E & M - Basic Physical Concepts

Electric force and electric field

Electric force between 2 point charges:

$|F| = k\,\frac{|q_1|\,|q_2|}{r^2}$

$k = 8.987551787 \times 10^9\ \mathrm{N\,m^2/C^2}$

$\epsilon_0 = \frac{1}{4\,\pi\,k} = 8.854187817 \times 10^{-12}\ \mathrm{C^2/N\,m^2}$

$q_p = -q_e = 1.60217733\,(49) \times 10^{-19}\ \mathrm{C}$

$m_p = 1.672623\,(10) \times 10^{-27}\ \mathrm{kg}$

$m_e = 9.1093897\,(54) \times 10^{-31}\ \mathrm{kg}$

Electric field: $\vec{E} = \frac{\vec{F}}{q}$

Point charge: $|E| = k\,\frac{|Q|}{r^2}, \quad \vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots$

Field patterns: point charge, dipole, $\parallel$ plates, rod, spheres, cylinders,. . .

Charge distributions:

Linear charge density: $\lambda = \frac{\Delta Q}{\Delta x}$

Area charge density: $\sigma_A = \frac{\Delta Q}{\Delta A}$

Surface charge density: $\sigma_{surf} = \frac{\Delta Q_{surf}}{\Delta A}$

Volume charge density: $\rho = \frac{\Delta Q}{\Delta V}$

Electric flux and Gauss' law

Flux: $\Delta\Phi = E\,\Delta A_\perp = \vec{E}\cdot\hat{n}\Delta A$

Gauss law: Outgoing Flux from S, $\Phi_S = \frac{Q_{enclosed}}{\epsilon_0}$

Steps: to obtain electric field

–Inspect $\vec{E}$ pattern and construct S

–Find $\Phi_s = \oint_{surface} \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$, solve for $\vec{E}$

Spherical: $\Phi_s = 4\,\pi\,r^2\,E$

Cylindrical: $\Phi_s = 2\,\pi\,r\,\ell\,E$

Pill box: $\Phi_s = E\,\Delta A$, 1 side; $= 2\,E\,\Delta A$, 2 sides

Conductor: $\vec{E}_{in} = 0, \quad E^{\parallel}_{surf} = 0, \quad E^{\perp}_{surf} = \frac{\sigma_{surf}}{\epsilon_0}$

Potential

Potential energy: $\Delta U = q\,\Delta V$ 1 eV $\approx 1.6 \times 10^{-19}$ J

Positive charge moves from high V to low V

Point charge: $V = \frac{k\,Q}{r} \quad V = V_1 + V_2 = \ldots$

Energy of a charge-pair: $U = \frac{k\,q_1\,q_2}{r_{12}}$

Potential difference: $|\Delta V| = |E\,\Delta s_{\|}|$,

$\Delta V = -\vec{E}\cdot\Delta\vec{s}, \quad V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s}$

$E = -\frac{dV}{dr}, \quad E_x = -\frac{\Delta V}{\Delta x}\Big|_{fix\ y,z} = -\frac{\partial V}{\partial x}$, *etc.*

Capacitances $\quad Q = C\,V$

Series: $V = \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \cdots, \quad Q = Q_i$

Parallel: $Q = C_{eq}\,V = C_1\,V + C_2\,V + \cdots, \quad V = V_i$

Parallel plate-capacitor: $C = \frac{Q}{V} = \frac{Q}{E\,d} = \frac{\epsilon_0\,A}{d}$

Energy: $U = \int_0^Q V\,dq = \frac{1}{2}\,\frac{Q^2}{C}, \quad u = \frac{1}{2}\,\epsilon_0\,E^2$

Dielectrics: $C = \kappa C_0, \quad U_\kappa = \frac{1}{2\,\kappa}\frac{Q^2}{C_0}, \quad u_\kappa = \frac{1}{2}\,\epsilon_0\,\kappa\,E_\kappa^2$

Spherical capacitor: $V = \frac{Q}{4\,\pi\,\epsilon_0\,r_1} - \frac{Q}{4\,\pi\,\epsilon_0\,r_2}$

Potential energy: $U = -\vec{p}\cdot\vec{E}$

Current and resistance

Current: $I = \frac{d\,Q}{dt} = n\,q\,v_d\,A$

Ohm's law: $V = I\,R, \quad E = \rho J$

$E = \frac{V}{\ell}, \quad J = \frac{I}{A}, \quad R = \frac{\rho\ell}{A}$

Power: $P = I\,V = \frac{V^2}{R} = I^2\,R$

Thermal coefficient of ρ: $\alpha = \frac{\Delta\rho}{\rho_0\Delta T}$

Motion of free electrons in an ideal conductor:

$a\,\tau = v_d \rightarrow \frac{q\,E}{m}\,\tau = \frac{J}{n\,q} \rightarrow \rho = \frac{m}{n\,q^2\,\tau}$

Direct current circuits $\quad V = I\,R$

Series: $V = I\,R_{eq} = I\,R_1 + I\,R_2 + I\,R_3 + \cdots, \quad I = I_i$

Parallel: $I = \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \cdots, \quad V = V_i$

Steps: in application of Kirchhoff's Rules

–*Label currents:* $i_1, i_2, i_3, \ldots$

–*Node equations:* $\sum i_{in} = \sum i_{out}$

–*Loop equations:* "$\sum(\pm\mathcal{E}) + \sum(\mp iR)=0$"

–*Natural:* "+" for loop-arrow entering − terminal

"−" for loop-arrow-parallel to current flow

RC circuit: if $\frac{d\,y}{dt} + \frac{1}{R\,C}\,y = 0, \quad y = y_0\exp(-\frac{t}{R\,C})$

Charging: $\mathcal{E} - V_c - R\,i = 0,\ \frac{1}{c}\,\frac{d\,q}{dt} + R\,\frac{d\,i}{dt} = \frac{i}{c} + R\,\frac{d\,i}{dt} = 0$

Discharge: $0 = V_c - R\,i = \frac{q}{c} + R\,\frac{d\,q}{dt}, \quad \frac{i}{c} + R\,\frac{d\,i}{dt} = 0$

Magnetic field and magnetic force

$\mu_0 = 4\,\pi \times 10^{-7}\ \mathrm{T\,m/A}$

Wire: $B = \frac{\mu_0\,i}{2\,\pi\,r}$ **Axis of loop:** $B = \frac{\mu_0\,a^2\,i}{2\,(a^2+x^2)^{3/2}}$

Magnetic force: $\vec{F}_M = i\,\vec{\ell}\times\vec{B} \rightarrow q\,\vec{v}\times\vec{B}$

Loop-magnet ID: $\vec{\tau} = i\,\vec{A}\times\vec{B}, \qquad \vec{\mu} = i\,A\,\hat{n}$

Circular motion: $F = \frac{m\,v^2}{r} = q\,v\,B, \quad T = \frac{1}{f} = \frac{2\,\pi\,r}{v}$

Lorentz force: $\vec{F} = q\,\vec{E} + q\,\vec{v}\times\vec{B}$

Hall effect: $V_H = \frac{F_M\,d}{q}, \quad U = -\vec{\mu}\cdot\vec{B}$

Sources of $\vec{B}$ and magnetism of matter

Biot-Savart Law: $\Delta\vec{B} = \frac{\mu_0}{4\,\pi}\,\frac{i\,\Delta\vec{\ell}\times\hat{r}}{r^2}, \quad B = \frac{\mu_0}{4\,\pi}\,\frac{q\,\vec{v}\times\hat{r}}{r^2}$

$\Delta B = \frac{\mu_0}{4\,\pi}\,\frac{i\,\Delta y}{r^2}\sin\theta, \quad \sin\theta = \frac{a}{r}, \quad \Delta y = \frac{r^2\,\Delta\theta}{a}$

Ampere's law: $\mathcal{M} = \oint_L \vec{B}\cdot d\vec{s} = \mu_0\,I_{encircled}$

Steps: to obtain magnetic field

–Inspect $\vec{B}$ pattern and construct loop L

–Find $\mathcal{M}$ and I_{encl}, and solve for $\vec{B}$.

Displ. current: $I_d = \epsilon_0\,\frac{d\,\Phi_E}{dt} = \epsilon_0\,\frac{d\,(E\,A)}{dt} = \frac{d\,Q_A}{dt}$

Magnetism in atom:

Orbital motion: $\mu = i\,A = \frac{e}{2\,m}\,L$

$L = m\,v\,r = n\,\hbar, \quad \hbar = \frac{h}{2\,\pi} = 1.06 \times 10^{-34}\ \mathrm{J\,s}$

$\mu_{orbit} = n\,\mu_B, \quad \mu_B = \frac{e\,\hbar}{2\,m} = 9.27 \times 10^{-24}\ \mathrm{J/T}$

Spin: $S = \frac{\hbar}{2}, \quad \mu_{spin} = \mu_B$

Magnetism in matter:

$B = B_0 + B_M = (1+\chi)\,B_0 = (1+\chi)\,\mu_0\,\frac{B_0}{\mu_0} = \kappa_m\,H$

Ferromagnetic: $\chi \gg 1$ **Diamagnetic:** $-1 \ll \chi < 0$

Paramagnetic: $0 < \chi \ll 1, \quad M = \frac{C}{T}B$