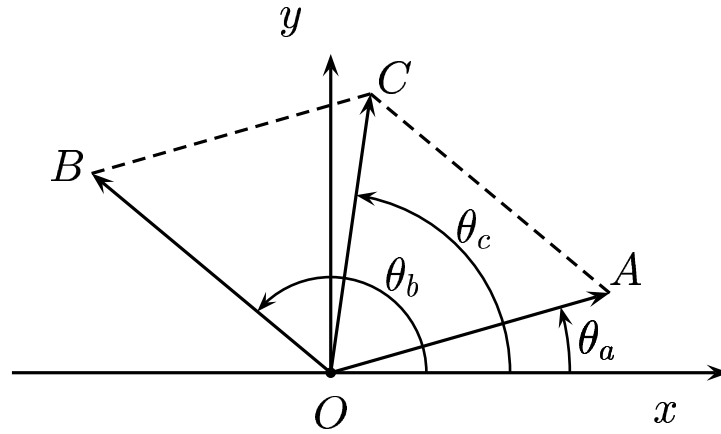


Using the definition of the scalar product of two vectors,



Find the square of the length \overline{OC} .

- A) $\overline{OC}^2 = \overline{OA}^2 + \overline{OB}^2 - 2 \overline{OA} \overline{OB} \cos \theta_c$
 B) $\overline{OC}^2 = \overline{OA}^2 + \overline{OB}^2 + 2 \overline{OA} \overline{OB} \cos \theta_c$
 C) $\overline{OC}^2 = \overline{OA}^2 + \overline{OB}^2 - 2 \overline{OA} \overline{OB} \cos(\theta_b - \theta_a)$
 D) $\overline{OC}^2 = \overline{OA}^2 + \overline{OB}^2 + 2 \overline{OA} \overline{OB} \cos(\theta_b - \theta_a)$

$\overline{OC}^2 = | \overline{OA} + \overline{OB} |^2 = \overline{OA}^2 + \overline{OB}^2 + 2 \overline{OA} \overline{OB} \cos(\theta_b - \theta_a)$, where $(\theta_b - \theta_a)$ is the angle between \overline{OA} and \overline{OB} .

If you look at the triangle $\triangle OAC$ the sides are \overline{OA} , \overline{OB} , and \overline{OC} . The included angle between sides \overline{OA} and \overline{OB} is $(180^\circ - \theta_b + \theta_a)$, since the four included angles in the parallelogram $OACB$ add to 360° .

Note that, $\cos(180^\circ - \alpha) = \cos(180^\circ) \cos(\alpha)$ and $\cos(180^\circ) = -1$. Then $\cos(180^\circ - \theta_b + \theta_a) = -\cos(\theta_b - \theta_a)$, and we have

$$\overline{OC}^2 = \overline{OA}^2 + \overline{OB}^2 - 2 \overline{OA} \overline{OB} \cos(180^\circ - \theta_b + \theta_a) .$$

This is the proof of the law of cosines.

Answer **D**.