



A uniform rod of length  $L$  and mass  $m$  is free to rotate about  $O$ . The rod is released from rest in the horizontal position (state A). Define state B to be when the rod passes the vertical position.

Determine kinetic energy released, from A to B. Denote the angular velocity of the rod at B be  $\omega$ .

- A)  $K_B = \frac{m L^2 \omega^2}{6}$  and  $U_A - U_B = \frac{m g L}{2}$ .
- B)  $K_B = \frac{m L^2 \omega^2}{24}$  and  $U_A - U_B = \frac{m g L}{2}$ .
- C)  $K_B = \frac{m L^2 \omega^2}{6}$  and  $U_A - U_B = m g L$ .
- D)  $K_B = \frac{m L^2 \omega^2}{24}$  and  $U_A - U_B = m g L$ .

$$U_A - U_B = K_B - K_A = K_B.$$

The amount of the potential energy which the rod has released in going from A to B is given by

$$U_A - U_B = \int g y d m = \int g y \frac{d m}{d y} d y = \frac{g m}{L} \int y d y.$$

Integrating  $y$  from 0 to  $L$  gives

$$U_A - U_B = \frac{m g L}{2}.$$

The kinetic energy of the rod at the state B is

$$K_B = \frac{I \omega^2}{2} = \frac{m L^2}{3} \frac{\omega^2}{2} = \frac{m L^2 \omega^2}{6}.$$

Answer **A**.

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