Sign-conventions and Huygen's principle in Geometric Optics (4/13/07)

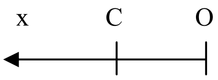
In this note we discuss

- a self consistent "sign-convention" table in geometric optics, and
- show how Huygen's principle can help us to solve lens problems which involve a sequence of lenses.

Please refer to your classnotes for details.

Caution: See also http://www.ph.utexas.edu/~itiq/303L/chiu/instructor/goptics.pdf. Unfortunately the word-html code may not display figures properly by your web browser. This note will be handed out in the class. Please refer to the handout for proper figures.

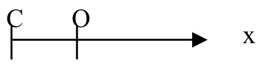
1. Image formed due to reflection on a spherical surface (mirrors). Introduce the x-coordinate in such a manner that <u>the location of a real image will always have a positive x-value</u>. For mirrors, the origin is chosen to be at the mirror. Here the x-axis must then be drawn to the left. The universal relationship which is applicable for both mirrors and lenses is given as: $\mathbf{(1/p)+(1/x_{image})=1/x_{focus}}$.



The object coordinate $x_{obj} = p > 0$. $x_{focus} = x_c /2$, where C is the center of the spherical mirror.

- For a concave mirror, $x_c>0$, so $x_{focus}>0$. It corresponds to a convergent mirror.
- For a convex mirror, , $x_c<0$, so $x_{focus}<0$. It corresponds to a divergent mirror.

2. **Image formed to due to refraction on a spherical surface** (transparent media)



Here the x-coordinate is from left to right. The origin is the intersection between the spherical surface and the x-axis. Denote the index of refraction of the medium to the left to be n_L and that to the right, n_R, the appropriate relationship is:

$$\mathbf{(n_L/p)+(n_R/x_{image})=(n_R-n_L)/x_c}.$$

- Convergent surface occurs when the right hand side: RHS = $(n_R-n_L)/x_c > 0$.
- Divergent surface occurs when the right hand side: RHS = $(n_R-n_L)/x_c < 0$.

3. **Lenses:** For lenses, the x-axis is again from left to right. The origin is located at the lens. The basic relationship is the above mentioned universal expression:

$$\mathbf{(1/p)+(1/x_{image})=1/x_{focus}}.$$

- Convergent lens is for the case where $x_f>0$
- Divergent lens is for the case where $x_f<0$.

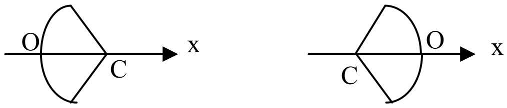
4. **Lens maker's formula**: Let the ray be going from left to right. It encounters surface #1 first and then #2. The lens maker's formula is given by:

$$\mathbf{(1/f)=(n_{lens}/n_{medium} - 1)[(1/x_1) - (1/x_2)]},$$

x_1 and x_2 are the x-coordinates of the center of surface #1 and surface #2 respectively.

5. Image position for a sequence of lenses: Transport of a wave front.

Define the <u>curvature of a spherical wave front</u> to be $C=(1/x_C)$. Consider a spherical wave front to be symmetric about the x-axis, where the origin is the intersection between the wave front and the x-axis.



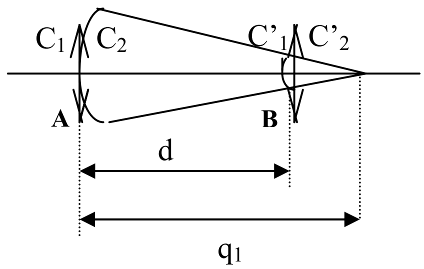
- Convergent wave front has $x_c>0$
- Divergent wave front has $x_c<0$.

<u>A new interpretation</u>: Now we would like to reinterpret the meaning of the usual lens formula.

- First rearrange the expression as follows: $(1/q)=(-1/p)+(1/f)$.
- Notice that $(1/q)$ is the final curvature C_f, of the wave front after the wave front has passed the lens.
- And $(-1/p)$ is the initial curvature C_i, of the wave front before it enters the lens.
- We write $C_f=(1/f)$, which is the modification to the initial curvature due to the passage through the lens.

<u>Transport of a wave front</u>: We can rewrite the above expression as: $\mathbf{C_f=C_i+C_{lens}}$**.** This is a **"local"** interpretation to the usual lens formula. The latter has been derived through a global geometric consideration. Our curvature expression will allow us to determine the image location for an object through a sequence of lenses.

Consider <u>a two lens problem</u>. The setup is depicted in the sketch.



- The left lens is the lens "A" and the right lens, the lens "B".
- Denote the curvature of the initial wave front before entering lens A as C_1 and the final curvature after it passes through the lens A as C_2. We have $C_2=C_1 + C_A$.
- Similarly at lens "B", we have $C_2'=C_1' + C_B$.
- What is C_1'? It is the curvature of the wave front, which <u>propagates freely</u> from A to B. By inspection, $C_1'=1/(q_1 - d)$.

<u>A sequence of lenses</u>. One may arrive at a general algorithm for a sequence of lenses.

- First a spherical wave front is originated by the object point, say at P.
- This wave front arrives at lens #1 and is then corrected by this lens #1.
- The corrected spherical wave front is <u>"propagating" freely</u>, from lens #1 to lens #2.
- This newly arrived wave front will then be corrected by lens #2.
- It then propagates freely to from lens #2 to lens #3, etc.