SummaryL of unit 2 (Ch17, Ch18, Ch22.6, update: 10/14/12)

Constant: $\frac{\mu_0}{4\pi} = 10^{-7} S.I.$ Integrals: $\int_{\alpha_1}^{\alpha_2} sin\alpha d\alpha = (cos\alpha_1 - cos\alpha_2)$, $\int_a^b r^n dr = \frac{r^{n+1}}{n+1}\big|_a^b$, $\int_a^b \frac{dr}{r} = ln(b/a)$.

Electric Potential: For 1 charged particle: $\Delta K + \Delta U = 0$, $\Delta U = -q\mathbf{E}\cdot\Delta\mathbf{l}$ (work against $F = q\mathbf{E}$).

Potential: $\Delta V = \frac{\Delta U}{q} = V_f - V_i = -\mathbf{E}\cdot\Delta\mathbf{l} = -E\Delta l\, cos\theta = -(E_x\Delta x + E_y\Delta y + E_z\Delta z)$. Units: V=Nm/C.

From V to **E**: $\mathbf{E} = -(\partial V/\partial x, \partial V/\partial y, \partial V/\partial z) \equiv -\bigtriangledown V$. For spherically symm. case: $E_r = -dV(r)/dr$.

Potential diff: $V_f - V_i = -\int_i^f \mathbf{E}\cdot d\mathbf{l}$. For Q at origin, define $V(\infty) = 0$. $V(r) = -\int_\infty^r \frac{kQ}{r'^2}dr' = kQ/r$.

For a potential function, path integral along any closed loop $\Delta V = 0$. This implies conservation of energy.

Potential energy U_A: work required to move q from ∞ to A, assume $U_\infty = 0$. Potential: $V_A = \frac{U_A}{q}$, $V_\infty = 0$.

System of many charged particles: $U = \Sigma_{i<j}U_{ij}$, where $U_{ij} = k\frac{q_iq_j}{r_{ij}}$, $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$. For 1-pair: $U_{12} = k\frac{q_1q_2}{r_{12}}$.

Potential functions: Various cases.

Metal: Within a metal region: $\Delta V = V_f - V_i = 0$. Connected metal-regions become an equal potential body.

Spherical shell(Q, R): What is V(r) for $r > R$ and for $r < R$?

Solid sphere with radius R: For $r > R$, what is V(r)?

For $r < R$, $V(r) - V(R) = -\int_R^r Edr$, with $E = kQ(r/R)^3/(4\pi r^2) = Cr$. Here C is defined by $kQ/R^2 = CR$.

Long rod (R, Q, linear charge density $\frac{\Delta Q}{\Delta y} = \lambda$). Verify that Gauss law leads to:

for $r > R$, $E = \frac{\lambda}{2\pi\epsilon_0}\cdot\frac{1}{r}$, and for $r < R$, $E = \frac{\lambda}{2\pi\epsilon_0}\cdot\left(\frac{r}{R}\right)^2\cdot\frac{1}{r}$. $V(r) = V(R) - \int_R^r Edr$.

Ring (Q, R): $V_{ring} = kQ/\rho$, where $\rho = \sqrt{R^2 + z^2}$. Verify that $E_z = -\partial V/\partial z$ agrees with Ch16, p639.

Verify that for large z, it reduces to point charge limit. Inspect the motions of $\pm$ test charges near z=0.

Parallel plate capacitor(Q, A, and gap s). Based on Gauss Law, verify that for one plate: $E_1 = \frac{Q/A}{2\epsilon_0}$.

Determine E_{gap}, V, force between plates, energy stored, show the energy density is given by $u \equiv \frac{U}{As} = \frac{1}{2}\epsilon_0 E^2$.

Parallel plate capacitor filled with dielectrics of κ: $E' = \frac{E}{\kappa}$. $\kappa = \frac{E}{E'} = \frac{Q}{Q - Q_{pol}}$, $Q_{pol} = Q\left(1 - \frac{1}{\kappa}\right)$.

Magnetic Field. Long wire: The thumb points along the current, direction of **B** pattern is given by RHR1.

Horizontal Component of $\mathbf{B}_{earth}$ is $\sim 2\times 10^{-5}$T, in much of US.

Biot-Savart law for source $q\mathbf{v}$, $B = \frac{\mu_0}{4\pi}\cdot(q\mathbf{v}\times\hat{\mathbf{r}})\frac{1}{r^2}$. For a current segment: $\Delta B = \frac{\mu_0}{4\pi}\cdot(I\Delta\mathbf{l}\times\hat{\mathbf{r}})\frac{1}{r^2}$.

In the Drude model, $\Delta Q\bar{\mathbf{v}} = \Delta Q\cdot\frac{\Delta\mathbf{l}}{\Delta t} = \frac{\Delta Q}{\Delta t}\Delta\mathbf{l} = I\Delta\mathbf{l}$, where $\Delta\mathbf{l}$ is the drift distance in time Δt.

In terms of electron number density n, $I = q\frac{\Delta N}{\Delta t} = q\frac{nA\bar{\mathbf{v}}\Delta t}{\Delta t} = qi$, where $i = nA\bar{v}$. **Caution**: Among Quest problems, the symbol i is often used for the conventional current. When in doubt, please ask for clarification.

$$\underline{\text{Cross Product}}:\ \mathbf{A}\times\mathbf{B} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix}\hat{i} + \begin{vmatrix} A_z & A_x \\ B_z & B_x \end{vmatrix}\hat{j} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}\hat{k} = A\,B\,sin\,\theta\,\hat{\mathbf{n}}.\ \text{Direction : RHR2.}$$

Circular Arc:(I, $s = r\theta$, with finite arc length): $\Delta\mathbf{B}$ at center, $\Delta\mathbf{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{IR\Delta\theta}{R^2}\hat{\mathbf{n}}$, $\hat{\mathbf{n}}$ by RHR2, RHR3.

Wire segment, Fig18.24 (I, a): Direction: $\Delta\mathbf{B} = -\hat{k}\Delta B_z$, RHR1. $\Delta B_z = \left(\frac{\mu_0}{4\pi}\right)\frac{I\Delta y sin\theta}{r^2}$. $r = \sqrt{a^2 + y^2}$. Use $\frac{\Delta y}{r^2} = \frac{\Delta\theta}{a}$. $B_z = \left(\frac{\mu_0}{4\pi}\right)\frac{I}{a}H$, $H = (cos\theta_1 - cos\theta_2) \stackrel{symm}{\longrightarrow} 2cos\theta_1 \stackrel{long}{\longrightarrow} 2$. For $\pm\frac{L}{2}$ case: $cos\theta_1 = \frac{L/2}{\sqrt{a^2 + (L/2)^2}}$, p722.

Ring (I, R): $B_z = \left(\frac{\mu_0}{4\pi}\right)\frac{I\cdot 2\pi R}{\rho^2}\cdot\frac{R}{\rho}$, $\rho^2 = R^2 + z^2$. At z=0, $\rho = R$. $B = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi I}{R}$.

At $z >> R$, $B = \left(\frac{\mu_0}{4\pi}\right)\frac{2\pi R^2 I}{\rho^3} \equiv \left(\frac{\mu_0}{4\pi}\right)\frac{2\mu}{z^3}$, where dipole moment $\vec{\mu} = A_{loop}I\,\hat{\mathbf{n}}$. $\hat{\mathbf{n}}$ is defined by RHR3.

Solenoid: $B = \mu_0\left(\frac{NI}{2L}\right)(cos\alpha_1 - cos\alpha_2) \stackrel{long}{\longrightarrow} \mu_0\frac{NI}{L}$. Clicker 19.3 (skip derivation). B_z vs z curve, Fig. 18.53.

Magnetic moment of a bar magnet.

Atomic model: $\mu_{orbit} = \pi R^2 I = \pi R^2\frac{ev}{2\pi R} = \frac{e}{2m}L$, $L = mvR$. Ground state of H-atom $L \sim \hbar \sim 10^{-34}$ J s

Exp-check: $\mu_{magnet} \sim \mu_{atom}\times(number\,of\,atoms)$. RHS agrees with LHS to within factor of 2 (see p.731).

Ampere's Law: $\oint_{\mathcal{L}}\mathbf{B}\bullet d\mathbf{l} = \mu_0 I_{\mathcal{L}}$. Application: 1) Cyl-symmetry: LHS= $2\pi rB$. Find RHS for long wire, solid wire, cyl-conducting shell. 2) Packed loops(LHSs): Solenoid(Bd), Toroid ($2\pi rB$), current sheet (2Bd).