

Jill has been walking for $8 \text{ min} \pm 2 \text{ min}$. She has been gone a distance of $128 \text{ m} \pm 16 \text{ m}$.

What is the uncertainty Δv in her speed $v = \frac{d}{t} = \frac{(128 \text{ m})}{(8 \text{ min})} = 16 \text{ m/min}$?

A) $\Delta v = 2 \text{ m/min}$

B) $\Delta v = 3 \text{ m/min}$

C) $\Delta v = 4 \text{ m/min}$

D) $\Delta v = 5 \text{ m/min}$

E) $\Delta v = 6 \text{ m/min}$

A first-order approximation is

$$\begin{aligned}\Delta v &= \left| \frac{\partial v}{\partial t} \right| \Delta t + \left| \frac{\partial v}{\partial d} \right| \Delta d \\ &= \left| -\frac{d}{t^2} \right| \Delta t + \left| +\frac{1}{t} \right| \Delta d \\ &= \frac{(128 \text{ m})}{(8 \text{ min})^2} (2 \text{ min}) + \frac{1}{(8 \text{ min})} (16 \text{ m}) \\ &= (4 \text{ m/min}) + (2 \text{ m/min}) \\ &= \boxed{6 \text{ m/min}}, \quad \text{since} \\ \frac{\partial v}{\partial t} &= \frac{\partial d}{\partial t} \frac{1}{t} = -\frac{d}{t^2} = -\frac{(128 \text{ m})}{(8 \text{ min})^2}, \quad \text{and} \\ \frac{\partial v}{\partial d} &= \frac{\partial d}{\partial d} \frac{1}{t} = +\frac{1}{t} = +\frac{1}{(8 \text{ min})}.\end{aligned}$$

Answer **E**.

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