

# A Spacecraft Voyage: Energy, Angular Momentum, and a little Chaos

## OBJECTIVES

In this activity you will study the dynamics of a spacecraft in orbit around the Earth. You will visualize energy and angular momentum to help understand the system's behavior. When the moon is taken into account, you will see how the spacecraft's orbit is affected. You will

- calculate and graph the kinetic energy, potential energy, and the sum of K+U as a function of time
- calculate and graph the angular momentum as a function of time
- display arrow objects representing the position, momentum, and angular momentum of the spacecraft

Using the VIDLE editor, open the shell file, “Model5\_Shell.py” and save it on your computer as “Model5.py”.

## BACKGROUND

Orbital motion around the Earth is due to the universal law of gravitation:

$$\vec{F}_{Earth} = -\frac{GM_E m_c}{r_{Ec}^2} \hat{r}_{Ec}$$

where the subscript “Ec” stands for “Earth to craft”. Integrating this force and setting potential energy equal to zero at  $r \rightarrow \infty$ ,

$$U_g = -\frac{GMm}{r}$$

If the spacecraft is very far from the Earth, its total energy,  $E=K+U$ , will be greater than or equal to zero. Since energy is conserved, a negative total energy can never escape and hence represents a bound system.

$$\begin{aligned} E \geq 0 &\Rightarrow \text{Unbound System} \\ E < 0 &\Rightarrow \text{Bound System} \end{aligned}$$

We will see examples of both types of systems in this model.

The angular momentum of the craft relative to the Earth is given by its radius crossed with its momentum:

$$\vec{L}_E = \vec{r}_{Ec} \times \vec{p}_c$$

This quantity can be changed by the application of a torque:

$$\frac{d\vec{L}}{dt} = \vec{\tau} = \vec{r}_{Ec} \times \vec{F}_g$$

We will consider when gravity can and cannot put a torque on the craft.

## THE PROGRAM

### 1. Forces

As usual, our program is an iterative calculation of motion. In the main time loop, we calculate the force on the craft due to the Earth. The position vector from Earth to craft is defined in the program as `rEc`, with associated variables `rEcmag` and `rEchat` for the distance and unit vector, respectively.

Using these variables, complete the line of code that calculates the force `FEarth` on the craft.

The next step in the iteration calculates the force of the moon on the craft, `Fmoon`, using the position vector from Moon to craft, `rMc`. This has already been done for you. The momentum principle and position update for the craft then follow, with two if statements checking to see if the craft has crashed into the Earth or the Moon, whereupon the calculation terminates.

## 2. Angular Momentum

Now we must calculate the angular momentum of the craft relative to the Earth's position. Luckily, VPython can calculate the cross product between two vectors for us! There is a way to manually calculate the cross product using vector components, but we will leave the details to VPython.

The function `cross(a,b)` returns the cross product between two vectors `a` and `b`. Using the `cross()` function, calculate the angular momentum, `L`, of the craft about the center of the Earth. You may want to refer to the definition given above in the Background section. Make sure you use the correct displacement vector.

For the sake of comparison,  $|\vec{r}||\vec{p}|$  will be graphed alongside  $|\vec{L}|$  when you run the program.

## 3. Energy

Finally, we calculate the energy of the craft. Taking the Earth-Moon-craft as our system, we need to take into account the change in internal energy of the system in order to observe conservation of energy. This internal energy is in the form of gravitational potential energy between pairs of interacting objects. Strictly speaking, there are three pairs of objects ( ${}_3C_2=3$ ), but we will assume the potential energy between the Earth and the Moon is constant, so it can be neglected.

Complete the lines defining `UEarth` and `UMoon`, which are, respectively, the potential energy between the Earth and the craft, and the potential energy between the Moon and the craft. Make sure you use the correct masses and distances.

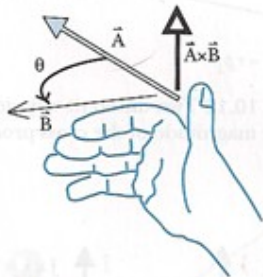
## 4. Reflection

At this point, you may begin answering the reflection questions for Model 5.

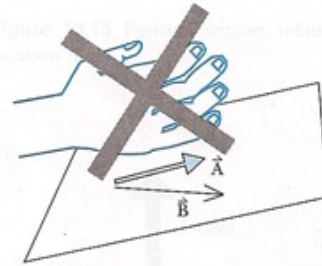
Remember that you only get one try per question on a reflection assignment, so be sure discuss with your group or an instructor before answering!



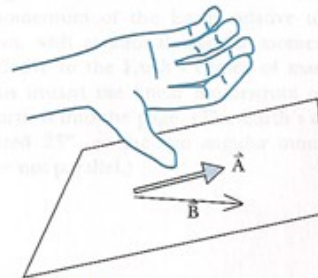
**Figure 10.8** Open right hand, fingers point in direction of  $\vec{A}$ .



**Figure 10.9** Bend fingers less than  $180^\circ$  toward lining up with  $\vec{B}$ . Thumb points in direction of  $\vec{A} \times \vec{B}$ .



**Figure 10.10** You can't bend your fingers backward. You must rotate the wrist into a position that lets you bend the fingers.



**Figure 10.11** After rotating the wrist, it is possible to bend the fingers.