Quantum Zeno Paradox: Survival and Decay[1]

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Outline

Quantum Zeno effect: The slowing down of time evolution of a quantum system due to repeated measurements.

- Unstable quantum system
- Evidence of "Equilibrium time" in hadron-nucleus collisions
- Zeno effect seen by a moving observer

Work of E.C.G.Sudarshan in collaboration with Misra, Valanju and myself on Quantum Zeno effect.

I. Unstable Quantum System

Unstable State: $|M\rangle$ at t=0. Survival amplitude:

$$a(t) = < M|e^{-iHt}|M > = \int_{\lambda_{th}}^{\infty} d\lambda \rho(\lambda) e^{-i\lambda t}$$

Energy spectrum: $\rho(\lambda) = < M|\lambda >< \lambda|M >.$

Survival probability: $Q(t) = |a(t)|^2.$

$$\dot{Q}(0) = \dot{a}^*(0)a(0) + \dot{a}(0)a^*(0) = 0.$$

Fourier integral implies: $\dot{a}^*(0) = -\dot{a}(0).$

Proposal: Quantum Zeno paradox

Misra & Sudarshan(77)

Monitor unstable quantum system over time T in T/N-intervals. In large N limit:

$$Q(T) = Q(T/N)^N \to 1.$$

- Repeated measurements will prolong survival.
- Large N limit: 100% survival, or nondecay

Analysis on Zeno Time

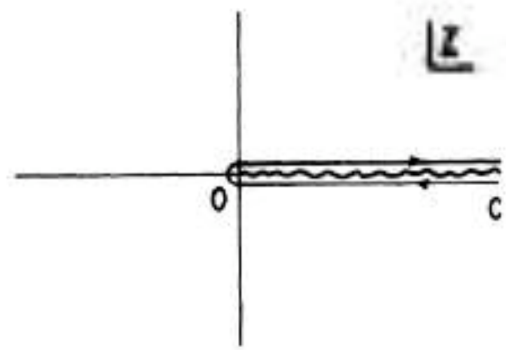
Chiu, Sudarshan & Misra (77)

Lee model:

- Basis states: $|V>$ continuum $|N\theta(E)>$
- V decays into $N\theta$, H_{int} assumed.

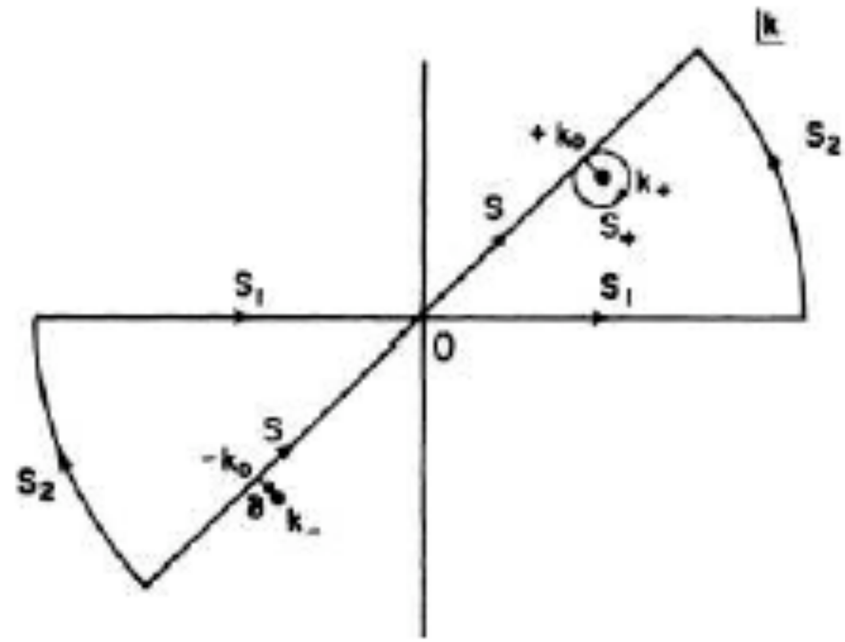
Survival Amplitude of V:

$$a(t) \equiv < V|e^{-iHt}|V> = \frac{i}{2\pi}\int_C \frac{e^{-izt}}{\gamma(z)}dz.$$



Complex k-plane: Equivalent contours

$$k = [ze^{i\pi/2}]^{1/2}$$



$S = S_+ + S_1 + S_2$. $S_2 = 0$, for $t \geq 0$.

$a(t) = a_{pole}(t) + a_{bg}(t)$, with $a_{pole}(t) \sim e^{-i(E_0 - i\Gamma/2)t}$.

Cancellation near t=0

Survival amplitude: $a(t) = a_{pole}(t) + a_{bg}(t)$

- Recall: $\dot{a}^*(0) = -\dot{a}(0)$, or $Re\,\dot{a}(0) = 0.$
- Pole term: $Re\,\dot{a}_{pole}(0) = -\Gamma/2$
- Cancellation between pole term & bg-term

Results based on Lee Model:

- Cancellation occurs in: $0 < t < T_1$, $T_1 \sim 1/(E_0 - E_{th})$
- Beyond this region the background terms fall off with $t^{-3/2}$ behavior, pole term begins to dominate.

Zeno time: Property of unstable quantum system

- The Zeno time: A delay-time up to $t=T_1$ before the pole term begins to dominate.
- Here $T_1 \sim 1/(E_0 - E_{th})$
- General nature of the expression T_1, suggests that Zeno time should be very common among unstable quantum systems.
- Among cases studied, for each case: $E_0-E_{th} >> \Gamma/2$, or Zeno time << Life time.

II. Evidence of the “equilibrium time” in hadron-nucleus Collision.

Valanju, Sudarshan & Chiu 80

- Emerging picture: Newly created quantum system is in a non-equilibrium state. “Equilibrium time”: the time for the system to reach equilibrium state is a Zeno time.
- Smallness of Zeno-time makes it difficult to demonstrate Zeno effect in unstable particle decay.
- Hadron-nucleus collision turns out to be a suitable process to demonstrate the Zeno effect.
- We followed the approach of Feinberg [66], to estimate the equilibrium time.
- Based on this equilibrium time, we demonstrate Zeno effect in pion productions from p-nucleus collisions.

Estimate of the equilibrium time

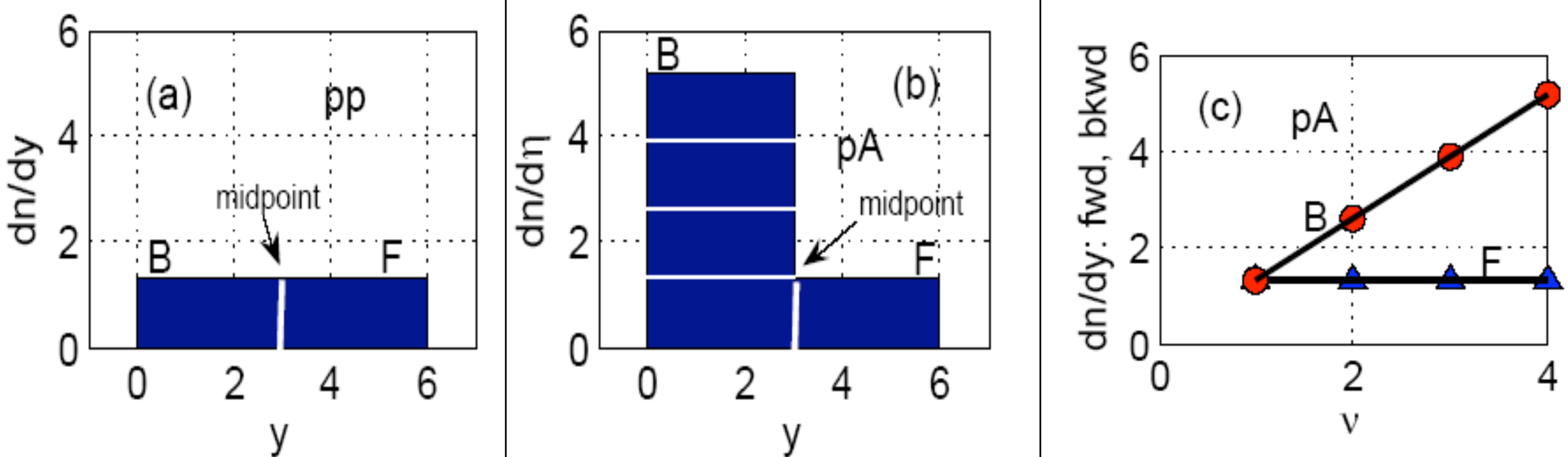
Electron bremsstrahlung process [Feinberg66].

- A large momentum change due to a precursor collision in the medium
- Subsequent radiation

After the collision, electron becomes “bare”. There is an equilibrium time (or survival time) before electron can emit a photon again. This time is $T_k = \gamma / k_0$.

- k_0 : Photon energy in rest frame of electron
- γ : The Lorentz factor of the electron in

Looking for "equilibrium time" effect in pp & pA collisions at high energies: Ideal scenario



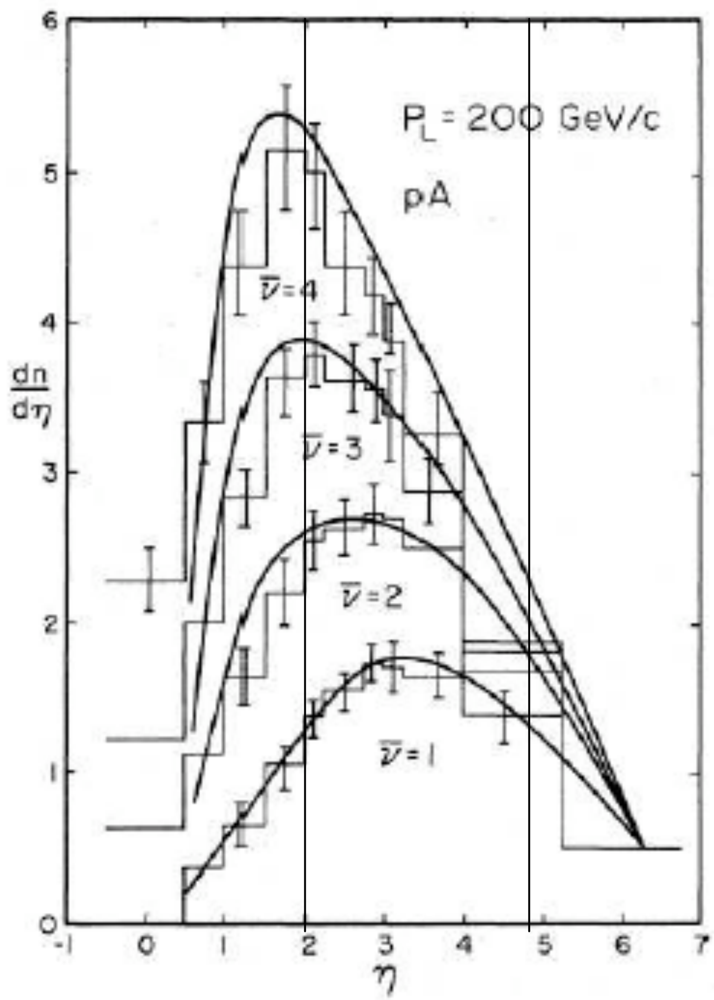
pp: B ~ F

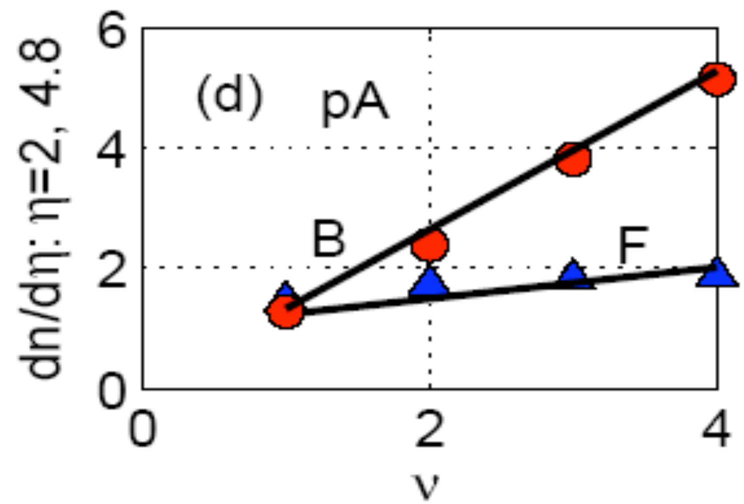
pA: B = ν F

pA: F –flat, B ∝ ν

Suppression of ν-dependent in forward spectrum is sensitive signal to detect equilibrium time effect.

Quantitative model: Comparison with data





- Our model uses equilibrium time $T_\pi \sim \gamma / M_{eff.}$ It reproduces main features of the data (Elias et al 77).
- Suppression of ν-dependence in forward data and our fits support equilibrium time interpretation.

III. Zeno effect seen by a moving observer*

Aharonov and Vardi (80): AV effect.

3 sets of spinor states

- A stationary state: $\psi_0 = |\sigma_x = +1 >$
- Rotating states: $\psi(t) = R_z(\theta(t))|\psi_0 >$, where $\theta(t) = \omega t$.
- Discrete fiducial states: $\mu(t_i) = \psi(t_i)$, with t_i=iT/N, i=1,2…N which track the rotation of $R_z(\theta(t))$

Invoke measurements on ψ_0 at t=t_i by $\mu(t_i)$, i=1.2..N.

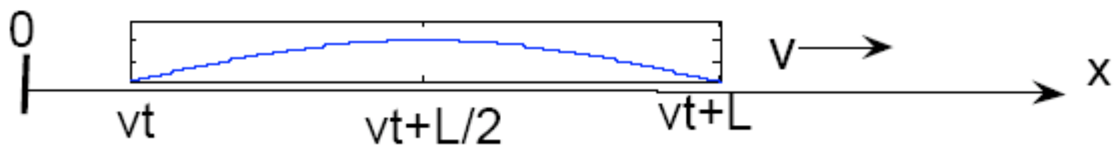
*Based on my recent discussions with George on the AV effect

AV effect & our objective

- In large N limit, $Q(T) = \Pi_i |< \mu_i|\psi_0 >|^2 \rightarrow 1$.
- AV effect: Quantum measurements cause the stationary spinor state to rotate in the same manner as the fiducial states do.
- Our approach:
  - Simulate the AV effect in an inertial frame.
  - We show this effect may be seen as Zeno effect by a moving observer.

A toy model

- Box-state:



$$\psi(x, vt) = f(x, vt), \text{ where } f(x, vt) = \sqrt{\frac{2}{L}} sin \frac{\pi(x - vt)}{L}.$$

- Fiducial states of measuring device (with velocity u):

$$\mu_i(x, ut_i) = f(x, ut_i).$$

where $t_i = iT/N$ with $i = 1, 2, \cdots$

Toy model -- cont'd

Survival amplitude at $t=t_i$

$$a(t_i) = \frac{2}{L}\int_{ut_i}^{L+ut_i} \mu_i(x, ut_i)\psi(x, vt) = cos\left[(v-u)\frac{t_i}{2}\right]$$

Effect of a quantum measurement:

- At the ith measurement, the box-state is collapsed to $\mu_i(x, ut_i)$ state.
- Assuming the new identity of $\mu_i(x, ut_i)$, the box-state continues to evolve.

Zeno effect case; v >0, u=0.

Survival amplitude at $t=t_i$: $a(t_i) = \cos(v\, t_i/2)$

Make N equal-time measurements over time T.
In large N limit,

$$a(T) = \left[1 - \frac{1}{2}\left(\frac{vT}{2N}\right)^2\right]^N \rightarrow e^{-\frac{(vT)^2}{8N}} \rightarrow 1$$

After invoking measurements, box becomes stationary relative to measuring device. The box is no-longer moving.--Zeno effect.

Case with AV effect: v=0, u>0.

- Survival amplitude at $t=t_i$, is given by $\cos(ut_i/2)$.
- Again invoke equal-time N measurements over time T.
- In large N limit, one obtains a(T)=1.

The overlap between the measuring state and the box state is unity. In other words, the box-state is now co-moving with the measuring device. --- AV effect.

Same case: v=0 & u>0, seen by a moving observer

Consider an observer co-moving with the measuring device.

- Without measurements: The box-state is moving backward with velocity -u.
- With measurements: backward moving box is now stopped --- Zeno effect.
- So AV effect is another manifestation of Zeno effect.

Summary

Quantum Zeno effect is a manifestation of general properties of nature (QM system+Hamiltonian). It is up to us to identify such effect. We have looked at 3 examples here.

- ***Unstable quantum system***. Model study shows there is the Zeno time associated with the decay of unstable particle.
  $T_1 \sim 1/(E_0 - E_{th})$.
- ***Equilibrium time***: Each quantum system is created in a non-equilibrium state. Some finite time is required to reach equilibrium. This is a Zeno time. Pion production in pp and pA collisions provides evidence for existence of such time scale.
- ***AV effect***: Quantum measurement causes the collapse of a quantum state to the fiducial state. When fiducial states are gradually varying, the quantum state follows along. This is AV effect, which is a Zeno effect seen by a moving observer.