SummaryL of unit 1 (update: 9/21/12)

Constants(SI): $m_e = 9\times10^{-31}$, $m_p = 1.7\times10^{-27}$, $e = 1.6\times10^{-19}$, $c = 3\times10^8$, $k = 9\times10^9$, $\epsilon_0 = 8.85\times10^{-12}$.

Electric Field. Electric force on q exerted by a field $\mathbf{E}$ is given by $\mathbf{F} = q\mathbf{E}$. $\mathbf{F} = m\mathbf{a}$.

Mom-Pr: $\Delta\mathbf{p} = \mathbf{F}\Delta t$. $\Delta\mathbf{v} = \mathbf{a}\Delta t$, NR. Energy-Pr: $\Delta K = K - K_0 = W = \int_0^{\mathbf{r}} \mathbf{F}\cdot d\mathbf{r'}$, where $K \approx \frac{1}{2}mv^2$, NR.

Special case (1d, NR, a=const): At $t = 0$, $v = v_0$. At t later ($\Delta t = t$), $v = v_0 + at$, $x = \frac{v_0+v}{2}t = v_0 t + \frac{1}{2}at^2$.

Field at $\mathbf{r}$ due to point charge Q at origin, $\mathbf{E} = (kQ/r^2)\hat{\mathbf{r}}$,

Force on q_2 at $\mathbf{r}$ due to q_1 at origin: $\mathbf{F}^{due1}_{on2} = \frac{kq_1q_2}{r^2}\hat{\mathbf{r}}$. Same sign charges repel, opposite sign charges attract.

Dipole field. Math: Small ϵ expansion: $(1+\epsilon)^a = 1 + a\epsilon + O(\epsilon^2) \approx 1 + a\epsilon$. Dipole moment: $\mathbf{p} = q\mathbf{s}$ along x, centered at 0. At $< x, 0 >$, $E^p_x = -\frac{d}{dx}E^q_x s = \frac{2kp}{x^3}$; at $< 0, y >$, $E^p_x = 2E^q_x cos\theta = \frac{-kp}{y^3}$.

Electric field and matter. Matter system electric properties: charged, neutral, polarized.

In a neutral system, the applied field induces a dipole moment: $\mathbf{p}_{induced} = \alpha\mathbf{E}_{applied}$, α is the polarizability.

Insulator medium: No mobile charges. When field applied to a neutral atom, it generates an induced dipole.

Conductor medium: Two types of conducting media are considered.

Ionization solution: Both positive ions and negative ions are mobile.

At equilibrium, field vector inside is 0. At the surface, the parallel component of the field vector is 0.

Drude model: Momentum Principle: $\Delta\mathbf{p} = F\Delta t = eE\Delta t$. $v_{drift} = p/m = eE\Delta t/m$, $\overline{v_{drift}} = \bar{p}/m = eE\Delta\overline{t_c}/m$. At room temperature $\bar{v}_{therm} \sim 10^3 km/s$, $\mathbf{v}_{drift} \sim 10^{-3} m/s$. $\bar{\mathbf{v}} = \overline{|\mathbf{v}_{drift} + \mathbf{v}_{therm}|} \sim |\mathbf{v}_{therm}|$.

Write $\mathbf{v}_{drift} = u\mathbf{E}$, where u is the mobility. At a given temperature, $\Delta t_c \sim const.$, or $u \sim const.$

Example: Ball and Wire. Based on Fig15.38, determine $F_{ball-wire}$. What is the polarizability of the wire? Hints: Being in a metal medium, we take the total field at the center of the wire is 0. The magnitue of the field at the center due to the ball is kQ/r^2. Verify at the center the field due to the charges at ends of the wire is $2kq/(L/2)^2$. What is the polarizability of the wire?

$\frac{1}{r^n}$ dep. forces: Verify $F_{q-p} \propto 1/r^3$, where the force is between charge q and dipole moment p. They are at distance r apart. Verify $F_{q-atom} \propto 1/r^5$. Determine n for F_{p-p}. Also determine n for F_{p-atom}.

E of distributed charges. (i)Divide charges into elememts: $\Delta q =< density > \times < geometric\ element >$ (ii) The projected component$\Delta\mathbf{E}$. (iii) Integral expression. (iv) Use derivative identity to integrate.

Rod, Fig16.9: Length L along y, centered at O. At $< x, 0, 0 >$, $\Delta E_x = k\frac{(Q/L)\Delta y}{\rho^2} sin\alpha$, $\rho^2 = x^2 + y^2$, $E_x = \frac{kQ}{L}I$, $I = \int_{-L/2}^{L/2} \frac{dy}{\rho^2} sin\alpha$. Math ID: $\frac{dy}{\rho^2} = \frac{d\alpha}{x}$. [Proof: $tan\alpha = -\frac{x}{y}$, $\frac{dy}{d\alpha} = \frac{d}{d\alpha}\left(\frac{-x}{tan\alpha}\right) = \frac{x}{sin^2\alpha} = \frac{\rho^2}{x}$]. $I = \int \frac{d\alpha}{x} sin\alpha = \left.\frac{-cos\alpha}{x}\right|_{\alpha_1}^{\alpha_2}$. For $y_2 = -y_1 = \frac{L}{2}$, $\alpha_2 = \pi - \alpha_1$, $E_x = \frac{kQ}{L}\frac{2cos\alpha_1}{x} = \frac{kQ}{x[x^2+(L/2)^2]^{1/2}}$.

Ring, Fig16.17: The ring is centered at O, with r. At $< 0, 0, z >$, $E_z = kq\frac{z}{(r^2+z^2)^{3/2}}$.

Disk, Fig16.24: Consider a flat ring of average radius r, width Δr, and charge $\Delta Q = \frac{Q}{A} \times (2\pi r\Delta r)$. At $< 0, 0, z >$, $\Delta E_z = k\left[\frac{Q}{A}2\pi r\Delta r\right]\frac{1}{\rho^2}cos\alpha$, with $cos\alpha = \frac{z}{\rho}$. Thus $E_z = \frac{1}{2\epsilon_0}\frac{Q}{A}I$, where $I = \int_0^R \frac{z}{\rho^3} r dr = \int_0^R \frac{z}{\rho^3}\rho d\rho = \left.\frac{-z}{\rho}\right|_0^R$, where $\rho d\rho = r dr$ was used, since $\rho^2 = r^2 + z^2$. $E_z = \frac{Q/A}{2\epsilon_0}\left.\frac{-z}{\rho}\right|_0^R = \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{(R^2+z^2)^{1/2}}\right]$.

Verify for large z, $E_z \sim kQ/z^2$. Hint: Use small argument approx.: $(1+\epsilon)^a \approx 1 + a\epsilon$.

Disk (one plate), $z << R$: $E_z = (Q/A)/(2\epsilon_0)$. Capacitor plates ($z << R$). $E_{gap} \approx \frac{Q/A}{\epsilon_0}$, neglect fringe.

Gauss law: $\Phi_S = \frac{Q_S}{\epsilon_0}$. $\Phi_S \equiv \Sigma_S(\mathbf{E}\cdot\Delta\mathbf{A}) = EA^{enclose}_\perp$, flux through S. Q_S are charges enclosed by S.

Spherical case, S is a sphere with radius r. $A^{enclose}_\perp = 4\pi r^2$. Derive E along r at S.

Planar case, S encloses charged plate. Plate area is A. $A^{enclose}_\perp = 2A$, $Q_S = Q$. Derive E $\perp$ to the plate.

Cylindrical case, S: cylinder with radius r and height L. $A^{enclose}_\perp = 2\pi rL$, $Q_S = Q$. Derive E along r at S.