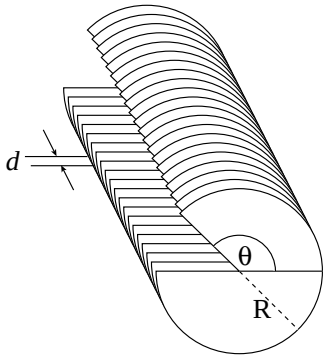
A variable air capacitor used in tuning circuits is made of N semicircular plates each of radius R and positioned d from each other. A second identical set of plates that is free to rotate is enmeshed with the first set.



Determine the capacitance as a function of the angle of rotation θ , where $\theta = 0$ corresponds to the maximum capacitance.

A) $C = \dfrac{\epsilon_0 \, N \, R^2 \, \theta}{d}$

B) $C = \dfrac{\epsilon_0 \, (2\,N) \, R^2 \, \theta}{d}$

C) $C = \dfrac{\epsilon_0 \, N \, R^2 \, (\pi - \theta)}{d}$

D) $C = \dfrac{\epsilon_0 \, (2\,N - 1) \, R^2 \, (\pi - \theta)}{d}$

Considering the situation of $\theta = 0$, the two sets of semicircular plates in fact form $2\,N - 1$ capacitors connected parallel, with each one having capacitance

$$C = \frac{\epsilon_0 \, A}{d/2} = \frac{\epsilon_0 \, \dfrac{\pi \, R^2}{2}}{d/2} = \frac{\epsilon_0 \, \pi \, R^2}{d} \, .$$

So the total capacitance would be $(2\,N - 1)\,\dfrac{\epsilon_0 \, \pi \, R^2}{d}$. *Note:* The common area of the two sets of plates varies linearly when one set is rotating, so the capacitance at angle θ is

$$C = \frac{\epsilon_0 \, (2\,N - 1) \, R^2 \, (\pi - \theta)}{d} \, .$$

Answer **D**.

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