SummaryL of unit 3 (update: 11/11/12)

E-field & circuits. Electron # current: $i = nA\bar{v}$, with $\bar{v} = uE$ (Drude model). Convent. current: I=$|q|$i. Steady flow, I=constant. At wire surface, charge density gradients lead to constant E within the wire.
Battery: $emf = \mathcal{E} = F_{NC}s/q$, F_{NC} is the Noncolumb Force. Node eqn: $i = i_1 + i_2 + \cdots$
Loop equation: $\Delta V_{round\,trip} = \Delta V_1 + \Delta V_2 + \cdots = 0$. Brightness of a bulb: $Power = i(e\Delta V) = I\Delta V$.
Capacitors & Resistors Capacitance: $C = \frac{Q}{\Delta V} = \frac{Q}{Es}$. ||-plate: $\frac{V}{s} = E = \frac{Q/A}{\epsilon_0}$, $C = \frac{\epsilon_0 A}{s}$.
Series: $Q = Q_1 = Q_2$, $V = V_1 + V_2$. $\frac{Q}{C} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$. Parallel: $V = V_1 = V_2$, $Q = Q_1 + Q_2$, $VC = V_1C_1 + V_2C_2$.
Ohm's Law1: $J \equiv \frac{I}{A} = \sigma E$, with $\sigma \equiv |q|nu$, since $I = (|q|nu)AE = \sigma AE$. $\rho \equiv \frac{1}{\sigma}$ (increases with temp.why?)
Ohm's Law2: $V = IR$, $R = \frac{\rho l}{A}$. Here V & I depend on conductor dimension, but in OL1 E & J do not.
Series: $I = I_1 = I_2$, $V = V_1 + V_2$, $IR = I_1R_1 + I_2R_2$. Parallel: $V = V_1 = V_2$, $I = I_1 + I_2$, $\frac{V}{R} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$.
RC-Charging: $\mathcal{E} - IR - \frac{q}{C} = 0$. $\frac{dI}{dt} = -\frac{I}{\tau}$, time-constant: $\tau \equiv RC$. $I = I(0)exp\left(-\frac{t}{\tau}\right)$, Find $I(0)$ and $q(t)$.
RC-Discharging: $IR - \frac{q}{C} = 0$. $-\frac{dq}{dt}R = \frac{q}{C}$. It leads to $q = q(0)exp\left(-\frac{t}{\tau}\right)$. Find $q(0)$ and $I(t)$.
Electric Power: $P = \frac{dW}{dt} = \frac{dq\Delta V}{dt} == I\Delta V$. Battery delivers $I\mathcal{E}$. R consums IV_R, C stores or releases IV_C.
Energy stored in C: In vacuum, $U = Q\left[\frac{Q/A}{2\epsilon_0}\right]d = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$, $u = \frac{U}{volume}$. With dielectrics ($\kappa > 1$), $E' = \frac{E}{\kappa}$, $V' = \frac{V}{\kappa}$, $C' = \frac{Q}{V'} = \kappa C$. (a) Fixed Q case: $U' = \frac{1}{2}\frac{Q^2}{C'}$, (b) Fixed V case: $U' = \frac{1}{2}C'V^2$, $u' = \frac{U'}{volume}$.
Magnetic force [E.M. constants: $\frac{\mu_0}{4\pi} = 10^{-7}\ \frac{T\cdot m^2}{C^2}$. $k = \frac{1}{4\pi\epsilon_0} \approx 9 \times 10^9 \frac{N\cdot m^2}{C^2}$.]
On $q\mathbf{v}$: $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$. On $I\Delta\mathbf{l}$: $\Delta\mathbf{F} = \Delta Nq\mathbf{v} \times \mathbf{B} = I\Delta\mathbf{L} \times \mathbf{B}$, since $\Delta Nq\mathbf{v} = \Delta Nq\frac{\Delta\mathbf{L}}{\Delta t} = q\frac{\Delta N}{\Delta t}\Delta\mathbf{L} = I\Delta\mathbf{L}$.
Circular motion: $\frac{mv^2}{r} = qvB$. $\omega = \frac{2\pi}{T} = \frac{v}{r} = \frac{qB}{m}$. Given q, B and m, ω is a fixed, independent on v & r.
Velocity selector. $\mathbf{E}$ is perpendicular to $\mathbf{B}$. At critical speed v_c, $F_E = F_M$. What is v_c in terms of E & B?
Parallel wires: Force on $I_2\Delta L$ is $\Delta F_2 = I_2\Delta L \times \mathbf{B}_1$. What is B_1 in terms of I_1 & d (distance between wires)?
Hall effect: Derive V_H based on the mechanical battery model, where $emf = \frac{F_{NC}h}{q}$, and $\mathbf{F}_{NC} = q\mathbf{v} \times \mathbf{B}$.
Motional emf. Metal bar of length L is moving with $\mathbf{v} \perp$ to $\mathbf{B}$. Determine $\mathcal{E}_{ind}$. Which end has + charges? Find mech. force which leads to a constant v. Verify the electric power consumed in R equals to mech-power.
Mag. force on loop. $\mathbf{F}_{loop} = I\Sigma_i\mathbf{L}_i \times \mathbf{B}_i = I(\mathbf{L}_1 \times \mathbf{B}_1 + \cdots + \mathbf{L}_4 \times \mathbf{B}_4)$. Show that if $\mathbf{B} = const.$, $\mathbf{F}_{loop} = 0$.
Mag. torque on loop: Show for a loop with area A where $\mathbf{B}$ is perpendicular to the area, torque on the loop is given by $\tau = \mu \times \mathbf{B}$, where $\mu = \hat{\mathbf{n}}IA$, where $\hat{\mathbf{n}}$ is determined by RHR3. Why $\tau = \mu_\perp \times \mathbf{B}$ is also valid?
Pattern of fields: Gauss law: See unit 1. Study cases using nonsymmetric S. Conducting medium: At the surface $E_\parallel = 0$, $E_\perp = \sigma/\epsilon_0$. Ampere's law: See unit 2. B-pattern leads to choice of path and direction of I.
Gauss Law for magnetism: No mag. monopole(s). $\Phi^B_S = \oint_S \mathbf{B} \bullet d\mathbf{A} = 0$. Mag. flux $\Phi^B = BA_\perp = \mathbf{B} \bullet \mathbf{A}$.
Faraday's law. $emf = \oint_{path} \mathbf{E}_{NC} \bullet d\mathbf{l} = -d\phi_B/dt$. Lenz-rule: Flux change in wire loop is opposed by B_{ind}.
Motional emf, a special case of Faraday's law: Verify they lead to the same emf (magnitude & direction)
Rotating 1-loop in B: $\frac{d\phi}{dt} = \frac{d}{dt}BA_\perp$. For $A_\perp = Acos\omega t$, $\omega = \frac{2\pi}{T} = 2\pi f$. It leads to $\mathcal{E} = -\frac{d\phi}{dt} = BA\omega sin\omega t$.
Inductance. Change variable: let $\phi = constI$. $\mathcal{E} = -N\frac{d\phi}{dt} = -(Nconst)\frac{dI}{dt} \equiv -L\frac{dI}{dt}$. So $L = Nconst = N\frac{\phi}{I}$.
Long solenoid: N turns, length d and cross section A: $B = \mu_0\frac{N}{d}I$. $L = N\frac{BA}{I} = N\left(\mu_0\frac{N}{d}\right)A$.
Energy stored in L: $P = \frac{dU}{dt} = \frac{dq(V)}{dt} = IV$. $U = \int Pdt = \int IVdt$, where $P_L = V_LI, V_L = L\frac{dI}{dt}$,
$U_L = \int_0^I I'\left(L\frac{dI'}{dt}\right)dt = \frac{1}{2}LI^2 = \frac{1}{2\mu_0}B^2Ad$, $LI^2 = (LI)I = (NBA)\left(\frac{Bd}{\mu_0 N}\right) = \frac{B^2}{\mu_0}Ad$ was used.
Energy density: $u = \frac{U}{volume}$. $u_E = \frac{U_C}{volume} = \frac{1}{2}\epsilon_0E^2$. $u_B = \frac{U_L}{volume} = \frac{1}{2\mu_0}B^2$, with $U_C = \frac{1}{2}\frac{q^2}{C}$, $U_L = \frac{1}{2}LI^2$.
LR circuit: $\mathcal{E} - L\frac{dI}{dt} - IR = 0$. Buildup: $I = I_0(1 - exp[-\frac{t}{\tau}])$, $\tau = \frac{L}{R}$. $L\frac{dI}{dt} = -IR$. Decay: $I = I_0\,exp[-\frac{t}{\tau}]$.
LC oscillator. Power-eqn: $I\left[L\frac{dI}{dt} + \frac{q}{C}\right] = \frac{d}{dt}\left[\frac{1}{2}LI^2 + \frac{1}{2}\frac{q^2}{C}\right]$, For $t \geq 0$, $U_C + U_L = U_{Cmax} = U_{Lmax} = const.$
LC circuit. Loop-eqn: $\frac{d^2q}{d^2t} = -\frac{q}{LC}$. Solution: $q = q_0cos(\omega t + \delta)$. $\omega = \frac{1}{\sqrt{LC}}$, δ the initial phase of oscillation.
LRC circuit: What is the loop equation? Sketch the oscillation pattern for I(t) and q(t). See Figure 23.49.
Maxwell equations: Relationship between fields ($\mathbf{E}$ and $\mathbf{B}$) & sources (q, $q\mathbf{v}$): G-L, mag-G-L, A-L, F-L.