Consider "Simple Harmonic Motion" (SHM) along the x-axis about the origin.

$$x = A\,\cos\left(\,\omega\,t + \phi\,\right)$$

and

$$v = -A\,\omega\,\sin\left(\omega\,t + \phi\,\right).$$

At $t = 0$, $x = x_0$, and $v = v_0$, select the correct expression for the amplitude A.

A) $A = x_0\,.$

B) $A = \dfrac{\omega}{v_0}\,.$

C) $A = \sqrt{x_0^2 + \left(\dfrac{v_0}{\omega}\right)^2}\,.$

D) $A = \sqrt{x_0^2 + \left(\dfrac{\omega}{v_0}\right)^2}\,.$

This is the situation of two equations and two unknown.

Using $\cos^2\phi = \sin^2\phi = 1$, one may eliminate ϕ.
In particular,

$$(A \cos\phi)^2 + (A \sin\phi)^2 = x_0^2 + \left(\frac{v_0}{\omega} \right)^2 = A^2 ,$$

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega} \right)^2} .$$

Answer **C** .

13.01-01˙SHM:˙From˙x0,v0˙to˙A˙and˙phi 2005-4-19