$$I_{sphere} = \int\int\int_{sphere} dm\ s^2$$

I use s as the variable instead of r to remind you that the distance from the origin (r) is different than the distance to the axis of rotation, which we'll say is the z-axis ($s = r\sin\theta$). As usual, we obtain the infinitesimal mass dm of a tiny volume element by multiplying the uniform density ρ by the infinitesimal volume dV. Then,

$$\begin{aligned} I_{sphere} &= \int\int\int_{sphere} s^2 \rho dV \\ &= \int\int\int_{sphere} r^2 \sin^2\theta \rho dV \end{aligned}$$

Here's where knowledge of calculus III comes in handy. The easiest way to integrate over a spherically symmetric volume is to use spherical coordinates r, θ, ϕ instead of the usual cartesian coordinates x, y, z. The rectangular volume element $dxdydz$ then becomes the spherical volume element $r^2 \sin\theta dr d\theta d\phi$.

$$\begin{aligned} I_{sphere} &= \int_0^{2\pi}\int_0^{\pi}\int_0^{R} r^2 \sin^2\theta \rho (r^2 \sin\theta dr d\theta d\phi) \\ &= 2\pi\rho \int_0^{\pi}\int_0^{R} r^4 \rho \sin^3\theta dr d\theta \\ &= \frac{2\pi\rho}{5} R^5 \int_0^{\pi} \sin^3\theta d\theta \\ &= \frac{2\pi\rho}{5} R^5 \frac{4}{3} \end{aligned}$$

But that still doesn't look like the result we're supposed to get. That's because we still have the density ρ in there. Let's get rid of it by using $\rho = M/(\frac{4}{3}\pi R^3)$.

$$\begin{aligned} I_{sphere} &= \frac{2}{5}\frac{4}{3}\frac{3}{4} M R^2 \\ &= \frac{2}{5} M R^2 \end{aligned}$$