An object is released from rest at time $t = 0$ and falls through the air, which exerts a resistive force such that the acceleration a of the object is given by $a = g - b\,v$, where v is the object's speed and b is a constant.

If limiting cases for large and small values of t are considered, which of the following is a possible expression for the speed of the object as an explicit function of time?

Determine the average velocity and the average speed (in units of m/s) from $t = 0$ s to $t = 2$ s.

A) $v = g\,t - b\,t^2$

B) $v = \dfrac{\left(g\,e^{bt}\right)}{b}$

C) $v = \dfrac{(g + a)\,t}{b}$

D) $v = v_0 + g\,t, \quad v_0 \neq 0$

E) $v = \dfrac{g\,\left(1 - e^{-bt}\right)}{b}$

What we have from the problem is that at time $t = 0$, the speed of the object is zero, and at time $t = \infty$, the acceleration is zero, corresponding to a speed $v = \dfrac{g}{b}$. Check the five choices, and it shows that the only possible answer is **E**.

Note: The answer can be directly obtained by integration.

$$a = g - b\,v \qquad \Longrightarrow \qquad \int_0^v \frac{dv}{\frac{b}{g}\,v - 1} = -g\int_0^t dt$$

$$\frac{g}{b}\,\ln\left(\frac{b}{g}\,v - 1\right)\Big|_0^v = -g\,t\,\Big|_0^t \qquad \Longrightarrow \qquad v = \frac{g}{b}(1 - e^{-b\,t})\,.$$

Answer **E**.

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