Basic physics content of LC circuit.

Following is a more detailed version on LC circuit. You may think of it as an expanded version of the 2 lines given in the summary page.

1. Basic equations.

Loop eqn: $V_L + V_c = 0$, where $V_L = L\frac{dI}{dt}$ and $V_c = \frac{q}{C}$. (1)

Energies: $U_L = \frac{1}{2}LI^2$, $U_c = \frac{1}{2C}q^2$. (2)

Power-eqn: $P = \frac{dU}{dt} = \frac{d(qV)}{dt} = IV$

Power-eqn = I × loop-eqn $= IV_L + IV_c = 0$,

Or $0 = I\left(L\frac{dI}{dt}\right) + \frac{dq}{dt}\cdot\frac{q}{C} = \frac{d}{dt}\left[\frac{LI^2}{2} + \frac{q^2}{2C}\right] = \frac{d}{dt}\left(U_L + U_c\right)$ (3)

We will see LC circuit is analogous to the mass spring system in mechanics. Here U_L and U_c is analogous to the Kinetic energy (KE) and potential energy (PE) there.

For the mass-spring system conservation of energy implies during osc.

$(PE)_{max} = KE + PE = K_{max} = U_0$

For the LC circuit case, there are oscillations between U_L + U_0.

Correspondingly we have

$U_{Lmax} = U_L + U_c = U_{cmax} = U_0$ (4)

2. Solutions to the LC loop eqn.

From eq(1) with $I = dq/dt$, the loop eqn for q is

$$V_L + V_C = L\frac{dI}{dt} + \frac{q}{C} = L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

Or $\quad L\frac{d^2q}{dt^2} = -\frac{q}{C} \quad (5)$

In mechanics, you recall the mass spring system satisfies the Hook's Law, $F = ma$ implies that

$$m\frac{d^2x}{dt^2} = -kx$$

We can write it in the form: $\frac{d^2x}{dt^2} = -\omega^2 x$

The solution gives SHM, $x = x_{max}\cos(\omega t + \delta)$

where δ is the initial phase. For our illustration here, $\delta = 0$.

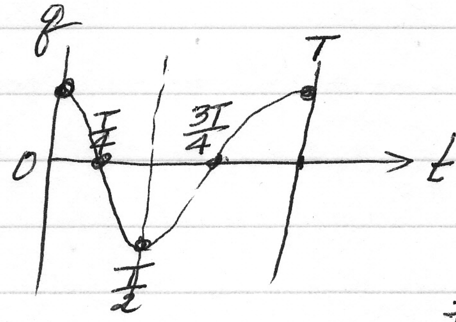
Analogous LC circuit solution is

$$q = q_{max}\cos\omega t \qquad (6)$$

$$I = \frac{dq}{dt} = -\omega q_{max}\sin\omega t \equiv -I_{max}\sin\omega t \qquad (7)$$

Graphic illustration of the solution is shown in Fig a + Fig b.

Fig a



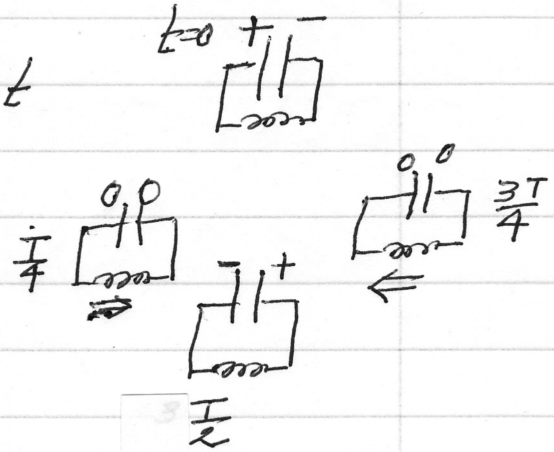
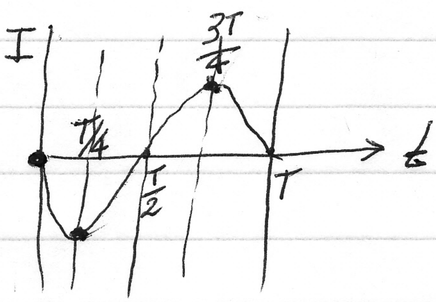


Fig b



Note: 1) q is the charge on the left plate

The reader should step through the sequence

$q = +q_{max}, 0, -q_{max}, 0, +q_{max}$

for $t = 0, \frac{T}{4}, \frac{T}{2}, \frac{3T}{4}, T$.

2) For I, $I = -I_{max}$ at $\frac{T}{4}$ where q has max negative slope.

$I = +I_{max}$ at $\frac{3T}{4}$ where q has max. positive slope.

Also note that

$$U_L + U_C = \frac{1}{2}L\left(I_{max}\sin\omega t\right)^2 + \frac{1}{2C}\left(q_{max}\cos\omega t\right)^2$$

$$= \frac{1}{2}L I_{max}^2 (\sin\omega t)^2 + \frac{q_{max}^2}{2C}(\cos\omega t)^2 = U_0(\sin^2\omega t + \cos^2\omega t) = U_0,$$

where we have used: $\frac{1}{2}L I_{max}^2 = \frac{1}{2}L\omega^2 q_{max}^2 = \frac{1}{2}L\left(\frac{1}{LC}\right)q_{max}^2 = \frac{q_{max}^2}{2C} = U_0$