$$I_{plate} = \int\int_{plate} dm\ r^2$$

As usual, we replace dm by the density σ times the infinitesimal volume element $dA = dxdy$. By the Pythagorean theorem, $r^2 = x^2 + y^2$.

$$\begin{aligned} I_{plate} &= \int_{-a/2}^{a/2}\int_{-b/2}^{b/2} r^2\sigma dA \\ &= \int_{-a/2}^{a/2}\int_{-b/2}^{b/2} (x^2+y^2)\sigma dxdy \\ &= \sigma\int_{-a/2}^{a/2} \frac{1}{3}\left(\left(\frac{b}{2}\right)^3 - \left(-\frac{b}{2}\right)^3\right) + y^2(\frac{b}{2}+\frac{b}{2})dy \\ &= \sigma\frac{b^3}{12}\left(\frac{a}{2}+\frac{a}{2}\right) + \sigma b\frac{1}{3}\left(\left(\frac{a}{2}\right)^3 - \left(-\frac{a}{2}\right)^3\right) \\ &= \frac{\sigma}{12}(b^3a + ba^3) \end{aligned}$$

And now using the formula for the density of the plate $\sigma = \frac{M}{ab}$,

$$\begin{aligned} I_{plate} &= \frac{1}{12}M\frac{b^3a+ba^3}{ab} \\ &= \frac{1}{12}M\left(a^2+b^2\right) \end{aligned}$$