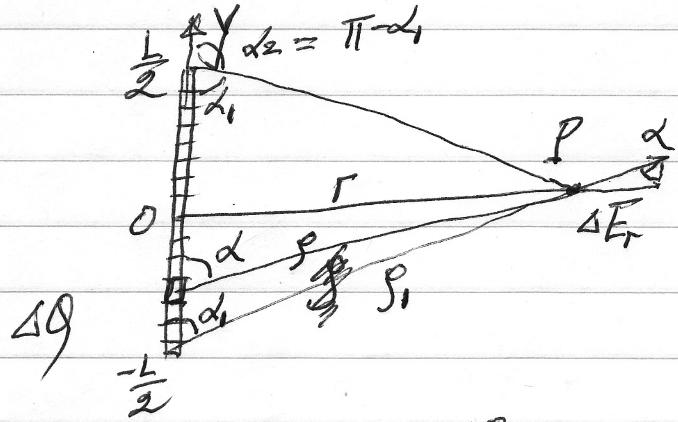
Ch 16: Fields derived through integrations

1. Field due to a charged rod



Geometry:

$\cos\alpha = \frac{|y|}{\rho}$, $\rho = \sqrt{r^2 + y^2}$

$\cos\alpha_1 = \frac{(L/2)}{\rho_1}$, $\rho_1 = \sqrt{r^2 + (\frac{L}{2})^2}$

Uniformly charged: $\frac{\Delta Q}{\Delta y} = \frac{Q}{L}$, $\Delta Q = \frac{Q}{L}\Delta y$

Symmetry: Resultant E is along r

$\sin\alpha = \frac{\Delta E_r}{\Delta E}$. $\therefore$ $\boxed{\Delta E_r = \Delta E \sin\alpha = \frac{k\left[\frac{Q}{L}\Delta y\right]}{\rho^2}\sin\alpha}$ (1)

$$E_r = \int dE_r = \int \frac{kQ}{L} \cdot \left(\frac{dy}{\rho^2}\right) \sin\alpha$$

Math ID - $\frac{dy}{\rho^2} = \frac{d\alpha}{r}$

$$= \boxed{\frac{kQ}{Lr}\int_{\alpha_1}^{\alpha_2} d\alpha \sin\alpha = \frac{kQ}{Lr}\left[\cos\alpha_1 - \cos\alpha_2\right]} \quad (2)$$

$\cos\alpha_2 = \cos(\pi - \alpha_1) = -\cos\alpha_1$

$$\therefore \boxed{E_r = \frac{kQ}{Lr} 2\cos\alpha_1 = \frac{kQ}{Lr} \cdot 2 * \frac{\frac{L}{2}}{\rho_1} = \frac{kQ}{r\rho_1}} \quad (3)$$

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Far field: $r \gg \frac{L}{2}$, $E_r = \frac{kQ}{r^2}$, since $\rho_1 = \sqrt{r^2 + (\frac{L}{2})^2} \sim r$

Near field: $r \ll \frac{L}{2}$, $E_r = \frac{kQ}{(L/2)} \cdot \frac{1}{r} \rightarrow \frac{1}{2\pi\epsilon_0} \cdot \frac{Q}{L} \cdot \frac{1}{r}$ (4)