

Barbara started from rest. Barbara rode a distance of $128 \text{ m} \pm 32 \text{ m}$ in a time of $8 \text{ min} \pm 2 \text{ min}$ from her starting point.

What is the uncertainty Δa in her acceleration

$$a = \frac{2d}{t^2} = \frac{2(128 \text{ m})}{(8 \text{ min})^2} = 4 \text{ m/min}^2$$

- A) $\Delta a = 2 \text{ m/min}^2$
- B) $\Delta a = 3 \text{ m/min}^2$
- C) $\Delta a = 4 \text{ m/min}^2$
- D) $\Delta a = 5 \text{ m/min}^2$
- E) $\Delta a = 6 \text{ m/min}^2$

A first-order approximation is

$$\begin{aligned}\Delta a &= \left| \frac{\partial a}{\partial t} \right| \Delta t + \left| \frac{\partial a}{\partial d} \right| \Delta d \\ &= 4 \frac{d}{t^3} \Delta t + 2 \frac{1}{t^2} \Delta d \\ &= 4 \frac{(128 \text{ m})}{(8 \text{ min})^3} (2 \text{ min}) + 2 \frac{1}{(8 \text{ min})^2} (32 \text{ m}) \\ &= (2 \text{ m/min}^2) + (1 \text{ m/min}^2) \\ &= 3 \text{ m/min}^2,\end{aligned}$$

since

$$\begin{aligned}\frac{\partial a}{\partial d} &= \frac{\partial}{\partial d} \left(2 \frac{d}{t^2} \right) = +2 \frac{1}{t^2}, \quad \text{and} \\ \frac{\partial a}{\partial t} &= \frac{\partial}{\partial t} \left(2 \frac{d}{t^2} \right) = -4 \frac{d}{t^3}.\end{aligned}$$

Answer bf B.

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