlled atomic collisions. The atoms are guided in their motion and their evolution yields the required entanglement of internal states. Other schemes to achieve this sort of entanglement starting from Bose-Einstein condensates (BECs) have been suggested [10]. In this Letter we present two general N-atom nonlinear processes. The first one is used to convert a Mott-insulator-like (MI) state [11] into a state with all particles in the (many-body) ground state of a single trap (BEC-like state); its reverse process converts the BEC state into a MI state. The second process is used to generate a Schrödinger cat state starting from a BEC state by controlling the splitting of the well. As an application of the processes we discuss a scheme for multiparticle interferometry with spatially separated paths.

In the first process, which is also stage I of the interferometry setup, we start with a collection of N atoms in the ground states of N independent traps (MI state). These separate atoms can be extracted from a reservoir using a quantum tweezer [12] recently proposed by our group. Alternatively, single atoms stored and detected in micro-optical traps (which have been experimentally reported [13] but are in excited states of the trap) can be Raman cooled individually to the ground state. The BEC state (stage I, Fig. 1) is achieved by bringing together the N wells adiabatically if the interaction between atoms is repulsive, as is shown in detail below. This is a consequence of the quantum adiabatic theorem, since the MI state is the ground state when the wells are far apart. The evolution is then represented by

$$|w_1 w_2 \dots w_N\rangle \rightarrow |\Psi_{\mathrm{I}}\rangle = |ww \dots w\rangle, \tag{1}$$

where the states are properly symmetrized bosonic states.

In the second process (stage II of multiparticle interferometry) the interaction is switched to attractive. This can be done by using a Feshbach resonance [14]. Starting from the BEC state, we slowly split the well into two approximately equal mictrotraps, which we label as L and R. The lowest energy states are then the ones having all atoms in the left or in the right well. Since initially the system is in the ground state, by separating the traps at some slow rate v when the wells are far apart we get a linear combination of these two nearly degenerate states; i.e., the system is in the Schrödinger cat state

$$|\Psi_{\mathrm{I}}\rangle \rightarrow |\Psi_{\mathrm{II}}\rangle = \alpha|LL \dots L\rangle + \beta e^{i\theta}|RR \dots R\rangle, \tag{2}$$

with α, β, and θ real. For perfectly symmetric traps, $\alpha = \beta$ and $\theta = 0$, but any asymmetry makes these parameters rate dependent, as is discussed in detail below.

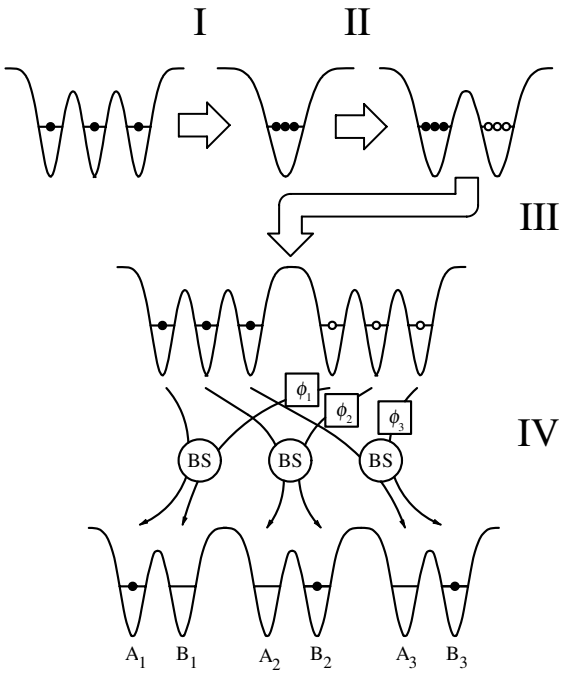

FIG. 1. Schematics of the multiparticle interferometry procedure. Stage I: creation of N atoms in the ground state of the trap starting with N individual atoms in N traps. Stage II: creation of the Schrödinger cat state. Stage III: spatial separation of the atoms. Stage IV: applying phases, combining on the beam splitters and measurement.

Additional processes are needed to realize multiparticle interferometry. During stage III, the interaction is switched back to repulsive and each of the two traps is separated to N wells. This stage can be seen as the inverse of stage I applied to the wells L and R. Again, if the separation is done adiabatically the system remains in the ground state which in this case corresponds to a single atom in each one of the wells. The state is now

$$|\Psi_{\mathrm{II}}\rangle \rightarrow |\Psi_{\mathrm{III}}\rangle = \alpha|L_1 \ldots L_N\rangle + \beta e^{i\theta}|R_1 \ldots R_N\rangle. \quad (3)$$

Subsequently, the atoms in wells derived from the original R well are subjected to additional phase shifts $\phi_1 \ldots \phi_N$, which can be applied, for example, by adjusting the depth of the wells adiabatically.

In the final stage, IV, of the scheme, we combine states L_i and R_i in a 50-50 beam splitter [15]. Notice that in the experiment only one of these two is occupied so the interatomic interaction plays no role in this stage. We denote the outputs of each beam splitter by A_i and B_i and assign a value of $+1$ to the measurement of an atom in channel A_i, and -1 to the measurement of atom in channel B_i. The probability, $P(+1)$, that the product of all measurements gives $+1$ (for instance $A_1B_2B_3$ in the case of three atoms) is $[1 - 2\alpha\beta\cos(\Delta + \theta)]/2$, where $\Delta = \sum_{i=1}^{N} \phi_i$. The probability for the product to be -1 is $P(-1) = 1 - P(+1)$; hence the expectation value over a large number of measurements is $-2\alpha\beta\cos(\Delta + \theta)$. We stress that a correlated measurement of less than N atoms does not show any dependence on phase and appears random.

In order to obtain the relevant parameters for the operation of our inteferometer, we study the evolution of an N particle system using optical microtraps. As an example, we numerically solve the Schrödinger equation in the case of three atoms in a quasi-1D configuration. This is achieved by strongly trapping the atoms in the perpendicular dimensions, effectively freezing these degrees of freedom. We scale the equations choosing units of length $L_u = 2\ \mu$m, of energy $E_u = \hbar^2/(2M_uL_u^2)$, and of time $t_u = \hbar/E_u$. The particle interaction is represented by a delta-function potential

$$U(x_1, x_2) = U_0\delta(x_1 - x_2). \quad (4)$$

The atoms are also subject to external potentials due to the optical traps, which in each stage are

$$V_{\mathrm{I,\,III}}(x, d) = \sum_{i=1}^{3}(1 + q_i)V(x, (i - 2)d),$$
$$V_{\mathrm{II}}(x, d) = \sum_{i=1}^{2}(1 + q_i)V(x, (i - 3/2)d), \quad (5)$$

with

$$V(x, d) = -V_0 \exp\left(-\frac{(x + d)^2}{2\sigma^2}\right). \quad (6)$$

The q_i parametrize the asymmetry between the intensities of the beams defining the different wells; we assume that these are of the order of 10^{-4}.

Let us consider first the evolution during the first and the third stages of the operation. There are four different energy scales in the problem. The first one is the energy difference between the energy levels localized in different wells, which we can estimate as $E_{\mathrm{asym}} \approx qV_0$. The second one is the energy required to move one of the atoms to an already occupied well, estimated to be $E_{\mathrm{int}} \approx U_0/\sigma_0$, where $\sigma_0 = (V_0/\sigma^2)^{1/4}$ is the width of the wave function in a well. The third scale is the energy $E_{\mathrm{exc}} \approx \sigma_0^{-2}$ required to put one of the atoms in an excited state of one of the traps. The last energy scale $E_D \approx (\pi/ND)^2$ is the energy required to excite the atoms out of the ground state when the distance between the wells is $D \approx 2\sigma$, at which time the trap can be approximated by a square well of width ND. We operate in the regime in which

$$E_{\mathrm{asym}} \ll E_{\mathrm{int}}, E_{\mathrm{exc}}, E_D. \quad (7)$$

Figure 2 shows the dependence of the adiabatic levels on the separation d during this stage. The presence of the small asymmetry in this stage does not affect the nature of the ground state, which is nondegenerate. Joining or separating the wells at a slow speed keeps the system in the ground state, i.e., the lowest curve in the figure. We can estimate the rate at which the adiabaticity is lost

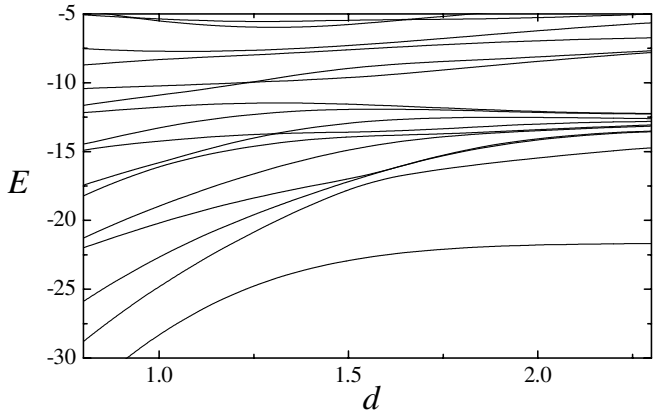

FIG. 2. Stage I: adiabatic energy levels for three atoms in three wells with repulsive interaction as a function of d. The other parameters of the potential are $V_0 = 10$, $\sigma = 0.5$, $U_0 = 10$, $q_3 = -q_1 = 10^{-4}$, $q_2 = 0$.

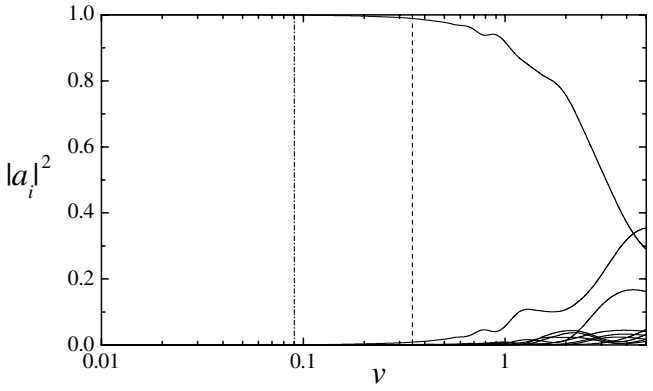

FIG. 3. Stage III: probabilities to find the system in the adiabatic states after a single well with three atoms is split into three wells with an atom per well ($d_{\text{final}} = 3.0$) as a function of the speed v. The energy levels are the ones shown in Fig. 2. For velocities smaller than denoted with the dashed line the probability to state in the ground state is larger than 0.99; for velocities larger than denoted with the dash-dotted line dephasing is less than 0.1. For stage I the dynamics are very similar except there is no limit on how slow the process could be done.

by applying the Landau-Zener formula [16], $v_{\text{ad}} \approx (\Delta E_{\text{gap}})^2/(dE/dx)$. The slope can be estimated as $\sqrt{NV_0}/\sigma^2$. The size of the gap depends on which of the three large energy scales in (7) is the smallest. In the example that we are presenting, all three are roughly the same order of magnitude. The probabilities $|a_i|^2 = |\langle\psi_i|\psi\rangle|^2$ to find the system in the states $|\psi_i\rangle$ at the end of the evolution is plotted in Fig. 3 as a function of the speed v. In our example, the critical rate is $v_{c\text{I},2} = 0.35$; the probability to find the system in other states is less than 0.01. For multiparticle interferometry it is critical not to accumulate an additional phase during the third process due to the asymmetry between the right and left set of wells. This gives rise to a lower bound for the allowed velocity, as explained below. For the parameters chosen in the figure this is $v_{c\text{I},1} = 0.09$.

Between these stages and stage I we need to change the sign of the effective interaction between the particles. For the cases we are considering, the particles remain the ground state with very high probability ($\sim 99\%$) even if this change is performed suddenly.

The adiabatic energy levels during stage II as a function of d are shown in Fig. 4; corresponding transition probabilities are plotted in Fig. 5. In this case we also have four energy scales, which can be approximated by $E_{\text{asym}} \approx NqV_0$, $E_{\text{int}} \approx (N-1)|U_0|/\sigma_0$, $E_{\text{exc}} \approx \sigma_0^{-2}$, and $E_D \approx (\pi/2D)^2$. Once again, we work in the regime in which (7) is valid. Separating the wells adiabatically maintains the system in the ground state and corresponds to all N atoms placed in the lowest of the two wells, which is not the desired state. In order to mix the lowest two energy states we need to evolve the system nonadiabatically with respect to the lowest gap but at a slow enough speed to remain adiabatic with respect to the larger gap. Below $v_{c\text{II},2} = 0.27$ the probability to tunnel to these excited states is less than 0.01 and entanglement is obtained with $2\alpha\beta = 0.99$ or larger. On the other hand, the asymmetry yields a dephasing between the two parts of the wave function $\theta = E_{\text{asym}} t_{\text{sep}}$, where the separation time is inversely proportional to the velocity v. Allowing a maximum dephasing $\phi_{\max}$, we must go faster than $v_{c\text{I},1} \approx qV_{0,\text{III}}N/\phi_{\max}$. This calculation assumes, however, that the asymmetry is constant. In a practical situation, q is driven by fluctuations in the laser power, and consequently the phase θ grows diffusively, as the square root of t_{sep} instead of linearly, making the condition less restrictive.

The only two conditions for the applicability of the method are related to the asymmetry of the potential. As long as condition (7) is met and as long as $v_{c,2}$ is larger than $v_{c,1}$, there is a range of velocities for which the operation is successful. The critical velocities have

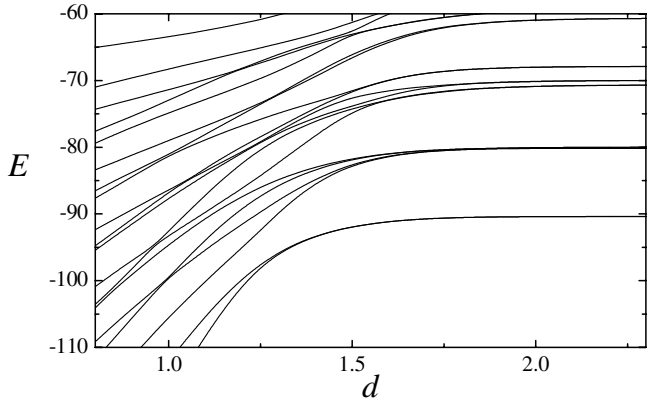

FIG. 4. Stage II: adiabatic levels of three atoms in two wells in the case of attractive interaction for different values of the separation d. The other parameters of the potential are $V_0 = 30$, $\sigma = 0.5$, $U_0 = -4$, $q_1 = 0$, $q_2 = 10^{-4}$.

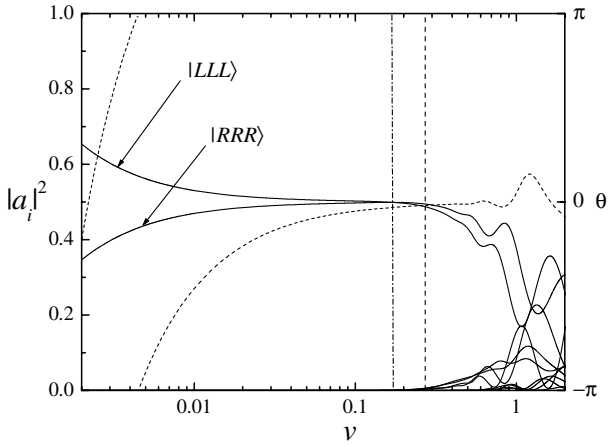

FIG. 5. Stage II: full lines are probabilities to find the system in the adiabatic states after the separation of one well with three atoms to two ($d_{\text{final}} = 3.0$) as a function of the speed v. The dashed line is θ. The interaction is attractive and the parameters are the ones used in Fig. 4. For velocities in the interval between vertical lines the desired state is prepared with probability of 0.99 and dephasing smaller than 0.1.

different dependence on N, so for fixed values of the parameters defining the potential and the interaction, there is a largest number of atoms for which this happens. However, by choosing a different set of parameters this condition can be relaxed. In particular, the fact that $E_D \propto N^{-2}$ during the preparation of the MI state can be overcome by separating the atoms in series instead of doing this in parallel (for $N = 2^n$, we can think of n steps in which each well is split into two).

Finally, numerical values for realistic experimental parameters are given. In the model described above the effective interaction between atoms is determined by the scattering length a and the strength of the confinement in the transverse direction. The frequency $\omega_\perp$ of the confinement in the case in which the system stays in the ground state of transverse motion may be expressed in terms of the dimensionless interaction parameter U_0 used above [8] as

$$\omega_\perp = \frac{U_0 \hbar}{4|a|M_u L_u}. \tag{8}$$

Hence it is desirable to use atoms with the largest product of mass and scattering length possible. In Table I we present the rescaled values used in the calculation for two workhorses of cold atom experiments, sodium and rubidium. The magnetic fields needed to observe Feshbach resonances in alkali atoms are typically hundreds of gauss [14]. In the proposed scheme for multiparticle interferometry one should work on the side of the resonance where the scattering length changes sign to avoid the losses associated with crossing the resonance.

TABLE I. Parameters of the numerical estimates in dimensional units. For the estimates we take scattering lengths of ^{23}Na $a_t = 65a_0$ and ^{87}Rb $a_t = 106a_0$ in triplet states with no magnetic field with a_0 being the Bohr radius [17]; we assume that near a Feshbach resonance the values will be of the same order of magnitude.

Parameter	Na	Rb	Units
$\omega_\perp$	79.9	13.0	2π kHz
$v_{c\mathrm{I},1}$	62.2	16.5	μm/s
$v_{c\mathrm{I},2}$	242	64.0	μm/s
$v_{c\mathrm{II},1}$	117	31.1	μm/s
$v_{c\mathrm{II},2}$	186	49.4	μm/s
$V_{0,\mathrm{II}}$	2.47	0.665	$h \times$ kHz

We acknowledge support from the NSF and from the R. A. Welch Foundation; M. G. R. also acknowledges support from the Sid W. Richardson Foundation.

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