Minimal field requirement in precessional magnetization switching

Di Xiao,[a] M. Tsoi, and Qian Niu
Department of Physics, The University of Texas at Austin, Austin, Texas 78712

(Received 9 May 2005; accepted 29 November 2005; published online 10 January 2006)

We investigate the minimal field strength in precessional magnetization switching using the Landau-Lifshitz-Gilbert equation in undercritically damped systems. It is shown that precessional switching occurs when localized trajectories in phase space become unlocalized upon application of field pulses. By studying the evolution of the phase space, we obtain the analytical expression of the critical switching field in the limit of small damping for a magnetic object with biaxial anisotropy in both the easy and hard plane. We also calculate the switching times for the zero damping situation by numerical means. We show that applying the field along the medium axis is good for both small field and fast switching times. [DOI: 10.1063/1.2161421]

I. INTRODUCTION

Magnetization reversal in magnetic particles and thin films has been a continuous and growing topic in the past several decades, motivated by its great application potential in magnetic data storage and random access memories (RAM). Recent development of fabrication techniques has made it possible to produce nanometer-sized magnetic objects with well-controlled shape, structure, and chemical composition. Magnetic anisotropy of these objects, including shape anisotropy and magnetocrystalline anisotropy, makes the dynamic magnetization processes highly nonlinear. A thorough understanding of micromagnetic dynamics is thus desirable as many efforts have already been made in this field (for a review, see Refs. 1 and 2).

In the past, magnetization reversal has been realized by applying a magnetic-field pulse mainly antiparallel to the initial magnetization. It will then undergo multiple rotations around the local effective field to reach the final equilibrium direction. The critical switching field can be derived by studying the energy landscape of the system under the influence of the applied field. We assume magnetization in the $\hat{x}$ and $-\hat{x}$ directions gives the local energy minima, separated by an energy barrier. In order to switch the magnetization from one direction to the other, the applied field has to be strong enough to overcome the energy barrier. It was first discussed in the uniaxial anisotropy case in the pioneering work by Stoner and Wohlfarth,[3] and recently extended to nonuniaxial and three-dimensional cases.[4,5] Energy dissipation is necessary in this process for the system to move from one minimum to another. We call this type of reversal the Stoner-Wohlfarth (SW) type. Typical reversal times for such processes are of the order of nanoseconds.

Recently, an approach toward ultrafast magnetization switching by precessional motion was proposed and observed experimentally.[6–12] Instead of applying a field pulse antiparallel to the initial magnetization, a perpendicular field pulse is applied and induces a large angle precession. If the pulse is terminated at 180° angle of the precession, whose period is usually of the order of picoseconds, the magnetization is reversed. However, the effective reversal times could be several nanoseconds due to the decay time of residual magnetic precession ("ringing") if the magnetization does not precisely end up in the final equilibrium state. One can overcome this difficulty by fine-tuning the pulse parameters so that the magnetization will move along the so-called ballistic trajectory, eliminating the ringing effect.[10] Thereby, the fundamental ultrafast limit of field-induced magnetization reversal is reached. In contrast to the SW-type reversal, the precessional switching is so fast that dissipative effects can be neglected. In other words, the system energy is *conserved* and the problem can be formulated using the Hamiltonian equations.

Such a precessional switching opens a way to reduce not only the reversal time but also the field strength required for switching. As pointed out by early numerical calculations,[13,14] precessional magnetization switching can be observed for fields well below the Stoner-Wohlfarth limit. Later this result was recovered by considering the precessional switching as a result of bifurcation, the long-term behavior of the dissipative system.[9] Recently, a number of studies based on the analytical solution of the Landau-Lifshitz-Gilbert equation have been carried out, trying to understand the magnetization dynamics.[15–19] In these studies, minimal field requirement for specific configuration has been discussed. However, a complete map of the minimal field strength based on an analytical solution still remains an open question.[20]

In this paper, we investigate the minimal field strength required for precessional switching of a magnetic object with biaxial anisotropy. We use phase-space analysis as our main tool to tackle this problem. It turns out that the mechanism of the precessional switching is directly related to the evolution of the localized and nonlocalized trajectories in the phase space. Unlike the SW-type switching, where the fixed points and their movement in the phase space are the central objects to study, in the precessional switching the states move along equienergy curves and we need to follow their motion globally. The minimal (critical) field is obtained when the localized trajectories become unlocalized. By studying the phase-space evolution, we are able to obtain the analytical

[a]Electronic mail: dxiao@physics.utexas.edu

expression for the critical field. This result provides useful information on the design of field pulses for precessional switching based devices.

The paper is organized as follows. In Sec. II we present a simple model describing magnetization dynamics and define the central problem. Then in Sec. III we consider the zero damping situation in which the switching field is applied perpendicular to the easy axis. We also calculate the switching times for this configuration. In Sec. IV with the assumption of very small damping we study the applied field perpendicular to the hard axis configuration and compare our results with the standard SW model. Finally, the paper is summarized in Sec. V.

II. MODEL

The simplest micromagnetic model is the so-called macrospin model, i.e., the magnetization is uniform and displays collective dynamics. This model is valid for small object size (reaching the single domain limit) and low magnetic coercivity. The motion of magnetization $\boldsymbol{M}$ with a phenomenological damping is governed by the Landau-Lifshitz-Gilbert (LLG) equation. This equation will be used in the following dimensionless form:

$$\frac{d\boldsymbol{m}}{dt} = -\boldsymbol{m} \times \boldsymbol{h}_{\text{eff}} + \alpha \boldsymbol{m} \times \frac{d\boldsymbol{m}}{dt}. \quad (1)$$

Here $\boldsymbol{m}=\boldsymbol{M}/M_s$ is the magnetization unit vector, $\boldsymbol{h}_{\text{eff}}=\boldsymbol{H}_{\text{eff}}/M_s$ is the scaled effective field, time is measured in units of $(\gamma M_s)^{-1}$, M_s is the saturation magnetization, γ is the absolute value of the gyromagnetic ratio, and α is the dimensionless damping parameter.

We consider a very thin film in the x-y plane with an in-plane uniaxial anisotropy, taking x the easy axis of magnetization. The demagnetizing field factors are practically equal to zero in the film plane and one perpendicular to the film plane, respectively. The magnetic energy density is written

$$w(\boldsymbol{m},\boldsymbol{h}) = \frac{1}{2}m_z^2 - \frac{1}{2}Km_x^2 - \boldsymbol{h}\cdot\boldsymbol{m}, \quad (2)$$

where $K>0$ accounts for the scaled in-plane x-axis anisotropy and $\boldsymbol{h}$ denotes applied field. Usually the value of K is about 0.01 for thin films because of the large demagnetizing field in the $\hat{z}$ direction. However, with proper scaling the above expression can be also used to describe the energy density of small magnetic particles with biaxial anisotropy.

Let us consider the energy dissipation rate. Recall that $\boldsymbol{h}_{\text{eff}}=-\partial w/\partial\boldsymbol{m}$. We find out the dissipation rate has the following form:

$$\frac{dw}{dt} = \frac{\partial w}{\partial \boldsymbol{m}}\cdot\frac{d\boldsymbol{m}}{dt} = -\frac{\alpha}{1+\alpha^2}|\boldsymbol{h}_{\text{eff}}\times\boldsymbol{m}|^2. \quad (3)$$

For small damping parameter α, short pulse period, or small applied field, the damping term in the LLG equation (1) can be neglected. We note that in real experiments the damping parameter is usually of the order of 10^{-2}, and the field pulse duration can be adjusted to hundreds of picoseconds, providing a very good example of this *zero* damping model. The dynamics is described by the following equation:

$$\frac{\partial\boldsymbol{m}}{\partial t} = -\boldsymbol{m}\times\boldsymbol{h}_{\text{eff}}. \quad (4)$$

In terms of the polar angle θ and the azimuthal angle ϕ, the energy density reads

$$w = \frac{1}{2}\cos^2\theta - \frac{1}{2}K\sin^2\theta\cos^2\phi - h_x\sin\theta\cos\phi - h_y\sin\theta\sin\phi - h_z\cos\theta, \quad (5)$$

and the equations of motion are

$$\dot{\theta} = -\frac{1}{\sin\theta}\frac{\partial w}{\partial\phi}, \quad \dot{\phi} = \frac{1}{\sin\theta}\frac{\partial w}{\partial\theta}. \quad (6)$$

For simplicity, we consider field pulses with zero rise and fall times. That is, the applied field remains constant for a pulse duration of T. We then define the problem as follows. Assume initially the magnetization is along the $+\hat{x}$ direction at $\theta=\pi/2$, $\phi=0$, denoted by $\boldsymbol{m}_0$. By applying a field pulse, we expect the magnetization to switch to the $-\hat{x}$ direction at $\theta=\pi/2$, $\phi=\pi$, denoted by $\boldsymbol{m}_1$. In order to achieve this, both field strength and pulse duration have to satisfy certain requirements. In this paper, we will focus on the field strength requirement.

III. ZERO DAMPING: SWITCHING FIELD IN THE *Y-Z* PLANE

We first study the $\alpha=0$ case. In this case, the magnetization cannot relax to a local minimum by dissipating energy, therefore the initial state $\boldsymbol{m}_0$ and the final state $\boldsymbol{m}_1$ must be connected by a constant energy curve during the application of the field pulse. This requirement leads to $\boldsymbol{h}\cdot(\boldsymbol{m}_1-\boldsymbol{m}_0)=0$; the applied field must be in the y-z plane.

A useful method to study the dynamics is phase-space analysis. In our problem, the phase space is obtained by mapping the constant energy curves on the unit sphere (the magnitude of $\boldsymbol{m}$ is a constant) to a 2D plane with the lines $\theta=0$, π corresponding to the north and south pole of the sphere. The phase space without applied field is shown in Fig. 1(a). The system has (a) two minima at $\theta=\pi/2$, $\phi=0$, π; (b) two maxima at $\theta=0$, π (ϕ has no definition); and (c) two saddle points at $\theta=\pi/2$, $\phi=\pm\pi/2$. The phase space is divided by the separatrix (trajectories that join the saddle points) into four regions. Inside the separatrix there are two regions, where the motion is periodic and localized around the corresponding minima. In a dissipative system, these two regions are called basin of attraction for these two minima. Outside the separatrix, in the top and bottom regions, the motion is unlocalized, i.e., the magnetization can move from the $+\hat{x}$ direction to the $-\hat{x}$ direction. Generally speaking, in a precessional switching the function of the applied field is to tilt the phase-space structure and to drag the initial state $\boldsymbol{m}_0$ out of the localized region so that the magnetization can precess globally.

To better illustrate this idea, we consider two special cases in which the applied field is along the $\hat{z}$ direction and the $\hat{y}$ direction, respectively.

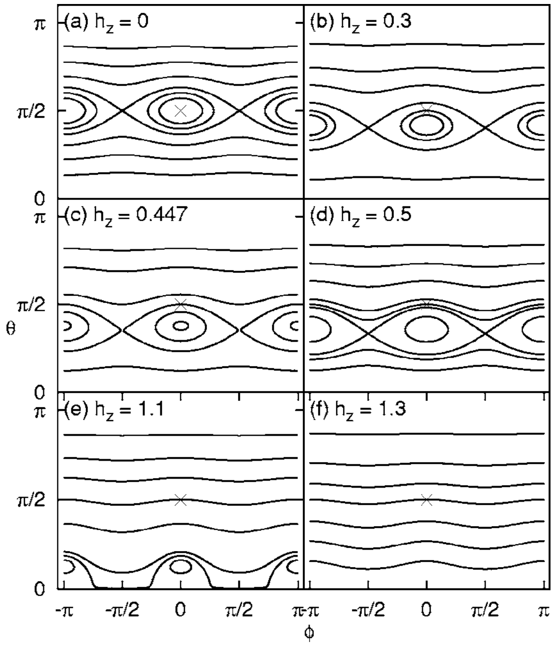

FIG. 1. The phase space at $K=0.2$ for different h_z. Point X denotes the initial state $\boldsymbol{m}_0$. In (a) $h_z=0$, $\boldsymbol{m}_0$ is a local minimum. For $0<h_z<1$, $\boldsymbol{m}_0$ gradually moves out from the localized region as shown in (b)-(d). In (e) $1<h_z<1+K$, the two saddle points collide at the north pole, ending up with only one saddle point. In (f) all trajectories are unlocalized in the range $h_z>1+K$.

In case (i), the field $\boldsymbol{h}$ is applied along the $\hat{z}$ direction. The phase space at $K<1$ is shown in Figs. 1(b)–1(f). In the range of $h_z<1$, the phase space has the same topological structure as $h_z=0$. There are (a) two minima at $\theta=\arccos[h_z/(1+K)]$, $\phi=0$, π; (b) two maxima at $\theta=0$, π; and (c) two saddle points at $\theta=\arccos h_z$, $\phi=\pm\pi/2$. In the range $0<h_z<\sqrt{K}$, the initial state $\boldsymbol{m}_0$ stays in the central localized region; no magnetization switching will occur [Fig. 1(b)]. If $h_z=\sqrt{K}$, $\boldsymbol{m}_0$ is on the separatrix and it could evolve into the other localized region; this is when the switching starts [Fig. 1(c)]. Note that on the separatrix the motion is, however, very slow when approaching the saddle points. As shown in Fig. 1(e), in the range $1<h_z<1+K$, the two saddle points at $\theta=\arccos h_z$, $\phi=\pm\pi/2$ disappear, and the point at $\theta=0$ becomes a saddle point, i.e., the separatrix passes the north pole. Finally, for $h_z>1+K$ there is only one minimum at $\theta=0$ and one maximum at $\theta=\pi$; all trajectories are unlocalized.

If $K>1$, i.e., the in-plane anisotropy of the easy axis is bigger than the demagnetizing field, the two wells around the energy minima are so deep that even after h_z passes 1, the initial state $\boldsymbol{m}_0$ still stays in the localized region, shown in Fig. 2(a). Calculation shows the minimal field strength that causes switching is $h_z=(1+K)/2$, bigger than $\sqrt{K}$.

Now let us consider the case in which the field $\boldsymbol{h}$ is applied along the $\hat{y}$ direction. The h_y field breaks the y axis symmetry. As a consequence, the two saddle points have different energies. The phase space at $K<1$ is shown in Fig. 3. In the range of $0<h_y<K$, there are (a) two minima at $\theta=\pi/2$, $\phi=\arcsin h_y/K$, $\pi-\arcsin h_y/K$; (b) two maxima at $\theta=\arcsin h_y$, $\pi-\arcsin h_y$, $\phi=-\pi/2$; and (c) two saddle points at $\theta=\pi/2$, $\phi=\pm\pi/2$. As h_y increases, the size of the localized regions shrinks. The precessional switching occurs at $h_y=K/2$ [Fig. 3(b)]. For $h_y>K$, the two minima collide with one of the saddle points and then disappear, with a new minima emerging at $\theta=\pi/2$, $\phi=\pi/2$. After h_y passes 1, there is only one minimum at $\theta=\pi/2$, $\phi=\pi/2$ and one maximum at $\theta=\pi/2$, $\phi=-\pi/2$. If $K>1$, the evolution of the phase space will be different, but the critical condition $h_y=K/2$ still holds.

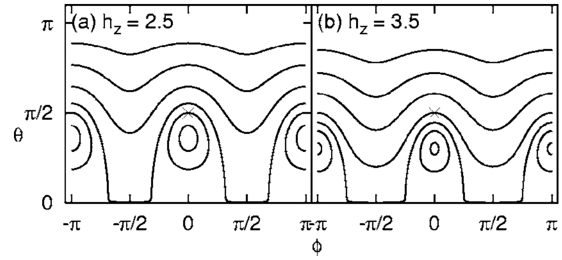

FIG. 2. The phase space at $K=5$ for different h_z. Point X denotes initial state $\boldsymbol{m}_0$. In (a) $1<h_z<(1+K)/2$, $\boldsymbol{m}_0$ stays in the localized region. In (b) $(1+K)/2<h_z<1+K$, $\boldsymbol{m}_0$ is on an unlocalized trajectory.

From the above discussion, we can see that precessional switching occurs during the transition of trajectories around the initial state $\boldsymbol{m}_0$ from localized to unlocalized. In other words, this is when $\boldsymbol{m}_0$ moves from inside to outside the separatrix, so the critical field is the one that makes $\boldsymbol{m}_0$ on the separatrix. Since points in the phase space move along trajectories of constant energy, the critical condition can be also stated as that the energy of the initial state equals the energy of one of the saddle points (but we need to check if they are on the same trajectory since there is more than one saddle point). We observe that this situation usually happens before collision of two fixed points, which is the critical condition for the SW-type reversal. This explains the smaller switching field in precessional switching.

Now we consider the general case in which the field $\boldsymbol{h}$ is applied in the y-z plane at an arbitrary angle. The energies of the initial state $\boldsymbol{m}_0$ and final state $\boldsymbol{m}_1$ are always $-K/2$. We are looking for saddle points with the same energy. The fixed points are given by

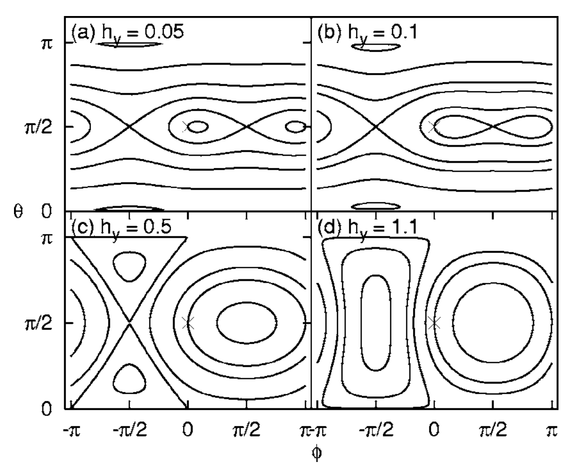

FIG. 3. The phase space at $K=0.2$ for different h_y. Point X denotes the initial state $\boldsymbol{m}_0$. In (a) and (b) $0<h_y<K$. At $h_y=K/2$, $\boldsymbol{m}_0$ is on the separatrix. In (c) $K<h_y<1$. In (d) $h_y>1$.

$$-\sin\theta\cos\theta(1+K\cos^2\phi)-h_y\cos\theta\sin\phi+h_z\sin\theta=0, \tag{7a}$$

$$K\sin^2\theta\sin\phi\cos\phi-h_y\sin\theta\cos\phi=0. \tag{7b}$$

From Eq. (7b) we see that solutions can be grouped into two categories. The first category includes solutions with $\sin\phi = h_y/(K\sin\theta)$. Inserting this expression into Eq. (7a) and solving it gives us fixed points

$$\cos\theta=\frac{h_z}{1+K},\ \sin\phi=\frac{h_y}{K\sqrt{1-h_z^2/(1+K)^2}}, \tag{8}$$

with the condition

$$\frac{h_y^2}{K^2}+\frac{h_z^2}{(1+K)^2}\leqslant 1. \tag{9}$$

In following discussion, we assume this condition always holds since we are interested in the minimal field strength. We will see later that the result is consistent with this assumption. The energy of the fixed points is

$$w=-\frac{K}{2}-\frac{1}{2}\left(\frac{h_y^2}{K}+\frac{h_z^2}{1+K}\right). \tag{10}$$

The energy is always smaller than $-K/2$ so they are not what we are looking for. Stability analysis shows that in fact they are energy minima.

Now we turn to the other category, which includes solutions with $\cos\phi=0$. In this category, fixed points occur on a great circle parametrized by setting $\phi=0$ and letting θ run from 0 to 2π. This makes the problem essentially a two-dimensional problem in the y-z plane.

Instead of numerics, this problem can be solved by a geometrical approach, first proposed by Slonczewski in the Stoner-Wohlfarth problem, then extended by Thiaville.[4,5] In essence, it considers the field $\boldsymbol{h}$ instead of the magnetization direction $\boldsymbol{m}$ as the main variable. We write $\boldsymbol{m}=\sin\theta\hat{y}+\cos\theta\hat{z}=(\sin\theta,\cos\theta)$ and its orthogonal vector $\boldsymbol{e}=(\cos\theta,-\sin\theta)$. The energy requirement is

$$w=\frac{1}{2}\cos^2\theta-\boldsymbol{h}\cdot\boldsymbol{m}=-\frac{K}{2}. \tag{11}$$

The extremum condition is

$$\frac{\mathrm{d}w}{\mathrm{d}\theta}=-\sin\theta\cos\theta-\boldsymbol{h}\cdot\boldsymbol{e}=0. \tag{12}$$

Since the system already has two minima at points given in Eq. (8), the saddle point we are looking for has to be the minimum along the $\hat{\theta}$ direction. This leads to

$$\frac{\mathrm{d}^2w}{\mathrm{d}\theta^2}=\sin^2\theta-\cos^2\theta+\boldsymbol{h}\cdot\boldsymbol{m}>0. \tag{13}$$

The critical switching field is determined by the intersections of the two lines defined in Eqs. (11) and (12). We find

$$h_y=\frac{1}{2}\sin\theta(K-\cos^2\theta), \tag{14a}$$

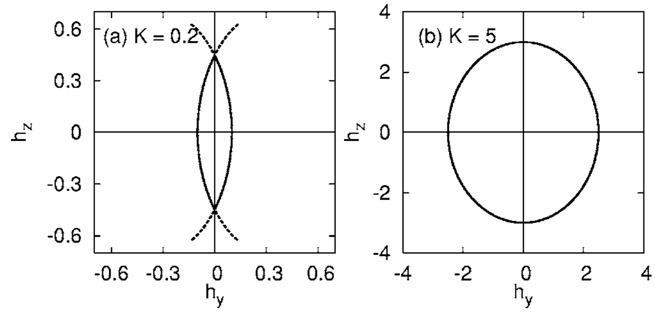

FIG. 4. Critical switching field in the y-z plane at $\alpha=0$ for (a) $K<1$ and (b) $K>1$. The intersections of the curve are in (a) $h_y=K/2$, $h_z=\sqrt{K}$, in (b) $h_y=K/2$, $h_z=(K+1)/2$. Fields shown by a dashed line correspond to the situation in which the initial state has the same energy with one of the saddle points but is not on the separatrix.

$$h_z=\frac{1}{2}\cos\theta(1+K+\sin^2\theta). \tag{14b}$$

Inserting the above expressions into Eq. (13) yields

$$\frac{d^2w}{d\theta^2}=1+\frac{1}{2}K-\frac{3}{2}\cos^2\theta>0. \tag{15}$$

For $K>1$, the above inequality always holds. For $K<1$ we have $|\cos\theta|<\sqrt{(2+K)/3}$. However, in this range the critical curve does not form a closed loop, shown in Fig. 4(a). The reason is that two points having the same energy does not necessarily mean that they are on the same trajectory. After a safety check, we can remove these "faked" solutions by requiring $|\cos\theta|<\sqrt{K}$.

From Fig. 4 we can see that if $K\gg 1$, the magnitude of the critical switching field along any direction in the y-z plane is about the same; for magnetic thin films where $K\ll 1$, the $\hat{y}$ direction is the best choice for small switching field. We also calculate the switching time in the y-z plane for the $K<1$ case by numerically integrating Eq. (6). The

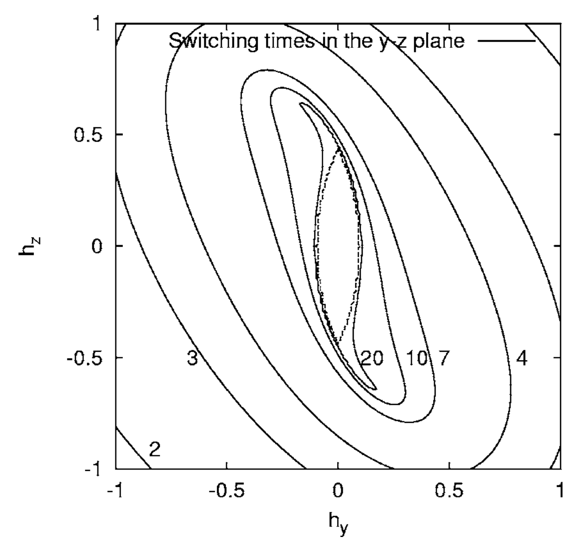

FIG. 5. Switching times contour in the y-z plane for $K=0.2$. The dashed line in the center is the critical switching field. From outside to inside, the switching times are 2, 3, 4, 7, 10, and 20, measured in $(\gamma M_s)^{-1}$. For typical configuration, the time scale is about 0.01 ns.

equitime contours are plotted in Fig. 5. As the magnitude of fields increases, the switching time decreases quickly from the center.

IV. SMALL DAMPING: SWITCHING FIELD IN THE *X-Y* PLANE

In this section, we study the small damping case. It is a formidable task to find the exact analytical solution to the equation of magnetization motion including damping. We thus turn to study the limiting case, where α is small enough so that we can still treat the problem as a Hamiltonian problem and α is also big enough so that after the application of the field pulse the magnetization can relax to the local minimum in finite time. Because of damping, the energies of the initial and final state need not be the same; a nonzero field component along the $\hat{x}$ direction is allowed.

We choose to apply the field in the x-y plane. In this way, we can compare our result with the SW model. The basic procedure is the same as the preceding section: we will search the saddle points with the same energy of the initial state, then map to the whole switching field.

It can be seen from Fig. 3 that during the phase-space evolution the saddle points at the south and north poles ($\theta = 0, \pi$) do not evolve in the precessional switching. Since all the other saddle points are in the x-y plane, we can set $\theta = \pi/2$, turning the problem into a two-dimensional problem in the x-y plane. We write $\boldsymbol{m} = \cos\phi\hat{x} + \sin\phi\hat{y} = (\cos\phi, \sin\phi)$ and its orthogonal vector $\boldsymbol{n} = (-\sin\phi, \cos\phi)$. The applied field is $\boldsymbol{h} = (h_x, h_y)$.

The energy requirement is

$$w = -\frac{K}{2}\cos^2\phi - \boldsymbol{h}\cdot\boldsymbol{m} = -\frac{K}{2} - h_x. \tag{16}$$

The extremum condition is

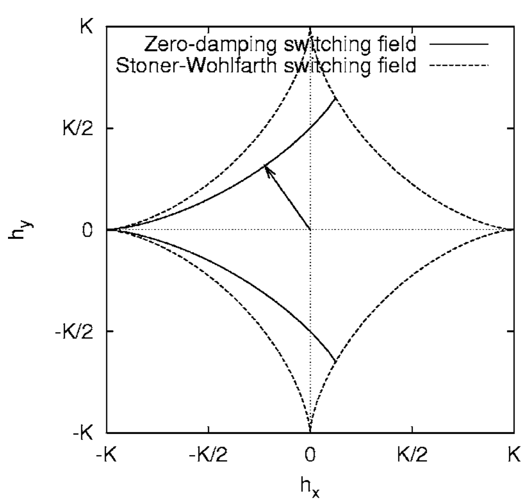

FIG. 6. Critical switching field in the x-y plane for switching from the $\hat{x}$ direction to the $-\hat{x}$ direction. The solid line shows the switching field of precessional switching at zero damping limit. The dashed line is the switching field of the Stoner-Wohlfarth model. For completion, we also draw the right half of the SW astroid. The arrow indicates the minimal field strength.

$$\frac{dw}{d\phi} = K\cos\phi\sin\phi - \boldsymbol{h}\cdot\boldsymbol{n} = 0. \tag{17}$$

Solving the above equations gives us the switching field in the x-y plane:

$$h_x = -\frac{K}{2}\cos\phi(1+\cos\phi), \tag{18a}$$

$$h_y = \frac{K}{2}\sin\phi(1-\cos\phi). \tag{18b}$$

In order to check the stability of the corresponding fixed point, we carry out the standard routine. With h_x, h_y taking the above form, the second derivatives of w are

$$\frac{\partial^2 w}{\partial\theta^2} = 1 + \frac{K}{2}(1-\cos\phi), \tag{19a}$$

$$\frac{\partial^2 w}{\partial\phi^2} = K(\cos\phi - 1)\left(\cos\phi + \frac{1}{2}\right), \tag{19b}$$

$$\frac{\partial^2 w}{\partial\theta\,\partial\phi} = 0. \tag{19c}$$

It is found that ϕ has to be in the range of $0 < \phi < 3/2\ \pi$ (upper branch in Fig. 6) or $-3/2\ \pi < \phi < 0$ (lower branch in Fig. 6) to make the fixed point a saddle point. Again, we still need to check if the saddle point and the initial state are actually on the same trajectory. We require that both h_x and h_y are monotonous functions of ϕ in the upper and lower branches, which yields the same constraint.

To compare our result with the result of the SW model, in which the critical switching field takes the form of $\boldsymbol{h} = K(-\cos^3\phi, \sin^3\phi)$, we plot both fields in Fig. 6. As shown in Fig. 6, if the field is inside the SW astroid, magnetization reversal cannot happen. For precessional switching, reversal can occur well below the SW limit, even for a field with component opposite to the desired switching direction. In this case, the minimal field strength can be as small as $0.38K(\cos\phi = 1/3)$, but the price one pays is the long "ringing" time for the magnetization to relax to the final direction due to the small damping parameter. Considering this, it is still better to apply the field mainly along the $\hat{y}$ direction with a small x component to compensate the energy loss during the motion. This critical switching field in the easy plane has been found by numerical calculations before.[10]

V. SUMMARY

The critical switching field of a monodomain magnetic object with biaxial anisotropy has been studied. In particular, switching fields in the planes perpendicular to the easy and hard axes are calculated using the geometrical method. It is found that in the limit of zero damping, applying the field along the medium axis is good for both small fields and fast switching times.

This method can be generalized for magnetic particles with a higher-order anisotropy if one follows the same steps outlined in this paper. The two-dimensional model may not be sufficient because one needs to consider more fixed points. A more complicated three-dimensional model would then be required.

ACKNOWLEDGMENTS

We acknowledge the NSF NIRT program (Grant No. DMR-0404252) for support of this work.

[1] *Spin Dynamics in Confined Magnetic Structures I*, edited by B. Hillebrands and K. Ounadjela (Springer, Berlin, 2001).
[2] *Spin Dynamics in Confined Magnetic Structures II*, edited by B. Hillebrands and K. Ounadjela (Springer, Berlin, 2003).
[3] E. C. Stoner, F.R.S. and E. P. Wohlfarth, Philos. Trans. R. Soc. London, Ser. A **240**, 599 (1948); reprinted in IEEE Trans. Magn. **27**, 3475 (1991).
[4] A. Thiaville, J. Magn. Magn. Mater. **182**, 5 (1998).
[5] A. Thiaville, Phys. Rev. B **61**, 12221 (2000).
[6] C. H. Back, D. Weller, J. Heidmann, D. Mauri, D. Guarisco, E. L. Garwin, and H. C. Siegmann, Phys. Rev. Lett. **81**, 3251 (1998).
[7] C. H. Back, R. Allenspach, W. Weber, S. S. P. Parkin, D. Weller, E. L. Garwin, and H. C. Siegmann, Science **285**, 864 (1999).
[8] M. Bauer, J. Fassbender, B. Hillebrands, and R. L. Stamps, Phys. Rev. B **61**, 3410 (2000).
[9] Y. Acremann, C. H. Back, M. Buess, D. Pescia, and V. Pokrovsky, Appl. Phys. Lett. **79**, 2228 (2001).
[10] J. Miltat, G. Albuquerque, and A. Thiaville, in *Spin Dynamics in Confined Magnetic Structures I*, edited by B. Hillebrands and K. Ounadjela (Springer, Berlin, 2001), pp. 1–33.
[11] H. W. Schumacher, C. Chappert, P. Crozat, R. C. Sousa, P. P. Freitas, J. Miltat, J. Fassbender, and B. Hillebrands, Phys. Rev. Lett. **90**, 017201 (2003).
[12] H. W. Schumacher, C. Chappert, R. C. Sousa, P. P. Freitas, and J. Miltat, Phys. Rev. Lett. **90**, 017204 (2003).
[13] L. He, W. D. Doyle, and H. Fujiwara, IEEE Trans. Magn. **30**, 4086 (1994).
[14] L. He and W. D. Doyle, J. Appl. Phys. **79**, 6489 (1996).
[15] C. Serpico, I. D. Mayergoyz, and G. Bertotti, J. Appl. Phys. **93**, 6909 (2003).
[16] G. Bertotti, I. Mayergoyz, C. Serpico, and M. Dimian, J. Appl. Phys. **93**, 6811 (2003).
[17] T. Devolder and C. Chappert, Eur. Phys. J. B **36**, 57 (2003).
[18] T. Devolder and C. Chappert, Solid State Commun. **129**, 97 (2004).
[19] Z. Z. Sun and X. R. Wang, Phys. Rev. B **71**, 174430 (2005).
[20] Part of the problem was solved by numerical calculations in Ref. 10.