Oscillation motion

$f = \frac{1}{T}, \quad \omega = \frac{2\pi}{T}$

SHM: $a = \frac{d^2 x}{dt^2} = -\omega^2 x, \quad \alpha = \frac{d^2\theta}{dt^2} = -\omega^2 \theta$

$x = x_{max}\cos(\omega t + \delta), \quad x_{max} = A$

$v = -v_{max}\sin(\omega t + \delta), \quad v_{max} = \omega A$

$a = -a_{max}\cos(\omega t + \delta) = -\omega^2 x, \quad a_{max} = \omega^2 A$

$E = K + U = K_{max} = \frac{1}{2} m (\omega A)^2 = U_{max} = \frac{1}{2} k A^2$

Spring: $m a = -k x$

Simple pendulum: $m a_\theta = m \alpha \ell = -m g \sin\theta$

Physical pendulum: $\tau = I \alpha = -m g d \sin\theta$

Torsion pendulum: $\tau = I \alpha = -\kappa \theta$

Gravity

$\vec{F}_{21} = -G \frac{m_1 m_2}{r_{12}^2} \hat{r}_{12}, \quad \text{for } r \geq R, \quad g(r) = G \frac{M}{r^2}$

$G = 6.67259 \times 10^{-11}\ \mathrm{N\,m^2/kg^2}$

$R_{earth} = 6370$ km, $\quad M_{earth} = 5.98 \times 10^{24}$ kg

Circular orbit: $a_c = \frac{v^2}{r} = \omega^2 r = \left(\frac{2\pi}{T}\right)^2 r = g(r)$

$U = -G \frac{m M}{r}, \quad E = U + K = -\frac{G m M}{2 r}$

$F = -\frac{dU}{dr} = -m G \frac{M}{r^2} = -m \frac{v^2}{r}$

Kepler's Laws of planetary motion:

i) elliptical orbit, $r = \frac{r_0}{1 - \epsilon\cos\theta} \quad r_1 = \frac{r_0}{1+\epsilon}, \quad r_2 = \frac{r_0}{1-\epsilon}$

ii) $L = r m \frac{\Delta r_\perp}{\Delta t} \longrightarrow \frac{\Delta A}{\Delta t} = \frac{1}{2} \frac{r \Delta r_\perp}{\Delta t} = \frac{L}{2m} = \text{const.}$

iii) $G \frac{M}{a^2} = \left(\frac{2\pi a}{T}\right)^2 \frac{1}{a}, \quad a = \frac{r_1 + r_2}{2}, \quad T^2 = \left(\frac{4\pi^2}{G M}\right) r^3$

Escape kinetic energy: $E = K + U(R) = 0$

Fluid mechanics

Pascal: $P = \frac{F_{\perp 1}}{A_1} = \frac{F_{\perp 2}}{A_2}, \quad 1\ \text{atm} = 1.013 \times 10^5\ \mathrm{N/m^2}$

Archimedes: $B = M g, \quad$ Pascal=N/m^2

$P = P_{atm} + \rho g h, \quad$ with $P = \frac{F_\perp}{A}$ and $\rho = \frac{m}{V}$

$F = \int P\, dA \longrightarrow \rho g \ell \int_0^h (h - y)\, dy$

Continuity equation: $A v = \text{constant}$

Bernoulli: $P + \frac{1}{2} \rho v^2 + \rho g y = \text{const}, \qquad P \geq 0$

Wave motion

Traveling waves: $y = f(x - v t), \quad y = f(x + v t)$

In the positive x direction: $y = A \sin(k x - \omega t - \phi)$

$T = \frac{1}{f}, \quad \omega = \frac{2\pi}{T}, \quad k = \frac{2\pi}{\lambda}, \quad v = \frac{\omega}{k} = \frac{\lambda}{T}$

Along a string: $v = \sqrt{\frac{F}{\mu}}$

General: $\Delta E = \Delta K + \Delta U = \Delta K_{max}$

$P = \frac{\Delta E}{\Delta t} = \frac{1}{2} \frac{\Delta m}{\Delta t} (\omega A)^2$

Waves: $\frac{\Delta m}{\Delta t} = \frac{\Delta m}{\Delta x} \cdot \frac{\Delta x}{\Delta t} = \frac{\Delta m}{\Delta x} \cdot v$

$P = \frac{1}{2} \mu v (\omega A)^2, \quad$ with $\mu = \frac{\Delta m}{\Delta x}$

Circular: $\frac{\Delta m}{\Delta t} = \frac{\Delta m}{\Delta A} \cdot \frac{\Delta A}{\Delta r} \cdot \frac{\Delta r}{dt} = \frac{\Delta m}{\Delta A} \cdot 2\pi r v$

Spherical: $\frac{\Delta m}{\Delta t} = \frac{\Delta m}{\Delta V} \cdot 4\pi r^2 v$

Sound

$v = \sqrt{\frac{B}{\rho}}, \quad s = s_{max}\cos(k x - \omega t - \phi)$

$\Delta P = -B \frac{\Delta V}{V} = -B \frac{\partial s}{\partial x}$

$\Delta P_{max} = B \kappa s_{max} = \rho v \omega s_{max}$

Piston: $\frac{\Delta m}{\Delta t} = \frac{\Delta m}{\Delta V} \cdot \frac{A \Delta x}{\Delta t} = \rho A v$

Intensity: $I = \frac{P}{A} = \frac{1}{2} \rho v (\omega s_{max})^2$

Intensity level: $\beta = 10 \log_{10} \frac{I}{I_0}, \ I_0 = 10^{-12}\ \mathrm{W/m^2}$

Plane waves: $\psi(x,t) = c \sin(k x - \omega t)$

Circular waves: $\psi(r,t) = \frac{c}{\sqrt{r}} \sin(k r - \omega t)$

Spherical: $\psi(r,t) = \frac{c}{r} \sin(k r - \omega t)$

Doppler effect: $\lambda = v T, \quad f_0 = \frac{1}{T}, \quad f' = \frac{v'}{\lambda'}$

Here $v' = v_{sound} \pm v_{observer}$, is wave speed relative to moving observer and $\lambda' = (v_{sound} \pm v_{source})/f_0$, detected wave length established by moving source of frequency f_0. $f_{received} = f_{reflected}$

Shock waves: Mach Number$= \frac{v_{source}}{v_{sound}} = \frac{1}{\sin\theta}$

Superposition of waves

Phase difference: $\sin(k x - \omega t) + \sin(k x - \omega t - \phi)$

Standing waves: $\sin(k x - \omega t) + \sin(k x + \omega t)$

Beats: $\sin(kx - \omega_1 t) + \sin(k x - \omega_2 t)$

Others: $a \cos(k x - \omega t) + b \sin(k x - \omega t)$

$y = \sin(k x - \omega t), \ z = \sin(k x - \omega t)$

Fundamental modes: Sketch wave patterns

String: $\frac{\lambda}{2} = \ell$, *Rod clamped middle:* $\frac{\lambda}{2} = \ell$,

Open-open pipe: $\frac{\lambda}{2} = \ell$, *Open-closed pipe:* $\frac{\lambda}{4} = \ell$

Temperature and heat

Conversions: $F = \frac{9}{5} C + 32^\circ, \qquad K = C + 273.15^\circ$

Constant volume gas thermometer: $T = a P + b$

Thermal expansion: $\alpha = \frac{1}{\ell} \frac{d\ell}{dT}, \quad \beta = \frac{1}{V} \frac{dV}{dT}$

$\Delta \ell = \alpha \ell \Delta T, \quad \Delta A = 2 \alpha A \Delta T, \quad \Delta V = 3 \alpha V \Delta T$

Ideal gas law: $P V = n R T = N k T$

$R = 8.314510$ J/mol/K $= 0.0821$ L atm/mol/K

$k = 1.38 \times 10^{-23}$J/K, $\quad N_A = 6.02 \times 10^{23}$, 1 cal=4.19 J

Calorimetry: $\Delta Q = c m \Delta T, \ \Delta Q = L \Delta m$

First law: $\Delta U = \Delta Q - \Delta W, \quad W = \int P\, dV$

Conduction: $H = \frac{\Delta Q}{\Delta t} = -k A \frac{\Delta T}{\Delta \ell}, \quad \Delta T_i = \frac{-H}{A} \frac{\ell_i}{k_i}$

Stefan's law: $P = \sigma A e T^4, \quad \sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$

Kinetic theory of gas

Ideal gas: $\Delta p_x = 2 m v_x, \quad F = \frac{\Delta p_x}{\Delta t} = \frac{m v_x^2}{d}$

Pressure: $P = \frac{N \overline{F}}{A} = \frac{m N}{V} \overline{v_x^2} = \frac{m N}{3 V} \overline{v^2}$

$P = \frac{2}{3} \frac{N}{V} \overline{K}, \quad \overline{K}_x = \frac{\overline{K}}{3} = \frac{1}{2} k T, \quad T = 273 + T_c,$

$P V = N k T, \quad n = N/N_A, \quad k = 1.38 \times 10^{-23}$ J/K,

$N_A = 6.02214199 \times 10^{23}$ #/kg/mole

Constant V: $\Delta Q = \Delta U = n C_V \Delta T$

Constant P: $\Delta Q = n C_P \Delta T$

$\gamma = \frac{C_P}{C_V}, \quad C_P - C_V = R$

$C_V = \frac{d}{2} R, \quad$ for transl.+rot+vib, $\quad d = 3 + 2 + 2$

Adiabatic expansion: $P V^\gamma = \text{constant}$

Mean free path: $\ell = \frac{v_{rms} t}{(v_{rel})_{rms} t} \frac{1}{\pi d^2 n_V} = \frac{1}{\sqrt{2} \pi d^2 n_V}$