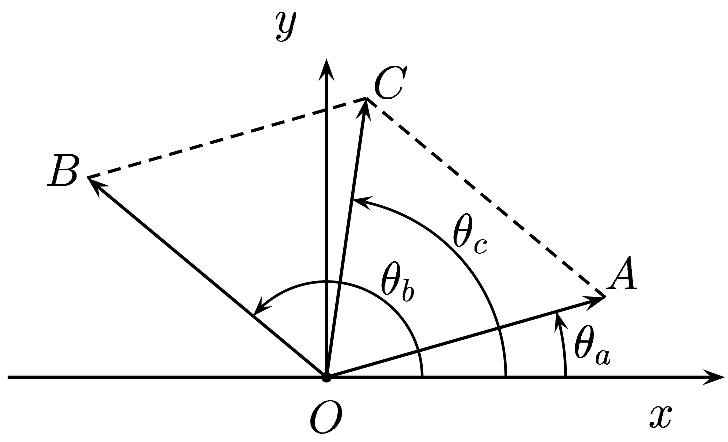
Using the definition of the scalar product of two vectors,



Find the square of the length $\overline{OC}$.

A) $\overline{OC}^2 = \overline{OA}^2 + \overline{OB}^2 - 2\,\overline{OA}\ \overline{OB}\cos\theta_c$

B) $\overline{OC}^2 = \overline{OA}^2 + \overline{OB}^2 + 2\,\overline{OA}\ \overline{OB}\cos\theta_c$

C) $\overline{OC}^2 = \overline{OA}^2 + \overline{OB}^2 - 2\,\overline{OA}\ \overline{OB}\cos(\theta_b - \theta_a)$

D) $\overline{OC}^2 = \overline{OA}^2 + \overline{OB}^2 + 2\,\overline{OA}\ \overline{OB}\cos(\theta_b - \theta_a)$

$\overline{OC}^2 = |\ \overline{OA} + \overline{OB}\ |^2 = \overline{OA}^2 + \overline{OB}^2 + 2\ \overline{OA}\ \ \overline{OB} \cos(\theta_b - \theta_a)$, where $(\theta_b - \theta_a)$ is the angle between $\overline{OA}$ and $\overline{OB}$.

If you look at the triangle $\triangle OAC$ the sides are $\overline{OA}$, $\overline{OB}$, and $\overline{OC}$. The included angle between sides $\overline{OA}$ and $\overline{OB}$ is $(180° - \theta_b + \theta_a)$, since the four included angles in the parallelogram $OACB$ add to 360°.

Note that, $\cos(180° - \alpha) = \cos(180°)\cos(\alpha)$ and $\cos(180°) = -1$. Then $\cos(180° - \theta_b + \theta_a) = -\cos(\theta_b - \theta_a)$, and we have

$$\overline{OC}^2 = \overline{OA}^2 + \overline{OB}^2 - 2\ \overline{OA}\ \ \overline{OB} \cos(180° - \theta_b + \theta_a)\,.$$

This is the proof of the law of cosines.

Answer **D**.

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