



A siphon, which is a flexible tube with a circular cross section, is used to drain water from a tank, see sketch.

Assume:

1. A steady flow within the tube, at least from the water surface through the bend to the exit end.
2. No friction for the water.
3. The water cannot sustain a negative pressure.

What is the maximum vertical distance  $h$  between the top of the bend  $B$  and the exit end  $C$ , beyond which water flow is not possible?

- A)  $h = b - \frac{P_{atm}}{\rho g}$ .    B)  $h = \frac{P_{atm}}{\rho g} - b$ .    C)  $h = \frac{P_{atm}}{\rho g}$ .    D)  $h = \frac{P_{atm}}{\rho g} + b$ .

Apply Bernoulli's principle,  $P_A + \frac{1}{2} \rho v_A^2 + \rho g y_A = P_B + \frac{1}{2} \rho v_B^2 + \rho g y_B$ , to the height of the tube above the water's surface. Since  $v_A = v_B$ , this gives  $\rho g (y_B - y_A) = P_A - P_B$ . The maximum height above the water surface  $y_B - y_A \equiv h - b$  occurs at

$$\rho g (y_B - y_A) = P_A = P_{atm}, \quad \text{so} \quad h - b = \frac{P_{atm}}{\rho g}, \quad \text{or} \quad h = \frac{P_{atm}}{\rho g} + b.$$

The vertical drop within the  $BC$  section is longer than the maximum height  $\frac{P_{atm}}{\rho g}$  air pressure can sustain. Since non-turbulent water flow is continuous, the water flow within the  $AC$  section will pull away from the sides of the tube; *i.e.*, the diameter of the water stream will be reduced,  $v_A d_A^2 = v_C d_C^2$ , thus  $v_C > v_A$  (similar to a water stream falling from an open faucet). Water's surface tension reduces the probability of bubbling and turbulence.

Answer **D**