Circuit Elements

- Capacitors;
- Resistors;
- RC circuit;
- Energy and Power

Capacitance

Electric field in a capacitor: $E = \dfrac{Q/A}{\varepsilon_0}$

$$\Delta V = -\int_i^f \vec{E} \bullet d\vec{l} \longrightarrow |\Delta V| = Es$$

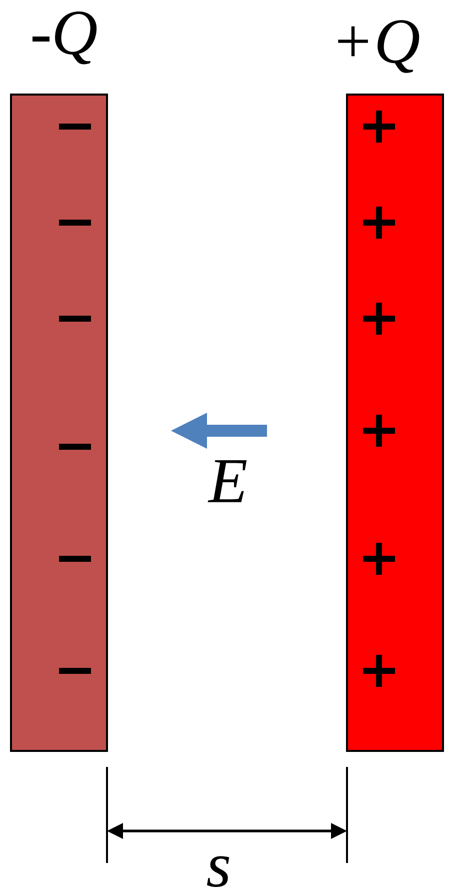
$$|\Delta V| = \frac{Q/A}{\varepsilon_0} s \longrightarrow Q = \frac{\varepsilon_0 A}{s} |\Delta V|$$

In general: $Q \sim |\Delta V|$

Definition of capacitance:

$$Q = C|\Delta V|$$

Capacitance

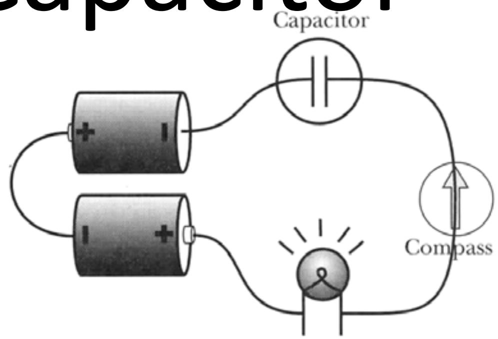


Capacitance of a parallel-plate capacitor:

$$C = \frac{\varepsilon_0 A}{s}$$

Energy Stored in a Capacitor

$$Q = C|\Delta V| \longrightarrow |\Delta V| = \frac{Q}{C}$$



$$dU_{electric} = dQ\Delta V = \frac{Q}{C}dQ$$

$$U_{electric} = \int_0^Q dU_{electric} = \int_0^Q \frac{Q}{C}dQ = \frac{1}{C}\int_0^Q QdQ$$

$$U_{electric} = \frac{1}{2}\frac{Q^2}{C} = \frac{C(\Delta V)^2}{2}$$

Alternative approach:

Energy density: $\frac{\varepsilon_0 E^2}{2}$

$$E = \Delta V / s$$

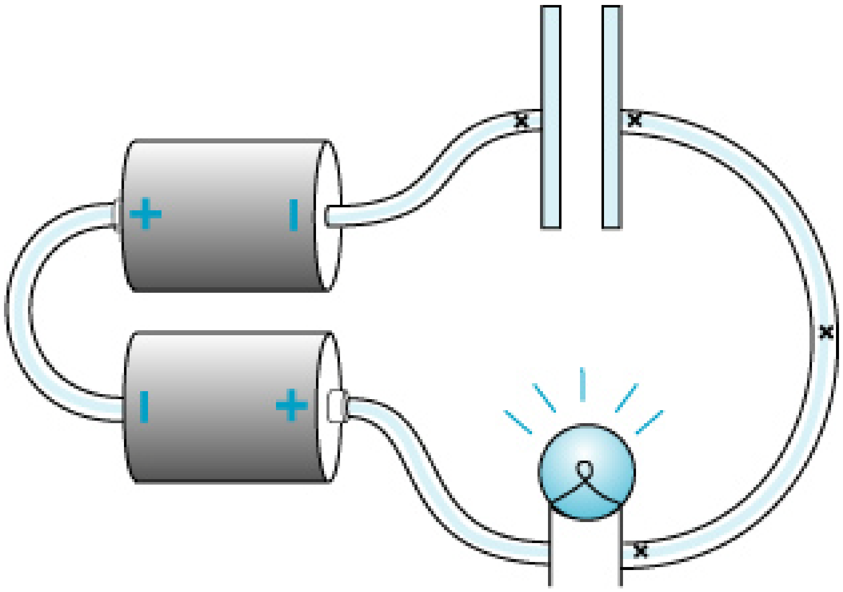
Energy: $\frac{\varepsilon_0 (\Delta V)^2}{2s^2} \times As = \frac{\varepsilon_0 A (\Delta V)^2}{2s}$

$$C = \frac{\varepsilon_0 A}{s}$$

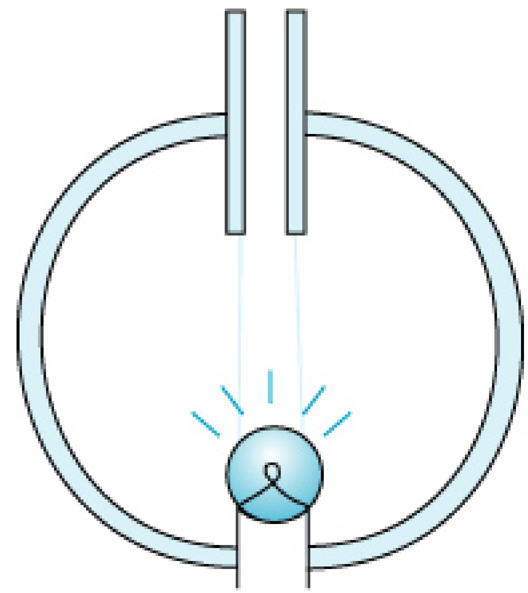
$$= \frac{C(\Delta V)^2}{2}$$

Capacitor: Charging and Discharging

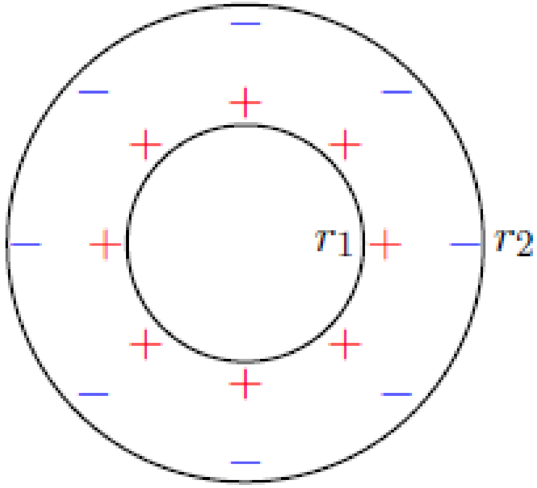
Charging



Discharging



As in the figure below, a a thin spherical metal shell of radius r_1 has a charge Q (on its outer surface) and is surrounded by a concentric thin spherical metal shell of radius r_2 which has a charge $-Q$ (on its inner surface).



Use the definition of capacitance

$$Q = C\,|\Delta V|$$

to find the capacitance of this spherical capacitor.

1. $C = \dfrac{4\pi\epsilon_0}{\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)}$

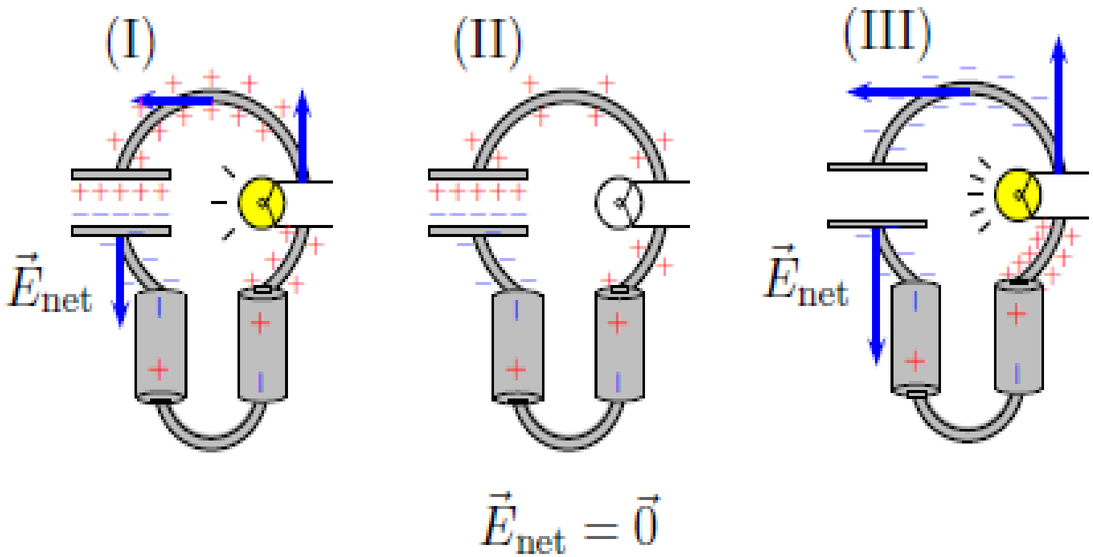
2. $C = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$

3. $C = \dfrac{1}{8\pi\epsilon_0}\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$

4. $C = \dfrac{8\pi\epsilon_0}{\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)}$

5. $C = 4\pi\epsilon_0\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$

When a particular capacitor, which is initially uncharged, is connected to a battery and a small light bulb, the light bulb is initially bright but gradually gets dimmer, and after 45 seconds it goes out.



Choose the diagram that represent the E field at 0.01 S, 8S, and 240 S after the connection

A parallel-plate capacitor is charged by connecting it to a battery.

If the battery is disconnected and then the separation between the plates is increased, what will happen to the charge on the capacitor and the electric potential across it?

1. The charge remains fixed and the electric potential increases.

2. The charge decreases and the electric potential remains fixed.

3. The charge remains fixed and the electric potential decreases.

4. The charge and the electric potential increase.

5. The charge and the electric potential remain fixed.

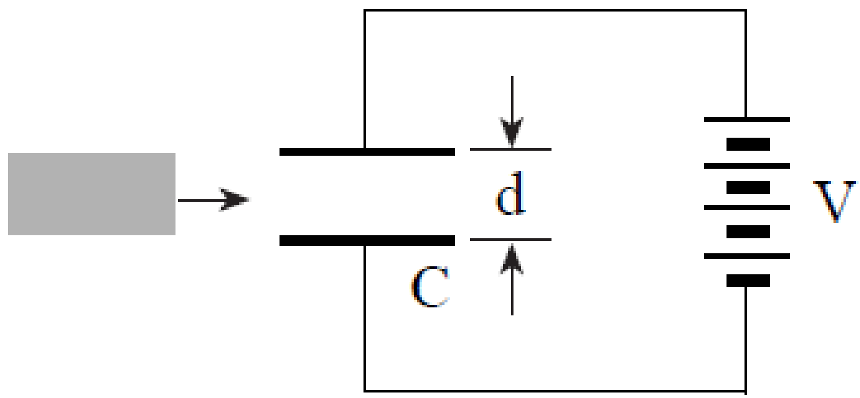
A 39 μF capacitor is charged to an unknown potential V_0 and then connected across an initially uncharged 7 μF capacitor.

If the final potential difference across the 39 μF capacitor is 51 V, determine V_0.

Consider the setup shown, where a capacitor with a capacitance C is connected to a battery with emf V and negligible internal resistance.

Before the insertion of the dielectric slab with dielectric constant κ, the charge on the capacitor is $Q = C\,V$ and the energy density is $u = \frac{1}{2}\,\epsilon_0\,E^2$.

Now, keeping the battery connected, insert the dielectric, which fills the gap completely.



Suppose the battery remains connected during the insertion of the slab. Determine the energy density u' within the gap in the presence of the dielectric.

1. $u' = \frac{u}{\kappa^2}$
2. $u' = u\,\kappa$ **correct**
3. $u' = u\,\kappa^2$
4. $u' = u$
5. $u' = \frac{u}{\kappa}$

Resistance

$$\Delta V = -\int_i^f \vec{E} \bullet d\vec{l}$$

$$|\Delta V| = EL \longrightarrow E = \frac{|\Delta V|}{L}$$

$$J = \frac{I}{A} = \sigma E \longrightarrow I = \sigma A E \longrightarrow I = \frac{\sigma A}{L}|\Delta V| = \frac{1}{R}|\Delta V| = \frac{|\Delta V|}{R}$$

Conventional current: $I = \dfrac{|\Delta V|}{R}$ $\longleftarrow$ Widely known as Ohm's law

George Ohm (1789-1854)

Resistance of a long wire: $R = \dfrac{L}{\sigma A}$

Units: Ohm, Ω

Resistance combines conductivity and geometry!

Microscopic and Macroscopic View

Microscopic

$$\overline{v} = uE$$

$$i = nA\overline{v} = nAuE$$

Macroscopic

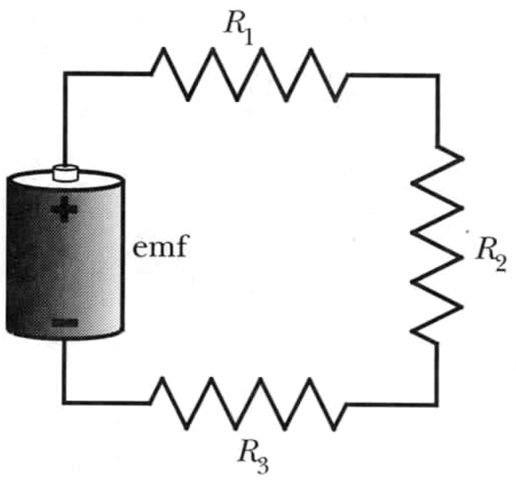
$$J = \sigma E$$

$$I = |q|nA\overline{v} = \frac{|\Delta V|}{R}$$

Can we write *V=IR* ?

Current flows in response to a ΔV

Series Resistance



$$\Delta V_{\text{batt}} + \Delta V_1 + \Delta V_2 + \Delta V_3 = 0$$

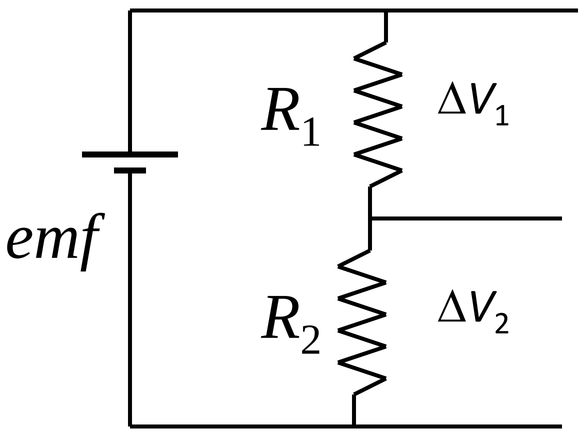
$$emf - R_1 I - R_2 I - R_3 I = 0$$

$$emf = R_1 I + R_2 I + R_3 I$$

$$emf = (R_1 + R_2 + R_3)\, I$$

$$emf = R_{\text{equivalent}}\, I \quad , \quad \text{where} \quad R_{\text{equivalent}} = R_1 + R_2 + R_3$$

Exercise: Voltage Divider



Know R , find $\Delta V_{1,2}$

Solution: $$I = \frac{|\Delta V|}{R} \longrightarrow |\Delta V| = IR$$

1) Find current: $$I = \frac{emf}{R_{equivalent}} = \frac{emf}{R_1 + R_2}$$

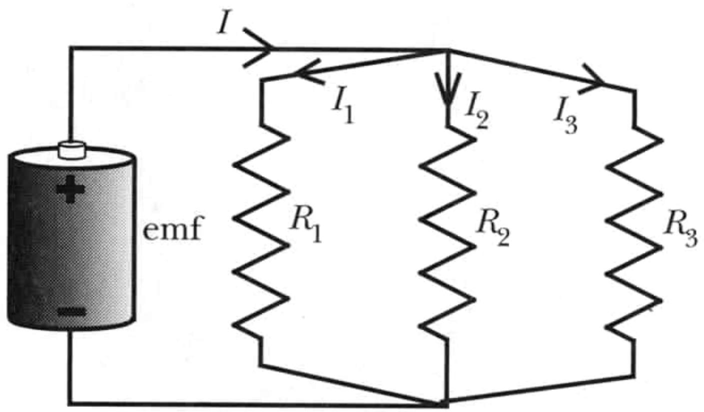
2) Find voltage:

$$|\Delta V_1| = IR_1 = emf\,\frac{R_1}{R_1 + R_2}$$

$$|\Delta V_2| = IR_2 = emf\,\frac{R_2}{R_1 + R_2}$$

3) Check: $|\Delta V_1| + |\Delta V_2| = emf \longrightarrow \quad emf\left[\frac{R_1}{R_1 + R_2} + \frac{R_2}{R_1 + R_2}\right] = emf$

Parallel Resistance



$$I = I_1 + I_2 + I_3$$

$$I = \frac{emf}{R_1} + \frac{emf}{R_2} + \frac{emf}{R_3}$$

$$I = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) emf = \frac{emf}{R_{equivalent}}$$

$$\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Work and Power in a Circuit

Current: charges are moving $\rightarrow$ work is done

Work = change in electric potential energy of charges

$$\Delta U_e = \Delta q \cdot \Delta V$$

Power = work per unit time:

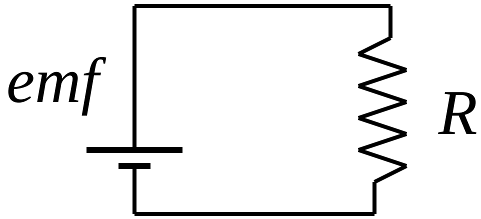
$$P = \frac{\Delta U_e}{\Delta t} = \frac{\Delta q \cdot \Delta V}{\Delta t} = \frac{\Delta q}{\Delta t} \Delta V$$

$\frac{\Delta q}{\Delta t} \rightarrow I$

Power for any kind of circuit component: $P = I\Delta V$

Units: $\mathrm{AV} = \frac{\mathrm{C}}{\mathrm{s}}\frac{\mathrm{J}}{\mathrm{C}} = \frac{\mathrm{J}}{\mathrm{s}} = \mathrm{W}$

Power Dissipated by a Resistor



Know ΔV, find P

$$P = I\Delta V$$

$$I = \frac{|\Delta V|}{R}$$

$$P = \frac{(\Delta V)^2}{R}$$

Know I, find P

$$P = I\Delta V$$

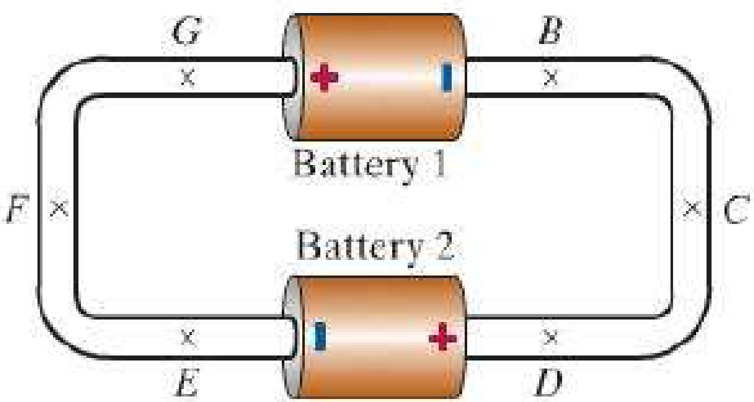
$$|\Delta V| = IR$$

$$P = I^2 R$$

In practice: need to know P to select right size resistor – capable of dissipating thermal energy created by current.

What is the power output of the battery?

A circuit constructed from two batteries and two wires, as shown in the Figure below. Each battery has an emf of 1.3 V. Each wire is 30 cm long and has a diameter of 0.0007 m. The wires are made of a metal that has 7×10^{28} electron/m^3 mobile electrons per cubic meter; the electron mobility is 5×10^{-5} (m/s)/(V/m). A steady current runs through the circuit. The locations marked by "x" and labeled by a letter are in the interior of the wire.

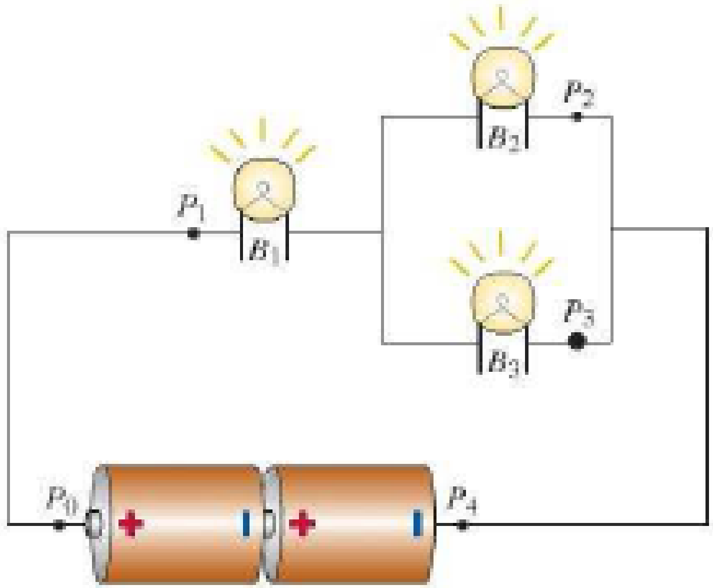


Figure

How many electrons per second enter the positive end of the battery 2?

$$i = n\,A\,u\,E$$

$$E = \frac{(\mathcal{E}_1 + \mathcal{E}_2)}{2\,L} = \frac{\mathcal{E}}{L}$$



Figure

Consider the circuit shown in the figure. Denote the total emf of the two cells unit by $\mathcal{E}$. The resistance of each of the three identical bulbs, by R. Determine the power in bulb1 and in bulb2. Denote the pair of the powers as: $[P_1, P_2]$.

$$\left[\frac{4}{9}\frac{\mathcal{E}^2}{R}, \frac{2}{9}\frac{\mathcal{E}^2}{R}\right]$$

$$\left[\frac{1}{4}\frac{\mathcal{E}^2}{R}, \frac{1}{16}\frac{\mathcal{E}^2}{R}\right]$$

$$\left[\frac{4}{9}\frac{\mathcal{E}^2}{R}, \frac{1}{9}\frac{\mathcal{E}^2}{R}\right]$$

$$\left[\frac{1}{4}\frac{\mathcal{E}^2}{R}, \frac{1}{8}\frac{\mathcal{E}^2}{R}\right]$$

Current flows through a light bulb connected to a circuit. Suddenly, a wire is connected across the terminals of the light bulb, in parallel with the light bulb in the circuit. What happens?

1. The light bulb continues to burn brightly, but all of the current flows through the wire.

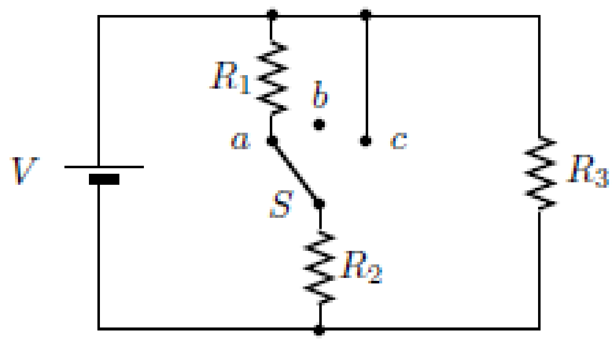
2. None of these.

3. All of the charge continues to flow through the light bulb.

4. Half of the charge flows through the light bulb, and half through the wire, causing the bulb to burn more dimly.

5. The light bulb extinguishes, and all the current flows through the wire.

In the figure below the switch S is initially in position **a** .



What happens to the current through R_3 when the switch is moved to the open position **b**? $R_1 = R_2 = R_3$. Neglect the internal resistance of the battery.

1. The current through R_3 is reduced to one-half its original value.

2. The current through R_3 remains the same.

3. The current through R_3 increases to three-halves its original value.

4. The current through R_3 increases to twice its original value.

5. The current through R_3 decreases to two-thirds its original value.

What happens when switch S is moved to position **c**, leaving R_2 and R_3 parallel?

1. The current through R_2 is half what it was with R_1 in the circuit.

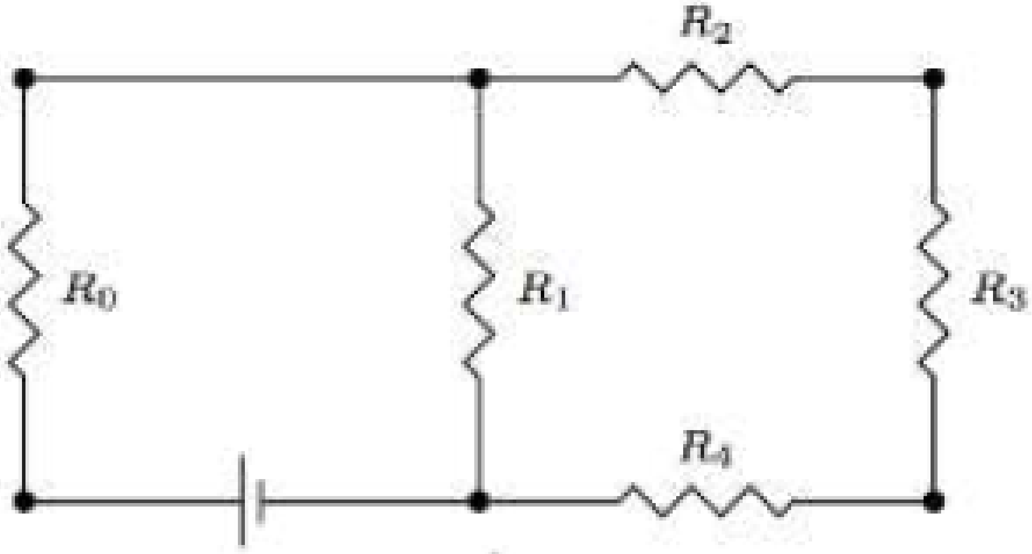
2. The current through R_3 increases.

3. The current through R_2 remains the same as when R_1 was in the circuit.

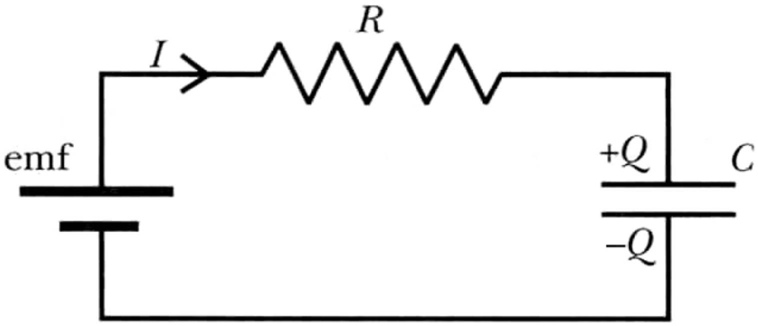
4. The current through R_2 and R_3 are now the same.

5. The current through R_3 decreases.

Calculate the *equivalent resistance* R_{eq} for the circuit below. Given that $R_0 = 186\ \Omega$, $R_1 = 58\ \Omega$ and $R_2 = R_3 = R_4 = 15\ \Omega$. {The equivalent resistance is the *total* resistance of the circuit, i.e. if you replaced all the resistors with a single resistor, it would have resistance R_{eq}.}



RC Circuit: Summary



Current in an RC circuit

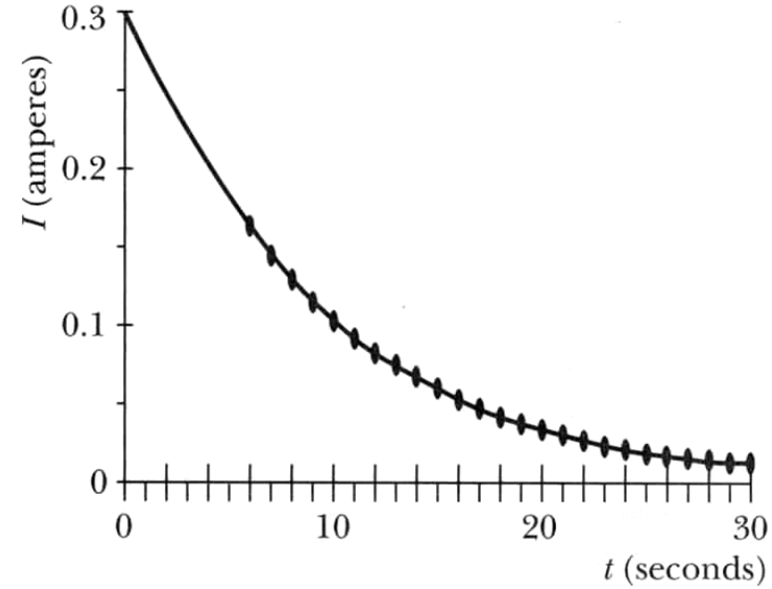
$$I = \frac{emf}{R} e^{-t/RC}$$

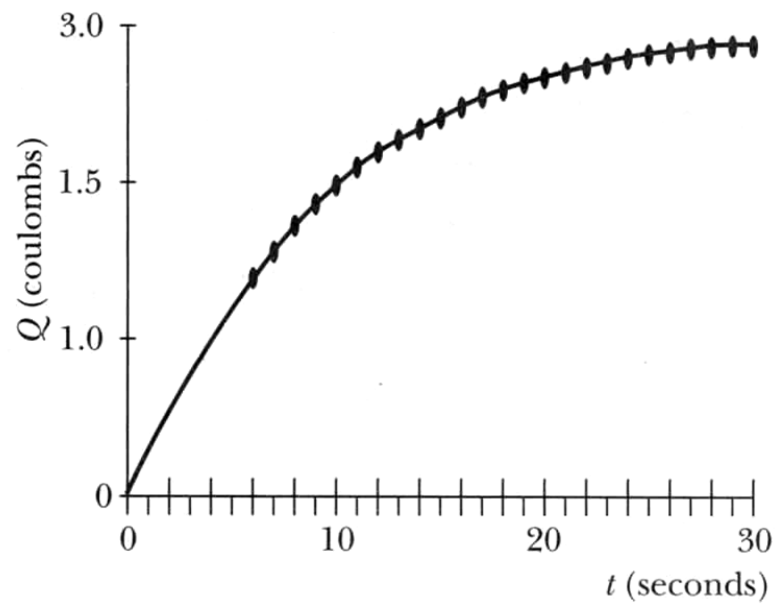
Charge in an RC circuit

$$Q = C\left(emf\right)\left[1 - e^{-t/RC}\right]$$

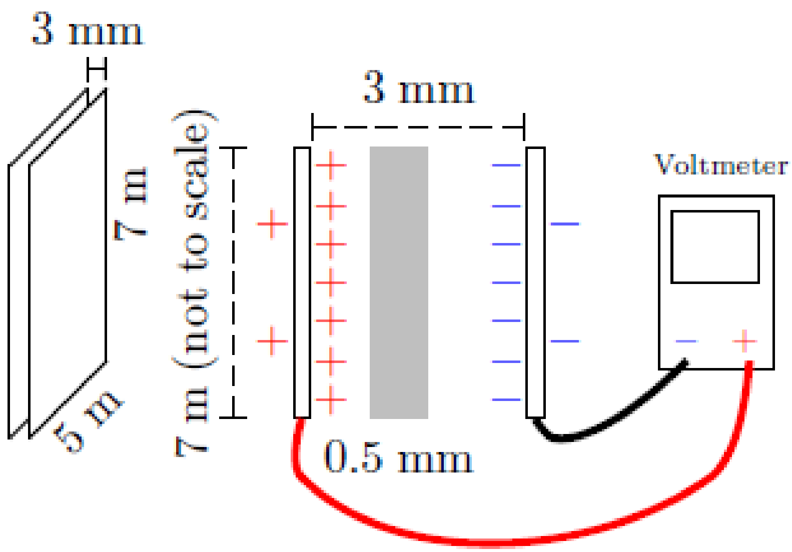
Voltage in an RC circuit

$$\Delta V = \left(emf\right)\left[1 - e^{-t/RC}\right]$$





A capacitor consists of two rectangular metal plates 5 m by 7 m, placed a distance 3 mm apart in air. The capacitor is connected to a 7 V battery long enough to charge the capacitor fully, and then the battery is removed. $\epsilon_o = 8.85 \times 10^{-12}$ F/m

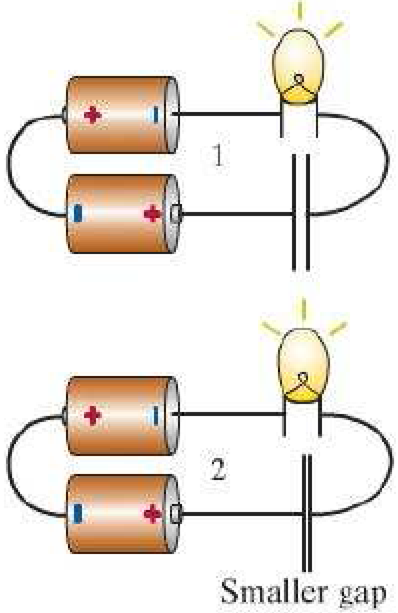


With the battery still disconnected, you insert a slab of plastic 5 m by 7 m by 0.5 mm between the plates, next to the positive plate, as shown in the figure. This plastic has a dielectric constant of 4.

After inserting the plastic, you connect a voltmeter to the capacitor. What is the initial reading of the voltmeter?

The voltmeter has a resistance of $1.1 \times 10^{8}\ \Omega$. What does the voltmeter read 3 seconds after being connected?

$$\Delta V = (6.125\ \text{V})e^{-\frac{t}{RC'}}$$



Consider the two circuits show in Figure above. The two circuits have identical setup, except that $gap_2 = \frac{gap_1}{3}$. At $t = 0$, the two capacitors have an initial charge of zero. The circuits are closed and the capacitors begin to get charged. Denote the capacitances of the two circuits by C_1 and C_2 and the potentials across the two capacitors at any time t by$V_1(t)$ and $V_2(t)$. Denote the electric fields in the capacitors after they are fully charged by E_1 and E_2.

Ia. $C_1 > C_2$.
Ib. $C_1 < C_2$.

IIa. $V_1(t) > V_2(t)$.
IIb. $V_1(t) < V_2(t)$.

IIIa. $E_1 > E_2$.
IIIb. $E_1 < E_2$.

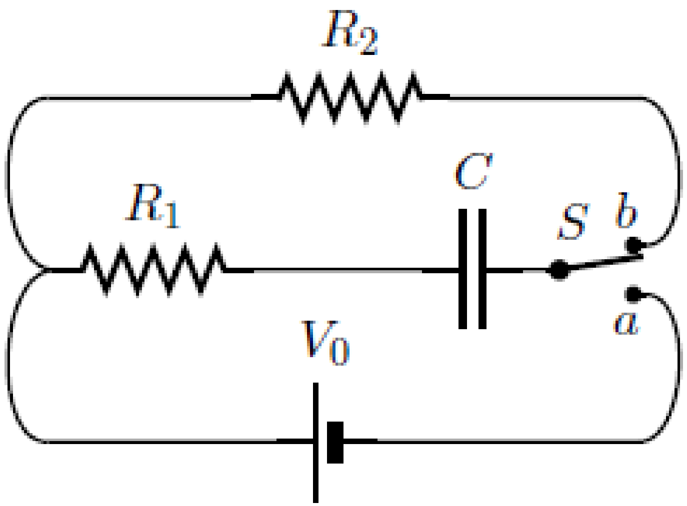
Ib, IIa, IIIb

Ia, IIa, IIIb

Ib, IIb, IIIa

Ib, IIb, IIIb

Consider the circuit shown below, where the capacitor is initially uncharged.



After S is switched to position a, the initial current through R_1 is

$$I_{t=0} = \frac{V_o}{R_1 + R_2}$$

$$I_{t=0} = \frac{R_1 + R_2}{R_1 R_2} V_o$$

$$I_{t=0} = \frac{V_o}{R_1}$$

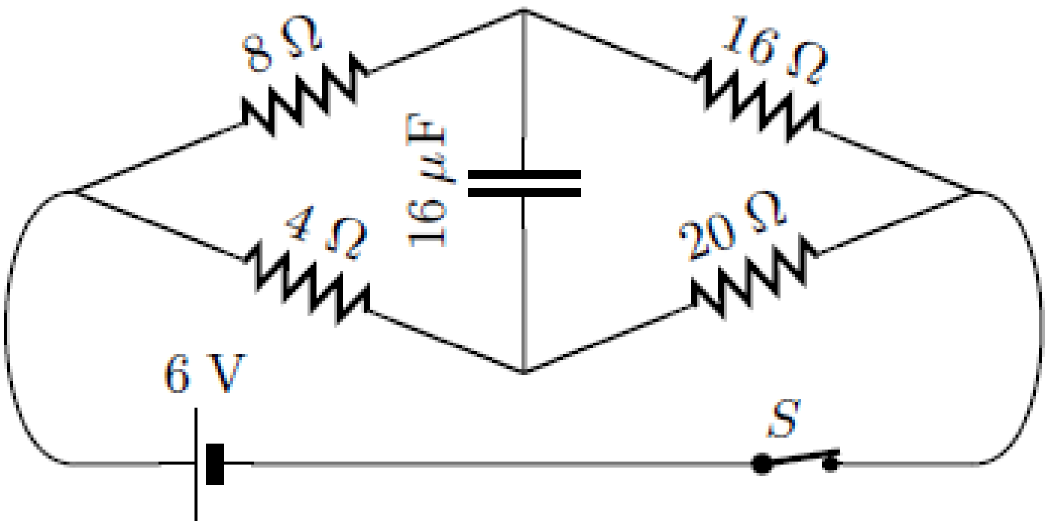
$$I_{t=0} = \frac{V_o}{R_2}$$

At a time $\frac{3}{2}\tau$ after S has been switched to position b, what is the power consumption of the circuit? $V_o = 11$ V, $C = 3$ μF, $R_1 = 8$ Ω, and $R_2 = 14$ Ω.

$$I(t) = \frac{V_o}{R_1 + R_2} e^{-t/\tau}$$

$$\left[\frac{V_o}{R_1 + R_2} e^{-3/2}\right]^2 (R_1 + R_2)$$

The circuit has been connected as shown in the figure for a “long” time.



What is the magnitude of the electric potential across the capacitor?

If the battery is disconnected, how long does it take for the voltage across the capacitor to drop to a value of $V(t) = \frac{\mathcal{E}_0}{e}$, where $\mathcal{E}_0$ is the initial voltage across the capacitor?