

Jack has been walking for $12 \text{ min} \pm 3 \text{ min}$. She has been walking at a speed of $4 \text{ m/min} \pm 2 \text{ m/min}$.

What is the uncertainty Δd in the distance he has traveled,
 $d \pm \Delta d = (v \pm \Delta v)(t \pm \Delta t) = 48 \text{ m} \pm \Delta d$?

Note that $(4 \text{ m/min})(12 \text{ min}) = 48 \text{ m}$.

A) $\Delta d = 44 \text{ m}$

B) $\Delta d = 36 \text{ m}$

C) $\Delta d = 54 \text{ m}$

D) $\Delta d = 48 \text{ m}$

E) $\Delta d = 24 \text{ m}$

$$\begin{aligned}
d \pm \Delta d &= (v \pm \Delta v) (t \pm \Delta t), \quad \text{as given, so} \\
&= v t \left(1 \pm \frac{\Delta v}{v} \right) \left(1 \pm \frac{\Delta t}{t} \right) \\
&= d \pm (t \Delta v + v \Delta t + \Delta v \Delta t), \quad \text{which reduces to} \\
\Delta d &= t \Delta v + v \Delta t, \quad \text{since} \\
\Delta v \Delta t &\approx 0 \quad \text{in first order.}
\end{aligned}$$

Using calculus notation, we have the same result

$$\begin{aligned}
\Delta d &= \left| \frac{\partial d}{\partial v} \right| \Delta v + \left| \frac{\partial d}{\partial t} \right| \Delta t \\
&= |t| \Delta v + |v| \Delta t \\
&= (12 \text{ min}) (2 \text{ m/min}) + (4 \text{ m/min}) (3 \text{ min}) \\
&= (24 \text{ m}) + (12 \text{ m}) \\
&= \boxed{36 \text{ m}}, \quad \text{since}
\end{aligned}$$

$$\frac{\partial d}{\partial v} = \frac{\partial}{\partial v} v t = t = 12 \text{ min}, \quad \text{and}$$

$$\frac{\partial d}{\partial t} = \frac{\partial}{\partial t} v t = v = 4 \text{ m/min}.$$

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Answer **B**.