

Joice has been riding for $8 \text{ min} \pm 2 \text{ min}$. Joice started riding from rest.

She has been riding at a constant acceleration of $4 \text{ m/min}^2 \pm 1 \text{ m/min}^2$.

What is the uncertainty Δd in the distance

$$d = \frac{1}{2} a t^2 = \frac{1}{2} (4 \text{ m/min}^2) (8 \text{ min})^2 = 128 \text{ m she traveled?}$$

A) $\Delta d = 64 \text{ m/min}$

B) $\Delta d = 86 \text{ m/min}$

C) $\Delta d = 96 \text{ m/min}$

D) $\Delta d = 108 \text{ m/min}$

E) $\Delta d = 144 \text{ m/min}$

A first-order approximation is

$$\begin{aligned}\Delta d &= \left| \frac{\partial d}{\partial a} \right| \Delta a + \left| \frac{\partial d}{\partial t} \right| \Delta t \\ &= \frac{1}{2} t^2 \Delta a + a t \Delta t \\ &= \frac{1}{2} (8 \text{ min})^2 (1 \text{ m/min}^2) + (4 \text{ m/min}^2) (8 \text{ min}) (2 \text{ min}) \\ &= (32 \text{ m}) + (64 \text{ m}) \\ &= 96 \text{ m/min},\end{aligned}$$

since

$$\begin{aligned}\frac{\partial d}{\partial a} &= \frac{\partial}{\partial a} \left(\frac{1}{2} a t^2 \right) = +\frac{1}{2} t^2, \quad \text{and} \\ \frac{\partial d}{\partial t} &= \frac{\partial}{\partial t} \left(\frac{1}{2} a t^2 \right) = +a t.\end{aligned}$$

Answer bf C.

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