

Given: $k \approx 9 \times 10^9 \text{ N m}^2/\text{C}^2$.

A charge of $162 \text{ C} \pm 18 \text{ C}$ is at a distance of $9 \text{ m} \pm 3 \text{ m}$ from the origin.

What is the uncertainty ΔE in the magnitude of the electric field

$$E = k \frac{q}{r^2} = (9 \times 10^9 \text{ N m}^2/\text{C}^2) \frac{(162 \text{ C})}{(9 \text{ m})^2} = 18 \times 10^9 \text{ N/C}, \text{ at the origin?}$$

A) $\Delta E = 9 \times 10^9 \text{ N/C}$

B) $\Delta E = 12 \times 10^9 \text{ N/C}$

C) $\Delta E = 14 \times 10^9 \text{ N/C}$

D) $\Delta E = 16 \times 10^9 \text{ N/C}$

E) $\Delta E = 24 \times 10^9 \text{ N/C}$

A first-order approximation is

$$\begin{aligned}\Delta E &= \left| \frac{\partial E}{\partial q} \right| \Delta q + \left| \frac{\partial E}{\partial r} \right| \Delta r \\ &= k_e \frac{1}{r^2} \Delta q + 2 k_e \frac{q}{r^3} \Delta r \\ &= (9 \times 10^9 \text{ N m}^2/\text{C}^2) \frac{1}{(9 \text{ m})^2} (18 \text{ C}) \\ &\quad + 2 (9 \times 10^9 \text{ N m}^2/\text{C}^2) \frac{(162 \text{ C})}{(9 \text{ m})^3} (3 \text{ m}) \\ &= (2 \times 10^9 \text{ N/C}^2) + (12 \times 10^9 \text{ N/C}^2) \\ &= 14 \times 10^9 \text{ N/C}^2 ,\end{aligned}$$

since

$$\begin{aligned}\frac{\partial E}{\partial q} &= \frac{\partial}{\partial q} \left(k_e \frac{q}{r^2} \right) = +k_e \frac{1}{r^2}, \quad \text{and} \\ \frac{\partial E}{\partial r} &= \frac{\partial}{\partial r} \left(k_e \frac{q}{r^2} \right) = -k_e 2 \frac{q}{r^3} .\end{aligned}$$

Answer bf C.

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