Faraday's law

$$\mathcal{E} = -N\,\frac{d\,\phi_B}{dt}, \qquad \phi_B = \int \vec{B}\cdot d\vec{A},$$
$$\mathcal{E} = \int \vec{E}\cdot d\vec{s}, \qquad \vec{E} = \frac{\vec{F}_M}{q}$$

Lenz law: Induced $\vec{B}$ opposes change of Φ_B

$$\frac{d\,\phi_B}{dt} = \frac{d\,(B\,A_\perp)}{dt} = \frac{d\,B}{dt}\,A_\perp + B\,\frac{d\,A_\perp}{dt}$$

Moving rods: $\frac{d\,A}{dt} = \ell\,v, \quad \frac{d\,A}{dt} = \frac{d}{dt}\left(\frac{1}{2}\,R\cdot R\,\theta\right)$

Rotating loop: $\frac{d\,A_\perp}{dt} = \frac{d}{dt}\,(A\,\cos\omega t)$

Cutting B lines $\rightarrow$ change $\phi_B \rightarrow E_{ind} \rightarrow \mathcal{E}_{ind}$

Maxwell equations:

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}, \qquad \oint \vec{B}\cdot d\vec{A} = 0\,,$$
$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\,\phi_B}{dt}, \quad \oint \vec{B}\cdot d\vec{s} = \mu_0\,[I + \epsilon_0\,\frac{d\,\phi_E}{dt}]$$

Inductance

Mutual: $\mathcal{E}_2 = -M_{21}\,\frac{d\,i_1}{dt}, \quad M_{21} = M_{12} = \frac{N_2\,\phi_{21}}{i_1}$

Self: $\mathcal{E} = -L\,\frac{d\,i}{dt}, \quad L = \frac{N\,\phi}{i}, \quad V_L = L\,\frac{d\,i}{dt}$

Long solenoid: $L = \frac{N\,B\,A}{i}, \quad B = \mu_0\,n\,i$

Energies: $U_L = \frac{1}{2}\,L\,i^2, \quad u_B = \frac{1}{2\,\mu_0}\,B^2$

$$U_C = \frac{1}{2\,C}\,q^2, \quad u_E = \frac{1}{2}\,\epsilon_0\,E^2$$

L C: $V_L + V_C = 0 \Rightarrow L\,\frac{d\,i}{dt} = -\frac{q}{C}\; q = q_0\cos(\omega\,t + \delta),$

$$\omega = \sqrt{\frac{1}{L\,C}}, \quad U_C + U_L = U_{C\,\max} = U_{L\,\max} = U_0$$

Decay Equations: $\frac{d\,y}{dt} = -a\,y, \quad y = y_0\,\exp(-a\,t)$

L R: $\mathcal{E} = V_L + R\,i, \quad \frac{d\,V_L}{dt} + \frac{R\,V_L}{L} = 0,$

$$V_L = \mathcal{E}\,\exp\left(-\frac{Rt}{L}\right), \quad i = \frac{\mathcal{E}}{R}\left[1 - \exp\left(\frac{-R\,t}{L}\right)\right]$$

L R C:

$$Q \approx Q_0\,e^{-\frac{R}{2\,L}\,t}\cos\omega_d\,t, \quad \omega_d = \sqrt{\frac{1}{L\,C} - \left(\frac{R}{2\,L}\right)^2}$$

Underdamped, critically damped & overdamped

A C Circuits

Impedance: $[\text{Ohm} \equiv \Omega] \quad Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$

Inductive $X_L = \omega\,L$, Capactive $X_C = \frac{1}{\omega\,C}$

Mean value: $\bar{f}(t) = \frac{1}{T}\int_0^T f(t)\,dt$

$$[\sin\omega\,t]_{rms} = \overline{[\sin^2\omega\,t]}^{\frac{1}{2}} = \overline{[\tfrac{1}{2}\,(1 - \cos 2\,\omega\,t)]}^{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

Electromagnetic waves

Properties of em waves:

$$E = E_m\cos(k\,z - \omega\,t), \quad B = \frac{E}{c}$$
$$v = \frac{d\,z}{dt} = \frac{\omega}{k} = \lambda\,f = \frac{\lambda}{T}, \quad n = \frac{c}{v}$$

speed of light: $c = \frac{1}{\sqrt{\epsilon_0\,\mu_0}} = 2.99792458\times 10^8$ m/s

$\vec{B} \perp \vec{E}$, **propagating along:** $\vec{E}\times\vec{B}$

$$u = u_E + u_B, \quad u_E = u_B$$

Poynting vector: $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}, \quad \bar{S} = \bar{I} = \frac{E_{rms}B_{rms}}{\mu_0}$

Intensity: $I = \frac{P}{A} = \frac{\Delta U}{A\,\Delta z}\,\frac{d\,z}{dt} = u\,c$

Energy conservation: $\int \vec{S}\cdot d\vec{A} = \frac{d\,U}{dt} + P_R$

Complete absorption: Momentum $p = \frac{U}{c}$

Pressure: $\mathcal{P} = \frac{F}{A} = \frac{\Delta p}{\Delta t}\,\frac{1}{A} = \frac{\Delta U}{c\,\Delta t}\,\frac{1}{A} = u = \frac{S}{c}$

Complete reflection: $p = \frac{2\,U}{c}, \quad \mathcal{P} = \frac{2\,S}{c}$

Reflection and Refraction

Index of refraction: $\frac{n_1}{n_2} = \frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1}$

Snell's law: $n_1\sin\theta_1 = n_2\sin\theta_2$

Critical angle: $n_2 > n_1, \quad n_2\sin\theta_c = n_1\sin 90^\circ$

Total reflection: $\theta > \theta_c$

Mirrors and lenses

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Ray tracing rules:

Mirror: At symm pt S, reflected symmetrically through center of sphere, undeflected. Parallel to axis, converges toward F (or diverges away from F), $f = \frac{R}{2}$.

Lens: Through center of lens, undeflected. Parallel to axis, converges toward F (or diverges away from F)

Image: $q > 0$ (real), $q < 0$ (virtual)

Focal point F: at $p = \infty, \quad q = f$

$f = \pm|f|$, "+" convergent, "−" divergent

Magnification: $M = \frac{h'}{h} = -\frac{q}{p}$

Refraction at spherical surface: $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$

R is coordinate of center with origin at S, with S the symmetry point of surface on the axis

Lens maker: $\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Two media: $M = \frac{h'}{h} = -\frac{q}{p}\,\frac{n_1}{n_2}$

Huygen's principles:

Points in wave front are sources of next wavelets

Forward tangent surface is next wave front

Interference

Maxima $\phi = 0, 2\,\pi, 4\,\pi, \cdots$; Minima $\phi = \pi, 3\,\pi, 5\,\pi, \cdots$

Double slits: $I_{average} = I_0\cos^2\left(\frac{\phi}{2}\right), \quad \phi = k\,\Delta\,.$

$\sin\theta = \frac{\Delta}{d},\ \tan\theta = \frac{y}{L},$ for small $\theta,\ \theta \approx \sin\theta \approx \tan\theta$

Phasor diagram: $\vec{A} = \vec{A}_1 + \vec{A}_2 + \vec{A}_3 + \cdots$

$$A_x = A_{1x} + A_{2x} + A_{3x} + \cdots, \quad A_y = A_{1y} + A_{2y} + \cdots$$
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

First minimum for N slits: $\phi = \frac{2\,\pi}{N}$

Thin film: $\phi = k\,\Delta + |\phi_{1_{reflected}} - \phi_{2_{reflected}}|, \quad \Delta = 2\,t$

$\phi_{reflected} = \pi$ (denser medium); =0 (lighter medium)

Diffraction

Single slit: $I = I_0\left[\frac{\sin\frac{\beta}{2}}{\frac{\beta}{2}}\right]^2, \quad \beta = k\Delta, \quad \Delta = a\,\sin\theta$

Resolution criterion: $\theta_{criterion} = 1.22\,\frac{\lambda}{D}$

Grating: Principle maxima $\Delta = m\,\lambda$

Polarization

Brewster $(n_1 < n_2)$**:** $n_1\sin\theta_{br} = n_2\sin(\frac{\pi}{2} - \theta_{br})$

Polarizer: $E_{transmit} = E_0\cos\theta, \quad I = I_0\cos^2\theta$

Unpolarized light: $\frac{\Delta I}{\Delta\theta} = \frac{I_0}{2\,\pi}$

Transmitted Intensity: $\Delta I' = \Delta I\,\cos^2\theta$

$$I' = \frac{I_0}{2\,\pi}\int_0^{2\,\pi}\cos^2\theta\,d\theta = \frac{I_0}{2}$$