

An object is released from rest at time $t = 0$ and falls through the air, which exerts a resistive force such that the acceleration a of the object is given by $a = g - bv$, where v is the object's speed and b is a constant.

If limiting cases for large and small values of t are considered, which of the following is a possible expression for the speed of the object as an explicit function of time?

Determine the average velocity and the average speed (in units of m/s) from $t = 0$ s to $t = 2$ s.

A) $v = gt - bt^2$

B) $v = \frac{ge^{bt}}{b}$

C) $v = \frac{(g+a)t}{b}$

D) $v = v_0 + gt, \quad v_0 \neq 0$

E) $v = \frac{g(1 - e^{-bt})}{b}$

What we have from the problem is that at time $t = 0$, the speed of the object is zero, and at time $t = \infty$, the acceleration is zero, corresponding to a speed $v = \frac{g}{b}$. Check the five choices, and it shows that the only possible answer is **E**.

Note: The answer can be directly obtained by integration.

$$a = g - bv \quad \Longrightarrow \quad \int_0^v \frac{dv}{\frac{b}{g}v - 1} = -g \int_0^t dt$$

$$\frac{g}{b} \ln \left(\frac{b}{g}v - 1 \right) \Big|_0^v = -gt \Big|_0^t \quad \Longrightarrow \quad v = \frac{g}{b}(1 - e^{-bt}).$$

Answer **E**.

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