

# Kaon physics: looking beyond the standard model

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When developing a strategy for future flavor physics experiments, knowledge of the present and future quantitative understanding of standard model predictions is vital. Since there has been substantial, very recent progress in determining the non-leptonic weak interactions of kaons, there may be value in providing an overview of these results and expected future capability. For experiments studying the properties of K mesons, accurate standard model predictions require quantitative control of first- and second-order weak phenomena, *i.e.*  $O(G_F)$  and  $O(G_F^2)$ . As the lightest states in QCD, the pseudoscalar mesons are most accessible to the techniques of lattice QCD and even the most challenging problems for this Euclidean-space approach (disconnected diagrams, two-particle final states and long-distance, second-order phenomena) are now being overcome for this favored system.

Both first- and second-order non-leptonic amplitudes are best viewed as receiving contributions from short distances (the Compton wave length of the  $W$  boson or top quark) and long distances (the one Fermi scale of QCD). For the past four decades the short distance contributions have been studied in electro-weak and QCD perturbation theory allowing an accurate description of such short distance phenomena as a simple, effective low energy theory made up of four-quark operators with known Wilson coefficients. The accuracy of this description is limited by three considerations: 1) The accuracy of the CKM matrix, an obvious target of future experiment. 2) The precision of the perturbative calculations, which is subject to systematic improvement by extending the calculation to higher order. 3) The use of perturbative methods at lower energies where the perturbative approach may converge poorly, a difficulty that can be avoided by including an active charm quark in the effective theory.

The long-distance component of these non-leptonic amplitudes is determined using lattice QCD. Historically, the most accessible quantity has been the  $O(G_F^2)$  parameter  $\epsilon_K$  which determines indirect CP violation in kaon decay. The needed long-distance quantity,  $B_K$ , is the  $K^0 - \bar{K}^0$  matrix element of the single  $\Delta S = 2$  four-quark operator in the effective low energy theory. Lattice techniques now determine  $B_K$  with a combined systematic and statistical accuracy of 4%, see for example Refs. [1, 2]. Newer lattice QCD methods, developed over the past ten years, have now been used to successfully calculate  $K \rightarrow \pi\pi$  decay amplitudes for the  $I = 2$  final state [3]. The implications of the Maiani-Testa “theorem” have been correctly circumvented and two pions carrying the physical decay momentum in a finite box properly simulated. The result for  $\text{Re}(A_2)$  agrees well with experiment and a first value for  $\text{Im}(A_2)$  has been obtained. This is now a straight-forward lattice QCD calculation and the present 20% errors can be easily reduced to 5% over the next couple of years using the next generation

of multi-petaflops computers.

Similar methods will also yield results with well understood errors for the  $I = 0$  amplitude  $A_0$  and a value for  $\epsilon'$  that can be compared with experiment. The presence of quark-disconnected diagrams makes this a much more difficult calculation but first numerical experiments have already demonstrated a signal above noise for  $\text{Re}(A_0)$  [4]. Here a forecast is more uncertain but over the next two years we anticipate first results for the complex amplitude  $A_0$  with physical final-state pions. A complete calculation including a continuum extrapolation may be possible within five years.

Given the inherent ease of calculations with kaons, it is also possible to envision calculations in lattice QCD of the so called “long-distance” contributions to the second order weak amplitudes entering  $\epsilon_K$  and the  $K_L - K_S$  mass difference  $\Delta m_K$ . In these calculations two first-order four-Fermi operators are included with QCD-scale space-time separations. Such lattice calculations are in principle possible with good control of all systematic errors [5] and the first numerical exploration is encouraging [6]. Such calculations are still in their infancy and contain difficulties at least as great as those needed for  $\epsilon'$ . However, in a fifteen year period, one should expect that the 5% long-distance contribution to  $\epsilon_K$  will be computed from first principles using lattice methods and that possible physics beyond the standard model will be revealed in a comparison between theory and experiment for  $\Delta m_K$ .

More work is needed before the theoretical results described above are sufficient to motivate a new round of kaon measurements increasing the accuracy of fundamental quantities such as  $\epsilon'$ . However, it may not be too early to think about what accuracy may be possible, when such next generation experiments might undertaken and what they would cost.

## References

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