

**Physics 362K, Fall 2004**  
**Review problems for Test 1**

1. If the charge  $Ze$  inside a nucleus is uniformly distributed in a sphere of radius  $R$ , the potential acting on an electron is

$$V(r) = \begin{cases} \frac{Ze^2}{8\pi\epsilon_0 R} \left( \frac{r^2}{R^2} - 3 \right), & r < R \\ -\frac{Ze^2}{4\pi\epsilon_0 r}, & r > R \end{cases}$$

- a) By what amount  $H'$  does this potential differ from a pure Coulomb potential  $V(r) = -Ze^2/4\pi\epsilon_0 r$ ?
- b) If the nucleus binds a single electron, show that the shift that  $H'$  produces in the ground state energy is

$$E^{(1)} = \frac{4}{5} \frac{Z^2 e^2}{8\pi\epsilon_0 a_0} \left( \frac{ZR}{a_0} \right)^2 \left[ 1 - \frac{5ZR}{6a_0} + \frac{3}{7} \left( \frac{ZR}{a_0} \right)^2 + \dots \right]$$

where  $a_0$  is the Bohr radius for a hydrogen atom. This shift is an example of the “nuclear volume effect”. It is one of the effects that cause the energy levels of different isotopes of the same element to differ slightly.

2. A particle of mass  $m$  accelerates towards the floor with gravitational acceleration  $g$ , and undergoes elastic reflections from the floor. Use the trial wavefunction  $\psi(x) = A \exp(-x/a)$  to estimate the energy of the quantum ground state of the particle. (*note added*: this wavefunction doesn't work. See solutions. Modify this problem by using the trial wavefunction  $\psi(x) = A x \exp(-x/a)$ .)

3. An electron in the  $n = 3$  state of hydrogen has orbital angular momentum  $\ell=2$ , and spin angular momentum  $s = 1/2$ . Its quantum states can be expressed in terms of the basis states  $|n, \ell, s, j, m_j\rangle$  or in terms of the basis states  $|n, \ell, s, m_\ell, m_s\rangle$ .

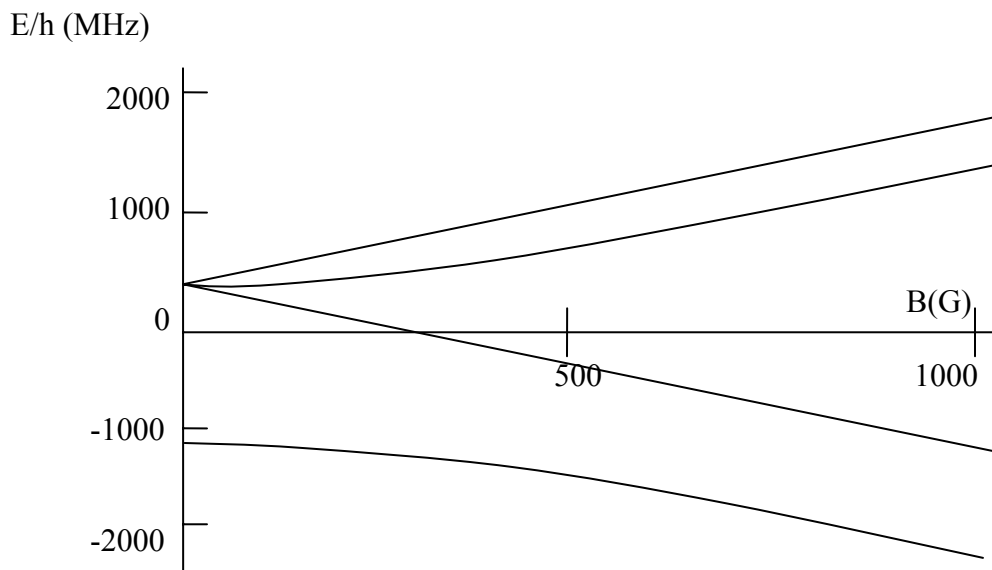
- a) List all possible states  $|n, \ell, s, j, m_j\rangle$ . Give the designation for each of these states in spectroscopic notation.
- b) Draw an energy level diagram of these states which takes into account both  $H^{(0)}$  and  $H_{fs}$ , where  $H^{(0)} = \mathbf{p}^2/2m_e - e^2/4\pi\epsilon_0 r$  and  $H_{fs}$  is the fine structure Hamiltonian (i.e. lowest order relativistic corrections to  $H^{(0)}$ ).
- c) List all possible states  $|n, \ell, s, m_\ell, m_s\rangle$ .
- d) Using table 4.7 in Griffiths, write the state  $|n=3, \ell=2, s=1/2, j=3/2, m_j=1/2\rangle$  as a superposition of the  $|n, \ell, s, m_\ell, m_s\rangle$  states.

4. A hydrogen atom is in the state  $|\psi\rangle = |n=3, \ell=2, s=1/2, j=3/2, m_j=1/2\rangle$ .
- What are the possible outcomes of a measurement of the energy  $H$  of the atom, and what are the probabilities of each outcome?
  - What is the expectation value of the magnitude squared  $|\mathbf{L}|^2$  of the orbital angular momentum of the electron?
  - What are the possible outcomes of a measurement of the z-component  $L_z$  of the orbital angular momentum of the electron, and what are the probabilities of each outcome?
  - What is the expectation value of the z-component  $S_z$  of the spin angular momentum of the electron?
  - Find the probability that a measurement of the electron's position would yield a result within the volume element  $d^3r$  located at position  $\mathbf{r}$ .

5. The Hamiltonian of a hydrogen atom in its 1s state and placed in a magnetic field  $\mathbf{B} = B \hat{z}$  can be written as

$$H = H^{(0)} + A\mathbf{I}\cdot\mathbf{S} - g_e\mu_B\mathbf{S}\cdot\mathbf{B} - g_p\mu_n\mathbf{I}\cdot\mathbf{B}/\hbar$$

where  $A$  is the hyperfine interaction constant,  $\mathbf{S}$  and  $\mathbf{I}$  are the electron and proton spins, and  $g_e$  and  $g_p$  are the electron and proton g-factors, respectively. When this Hamiltonian is diagonalized, the following (approximate) diagram results for the energies as a function of  $B$ .



This is known as a Breit-Rabi diagram. Denote the total spin by  $\mathbf{F} = \mathbf{I} + \mathbf{S}$ .

- What are the “good” quantum numbers at low magnetic field? At high magnetic field? What quantum numbers are good at all magnetic fields? Justify your answers.
- Label the state energies on this graph with their good quantum numbers. Justify your answers.