C-Summary: Special Theory of Relativity Update:4/16/09

Basic hypothesis: In all inertial frames,

speed of light is the same,

principle of relativity is valid.

Lorentz transformation:

$$x' = (x - ut)\gamma \quad (1)$$

$$t' = (t - \frac{\beta}{c}x)\gamma \quad (2)$$

$$\beta = \frac{u}{c}, \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Velocity addition:

$$\frac{dx'}{dt'} = \frac{dx'/dt}{dt'/dt} = \left(\frac{dx}{dt} - u\right) \Big/ \left(1 - \frac{\beta}{c}\frac{dx}{dt}\right)$$

$$v' = \frac{v-u}{1-\frac{uv}{c^2}}$$

Time dilation: Put Σ'-clock at $x' = 0$.

From (1), $x = ut$. From (2), $t' = [t - \frac{\beta}{c}(ut)]\gamma$

$= (1-\beta^2)t\gamma = \frac{t}{\gamma}$. Periods: $\Delta T' = \Delta T\gamma$.

Length contraction:

Measure in Σ frame. Using (1)

for $\Delta t = 0$, $\Delta x' = \Delta x\gamma$, $\Delta x = \frac{\Delta x'}{\gamma}$.

Doppler Shift: $f' = \frac{\text{wave velocity rel.to detector}}{\text{modified wavelength}}$.

Approaching:

Non-rel: Sound speed = c_s.

$f' = \frac{c_s + v_{det}}{T_{em}(c_s - v_{em})}$, $\quad \frac{1}{T_{em}} = f_0$

Light speed: c

Relativity: $f' = \frac{c}{\gamma T_{em}(c - v_{em})} = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} f_0$

Receding: $f' = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} f_0$

Lorentz transformation in matrix form:

Coordinate transformation

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = M(\beta) \begin{pmatrix} x \\ ct \end{pmatrix}$$

$$\text{with } M(\beta) = \begin{pmatrix} \gamma & -\eta \\ -\eta & \gamma \end{pmatrix}, \quad \eta = \beta\gamma$$

Boosting a rest frame particle

$$\begin{pmatrix} cp \\ E \end{pmatrix} = B(\beta) \begin{pmatrix} 0 \\ m_0 c^2 \end{pmatrix}$$

$$B(\beta) = \begin{pmatrix} \gamma & +\eta \\ +\eta & \gamma \end{pmatrix} = M(-\beta), \quad \eta = \beta\gamma$$

$$cp = \eta m_0 c^2$$

$$E = \gamma m_0 c^2$$

Identity: $\gamma^2 - \eta^2 = \gamma^2(1-\beta^2) = 1$

$$E^2 = (cp)^2 = (\gamma^2 - \eta^2)(m_0 c^2)^2$$

$$E = \sqrt{(cp)^2 + (m_0 c^2)^2}, \quad \text{See OM Eq.(36.51)}$$

Kinetic energy:

$$K = E - m_0 c^2 \xrightarrow{\text{NR}} \frac{1}{2} m_0 v^2$$