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Citation: AIP Conf. Proc. **193**, 227 (1989); doi: 10.1063/1.38733
View online: http://dx.doi.org/10.1063/1.38733
View Table of Contents: http://proceedings.aip.org/dbt/dbt.jsp?KEY=APCPCS&Volume=193&Issue=1
Published by the American Institute of Physics.

Localized Profiles of Optical Beams in Plasma

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Abstract. Self-focusing of an intense optical beam in a plasma is studied, including the nonlinear effects of both the relativistic electron mass and the ponderomotive potential due to the electromagnetic wave. An exact steady asymptotic solution of beam propagation in a localized solitary wave form is obtained in slab geometry. Amplitude - width scaling relations are obtained, which imply that the width is limited to be less than square root of three of the collisionless skin depth. In the nonrelativistic limit, keeping only the relativistic mass effects, our solution reduces to the solution obtained by Schmidt and Horton. The asymptotic nature of the solitary wave is tested using a recently developed numerical particle simulation code.

I. INTRODUCTION

The nonlinear self-focusing of intense electromagnetic radiation in a dielectric media has been studied for well over twenty years now.[1] The development of powerful lasers and various applications of them, has prompted a considerable interest in self-focusing processes in plasma. In particular, the concepts of laser ignited fusion and laser-plasma particle accelerators such as the beat wave accelerator[2] and the plasma fiber accelerator[3] require transport of the laser beam with minimal loss in intensity over a considerable distance. A mechanism that allows for such a transport of the beam, without significant depletion, is discussed here.

As an intense laser beam enters plasma, an initial transient phase is expected. The interesting question is, what kind of a stationary state will the system assume after the initial transient. In particular, what is the asymptotic or steady form of the beam profile as it traverses through the plasma. Soliton-like or solitary wave profiles (either single or multiple) have emerged from several studies. An asymptotic profile of solitary nature would indeed be welcome for the above mentioned applications[1-3] (particularly for the plasma fiber accelerator[3]) because for such a profile the beam propagates without transverse spreading and thus, without losing its intensity or profile in this way.

In this paper we obtain an asymptotic profile for short laser pulses propagating in a cool plasma. The advantage of using a short laser pulse is that the ions, being massive, do not have time to respond and thus can be taken to be immobile. Therefore, the laser plasma system should be free of the parametric instabilities associated with the ion motion. The self-focusing process for a quasineutral plasma[4] is absent due to the short time scale. Since we are studying a cool plasma, only ponderomotive and relativistic effects are considered; the thermal self-focusing[5] effect should be negligible.

In Sec. II we present the basic evolution equations for the laser-plasma system. In Sec. III we look for an asymptotic profile of the laser beam by taking an ansatz which makes the evolution equations separable. The equations are solved analytically in slab approximation. Comparisons to earlier work on the subject are made. In Sec. IV the asymptotic profile obtained analytically is tested using a particle simulation program. In Sec. V the results derived are summarized.

II. EVOLUTION EQUATIONS

The basic set of equations describing the laser-plasma system consists of Maxwell's equations and the equation of motion for relativistic electrons. The electron pressure gradient is neglected in comparison with the ponderomotive force and the electrons are treated as cold. The assumption of immobile ions, justified as previously mentioned by the shortness of the laser pulse, allows us to write the charge density and plasma current in terms of the equilibrium density n_0 and the electron density perturbation δn_e:

$$\begin{aligned} &\Sigma e_j n_j = -e\delta n_e \\ &\mathbf{J} = -e(n_0 + \delta n_e)\mathbf{v} \quad . \end{aligned} \tag{1}$$

Expressing the electromagnetic fields in terms of the potentials we obtain

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} - c^2\nabla^2\mathbf{A} + c\nabla\frac{\partial\Phi}{\partial t} = 4\pi c\mathbf{J} \quad , \tag{2}$$

where the Coulomb gauge, $\nabla \cdot \mathbf{A} = 0$, is chosen because it allows for a clear separation of the slowly and rapidly varying components of the electric field.

From here on we will use a normalized vector potential: $\mathbf{A} \rightarrow \mathbf{A}_n \equiv \frac{e\mathbf{A}}{mc^2} \equiv \sqrt{I_n}$. To single out the rapid laser variations, we take a trial function of the form

$$\mathbf{A_n} = a_n(\mathbf{r},t)e^{i(k_0 z - \omega_0 t - \psi(\mathbf{r},t))}(\hat{\mathbf{x}} + \mathbf{i}\hat{\mathbf{y}}) \quad , \tag{3}$$

where $a_n(\mathbf{r},t)$ and $\psi(\mathbf{r},t)$ are real functions of space and time, k_0 and ω_0 are the (constant) wavenumber and frequency of the laser wave in uniform, unperturbed plasma, and we have chosen the coordinate system so that the z-axis coincides with the direction of propagation. The wave is taken to have circular polarization.

Next we apply the slowly varying envelope approximation: the characteristic spatial length of the structure in our system is assumed much greater than the wavelength of the wave and the characteristic time period involved is assumed to be much longer than the laser oscillation period:

$$\begin{gathered} |\frac{\partial a_n}{\partial z}| \ll k_0 a_n \ , \ |\frac{\partial a_n}{\partial t}| \ll \omega_0 a_n \\ |\frac{\partial \psi}{\partial z}| \ll k_0 \ , \ |\frac{\partial \psi}{\partial t}| \ll \omega_0 . \end{gathered} \tag{4}$$

The electron velocity is approximately given by

$$\mathbf{v} = \frac{\mathbf{p}}{m\gamma} = \frac{e}{mc}\frac{\mathbf{A}}{\sqrt{1+I_n}} \ , \tag{5}$$

and the plasma current can be written as

$$\mathbf{J} = -en_e\mathbf{v} = -\frac{{\omega_{p}}_0^2}{4\pi c}\frac{N_e}{\sqrt{1+I_n}}\mathbf{A} \ , \tag{6}$$

where $N_e \equiv 1 + \delta n_e/n_0$ and only electrons contribute to the current. According to the slowly varying envelope approximation no significant development takes place in the time scale of the rapid laser oscillations. We thus average the wave equation over the laser oscillation period $T_0 = \frac{2\pi}{\omega_0}$. The real terms of the wave equation then yield an equation describing the evolution of the amplitude,[6]

$$\begin{aligned} \frac{\partial^2 a_n}{\partial t^2} =& a_n(\omega_0 + \frac{\partial \psi}{\partial t})^2 + c^2\nabla^2 a_n - \\ & c^2 a_n\{(k_0 - \frac{\partial \psi}{\partial z})^2 + |\nabla_T \psi|^2\} - {\omega_{p}}_0^2 \frac{N_e}{\sqrt{1+a_n^2}} a_n \quad , \end{aligned} \tag{7}$$

while the imaginary terms yield an equation for the phase shift,[6]

$$\begin{aligned} \frac{\partial}{\partial t}\left(a_n^2 \frac{\partial \psi}{\partial t}\right) =& -\omega_0 \frac{\partial a_n^2}{\partial t} - c^2 k_0 \frac{\partial a_n^2}{\partial z} + \\ & c^2\left(\nabla a_n^2\right)\cdot(\nabla \psi) + c^2 a_n^2 \nabla^2 \psi \quad , \end{aligned} \tag{8}$$

where $\lambda_c \equiv c/\omega_{pe}$ is the collisionless skindepth, and ∇_T is the transverse part of the gradient, $\nabla = \nabla_T + \frac{\partial}{\partial z}\,\hat{\mathbf{z}}$. The last term on the right hand side of equation (7) represents all the relevant nonlinearities, i.e. the ponderomotive force [acting through the normalized electron density N_e as will be given below by Eq. (9)]

and the relativistic electron mass effects (appearing as the inverse square root factor).

We shall look for a stationary state, assuming that the outward laser ponderomotive force exerted on the electrons is balanced by the electrostatic field produced by the charge separation when the electrons are driven outward. Under these circumstances the electron density perturbation can be expressed in the form[6,7]

$$N_e \equiv 1 + \frac{\delta n_e}{n_0} = 1 + \lambda_c^2 [\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}] \sqrt{1 + I_n} \ , \tag{9}$$

where we have also assumed axial symmetry, $\frac{\partial}{\partial \theta} = 0$. It is important to notice that this particular model does not have a mechanism for preventing negative — and thus unphysical — values for electron density. Therefore, once a solution is obtained using this model, it is necessary to check if the solution corresponds to physically meaningful values of electron density. For the stationary state described, the field equations become

$$2k_0 \frac{\partial \psi}{\partial z} - |\nabla \psi|^2 + \frac{1}{a} \nabla^2 a - \frac{1}{\lambda_c^2} \frac{N_e}{\sqrt{1 + a^2}} + \left(\frac{\omega_0^2}{c^2} - k_0^2 \right) = 0 \ , \tag{10}$$

and

$$-k_0 \frac{\partial a^2}{\partial z} + (\nabla a^2) \cdot (\nabla \psi) + a^2 \nabla^2 \psi = 0 \ . \tag{11}$$

Here (and henceforth) we have dropped the subscript n for convenience.

III. ASYMPTOTIC LASER PROFILE

We look for a stationary and asymptotic intensity profile independent of z for the laser beam under the combined influence of the ponderomotive and relativistic effects. We choose the following ansatz for the amplitude and phase:

$$\begin{aligned} a(r,z) &= a(r) \quad , \\ \psi(r,z) &= f(z) + g(r) \quad , \end{aligned} \tag{12}$$

where we have still allowed for phase modulation in z. Equations (10) and (11) are separable under this ansatz. Equation (10) yields

$$-2k_0\frac{df}{dz} - \kappa_0^2 + \left(\frac{df}{dz}\right)^2 = C_1$$
$$= \frac{1}{a}\frac{1}{r}\frac{d}{dr}r\frac{da}{dr} - \left(\frac{dg}{dr}\right)^2 - \frac{1}{\lambda_c^2}\frac{N_e}{\sqrt{1+a^2}} \quad , \tag{13}$$

where $\kappa_0^2 \equiv \frac{\omega_0^2}{c^2} - k_0^2$, and C_1 is the separation constant. The phase equation (11) yields

$$-\frac{d^2f}{dz^2} = C_2 = \frac{1}{r}\frac{d}{dr}r\frac{dg}{dr} + \frac{1}{a^2}\frac{da^2}{dr}\frac{dg}{dr} \tag{14}$$

where C_2 is the separation constant.

Equation (14) requires the z-dependent part of the phase shift ψ to have the form

$$f(z) = -\frac{1}{2}C_2 z^2 + C_3 z \quad , \tag{15}$$

where the (arbitrary) constant phase shift has been dropped. Now substituting Eq. (15) into Eq. (13) implies that $f(z)$ is given by the linear expression

$$f(z) = k_0 z \pm \sqrt{\frac{\omega_0^2}{c^2} + C_1}\ z \quad . \tag{16}$$

The assumption of slow modulations, $|\frac{\partial\psi}{\partial z}| \ll k_0$, implies that we have to choose square root with the negative sign to retain consistency. Thus

$$f(z) = k_0 z - \sqrt{\frac{\omega_0^2}{c^2} + C_1}\ z \quad , \tag{17}$$

where for consistency C_1 should be much less than k_0^2 in an underdense plasma. The constant C_1 can thus be interpreted as a measure of the z-dependence of the phase modulation. The radial portions of Eqs. (13) and (14) can now be written as

$$\frac{1}{a}\frac{1}{r}\frac{d}{dr}r\frac{da}{dr} - \left(\frac{dg}{dr}\right)^2 - \frac{1}{\lambda_c^2}\frac{N_e}{\sqrt{1+a^2}} = C_1 \quad , \qquad \text{and}$$
$$\frac{1}{r}\frac{d}{dr}r\frac{dg}{dr} + \frac{1}{a^2}\frac{da^2}{dr}\frac{dg}{dr} = 0 \quad . \tag{18}$$

Equations (18) in the slab limit are

$$\frac{1}{a}\frac{d^2}{dx^2}a - \left(\frac{dg}{dx}\right)^2 - \frac{1}{\lambda_c^2}\frac{N_e}{\sqrt{1+a^2}} = C_1 \quad , \tag{19}$$

and

$$a^2\frac{dg}{dx} = C_4 \quad , \tag{20}$$

where the phase equation was integrated once over x, bringing about the integration coefficient C_4, which can be interpreted to be a measure of the amplitude dependent transverse phase modulation. Combining Eqs. (19), (20) and (9) yields a differential equation for a only:

$$\frac{1}{a}\frac{d^2}{dx^2}a - \frac{C_4^2}{a^4} - \frac{1}{\lambda_c^2}\frac{1}{\sqrt{1+a^2}} - \frac{1}{\sqrt{1+a^2}}\frac{d^2}{dx^2}\sqrt{1+a^2} = C_1 \quad . \tag{21}$$

An equation of the form of Eq. (21) can be derived from the Hamilton's principle. Treating Eq. (21) as the "equation of motion" for the laser-plasma system with the coordinate x playing the role of time, we write the Lagrangian of the system in the form

$$L = g(a)\frac{a'^2}{2} - V(a) \ , \tag{22}$$

where $g(a)$ is a metric and $V(a)$ is the potential of the system, both yet to be determined, and where prime stands for the derivative with respect to the time-like variable. Lagrange's equation then yields

$$g(a)a'' + \frac{1}{2}\frac{dg}{da}a'^2 + \frac{\partial V}{\partial a} = 0 \ , \tag{23}$$

We find an integrating factor $\mu(a)$ by requiring that Eq. (21) multiplied by $\mu(a)$ should coincide with Eq. (23). We thus obtain

$$\begin{aligned} \mu(a) &= a \quad \text{and} \\ g(a) &= \frac{1}{1+a^2} \ , \end{aligned} \tag{24}$$

and from Eq. (21) the potential is seen to be

$$V(a) = \frac{1}{2}\frac{C_4^2}{a^2} - \frac{1}{\lambda_c^2}\sqrt{1+a^2} - \frac{1}{2}C_1 a^2 \ . \tag{25}$$

Applying Noether's theorem we can now write down the first integral of Eq. (21),

$$\begin{aligned} \mathcal{E} &= \frac{\partial L}{\partial a'}a' - L(a,a') \\ &= \frac{1}{2}g(a){a'}^2 + V(a) \ . \end{aligned} \tag{26}$$

Thus,

$$\mathcal{E} = \frac{{a'}^2}{2(1+a^2)} + \frac{C_4^2}{2a^2} - \frac{1}{\lambda_c^2}\sqrt{1+a^2} - \frac{C_1}{2}a^2 \ . \tag{27}$$

Exact Solutions

Seeking analytical solutions of Eq. (27) with finite total power we impose the boundary conditions: $a, a' \rightarrow 0$ as $x \rightarrow \infty$. This requires that the "energy" $\mathcal{E}$ and the coefficient C_4 have the following values:

$$\begin{aligned} C_4 &\equiv 0 \\ \mathcal{E} &\equiv -\frac{1}{\lambda_c^2} \quad . \end{aligned} \tag{28}$$

The amplitude equation now becomes

$${a'}^2 = C_1 a^4 + \left(C_1 - \frac{2}{\lambda_c^2}\right)a^2 - \frac{2}{\lambda_c^2} + \frac{2}{\lambda_c^2}\left(1+a^2\right)^{\frac{3}{2}} \quad . \tag{29}$$

If we let $\xi = x/\lambda_c$ and change variables according to $y(\xi)^2 = \sqrt{a(\xi)^2+1} - 1$, Eq. (29) takes on an elementary form, which has the solution

$$y = \frac{\pm 16\kappa^2 E}{E^2 \mp 4E(2\kappa^2 - 1) + 4} \ , \tag{30}$$

where $E \equiv exp(-2\kappa(\xi + C_8))$ and $\kappa^2 \equiv \lambda_c^2 C_1 + 1$. Unraveling the change of variable leads to

$$I = a^2 = \frac{\pm 32(E \pm 2)^2 \kappa^2 E}{\left[(E\pm 2)^2 \mp 8\kappa^2 E\right]^2} \ . \tag{31}$$

Since the intensity is positive, the upper sign in the expression (31) is relevant. The integration constant C_8 imbedded within E corresponds to merely a shift of the solution along the x-axis. From Eq. (31) the profile centered at $\xi = 0$ is

$$a = \frac{2\kappa \operatorname{sech}(\kappa\xi)}{1 - \kappa^2 \operatorname{sech}^2(\kappa\xi)} \,. \tag{32}$$

The parameter κ is now seen to be related to the inverse width of the profile in units of λ_c^{-1}. Further, κ is directly related to both the peak intensity of the profile at $\xi = 0$, and to the total power of the beam:

$$a_m = a(\xi = 0) = \frac{2\kappa}{1 - \kappa^2} \,. \tag{33}$$

The total power of the laser beam in dimensionless variables is given by

$$\begin{aligned} P &\equiv \int_{-\infty}^{+\infty} I(\xi) d\xi \\ &= 4[\frac{\kappa}{1-\kappa^2} + \frac{\arctan\left(\frac{\kappa}{\sqrt{1-\kappa^2}}\right)}{(1-\kappa^2)^{3/2}}] \,. \end{aligned} \tag{34}$$

Equation (34) is a scaling relation between the width and the power of the laser profile. The presence of a relation between the amplitude and the width is typical of soliton-like structures, although the relation (33) is not linear and is thus different from the KdV -soliton.

Physically reasonable values for κ^2 are given by

$$0 < \kappa^2 < 1 \,. \tag{35}$$

The lower limit excludes trivial solutions with zero amplitudes, and the upper limit keeps the peak amplitude finite. In Fig. 1(a) we have plotted profiles for various values of maximum amplitude a_m. In Fig. 1(b) are the corresponding potentials. The solitary type profile corresponds to the homoclinic orbit that the potential is found[8] to exhibit when $0 < \kappa^2 < 1$. There could, however, exist also oscillatory type solutions.[8]

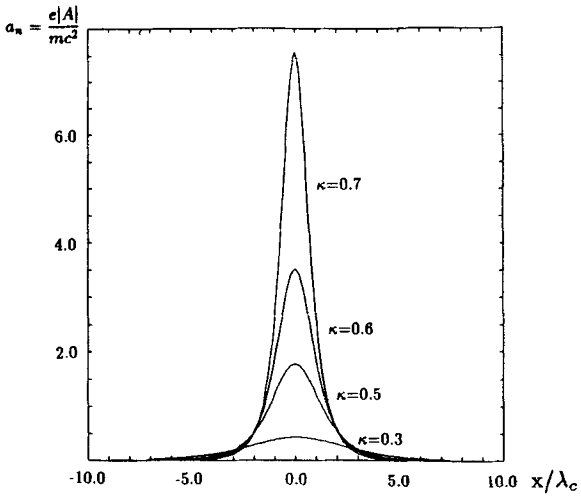

Figure 1(a)

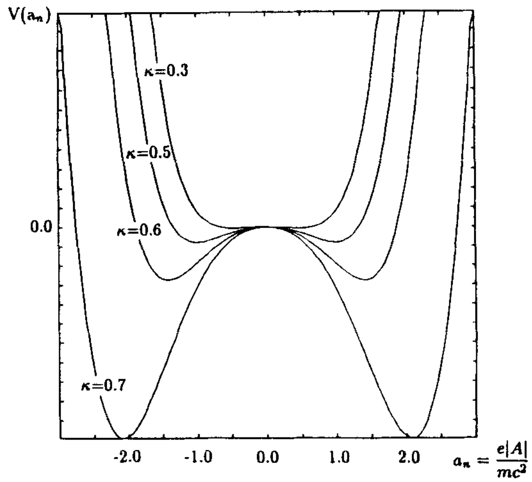

Figure 1(b)

Fig. 1 - Solitary profiles - 1(a) The asymptotic amplitude profile for various values of the beam width parameter κ. The normalized peak amplitude, defined as $a_0 \equiv \frac{eE_0}{mc\omega_0}$, is given by $a_0 = \frac{2\kappa}{1-\kappa^2}$: (1) $\kappa = 0.3$, $a_0 = 0.66$, (2) $\kappa = 0.5$, $a_0 = 1.33$, (3) $\kappa = 0.6$, $a_0 = 1.88$, (4) $\kappa = 0.7$, $a_0 = 2.75$. 1(b) The characteristic potential of the system V(a) describing the beam profile plotted for the same values of κ as in 1(a).

Another restriction on the possible values of κ^2 can be found by requiring that the solution correspond to physical values of the electron density given by Eq. (9). As mentioned when introducing the model for electron density, Eq. (9), it is necessary to check if these solutions correspond to physically meaningful values of electron density. From Eq. (9) the electron density perturbation $\frac{\delta n_e}{n_0}$ can be rewritten in terms of the variable $y(\xi)$ as

$$\frac{\delta n_e}{n_0} = y'' \sinh y + y'^2 \cosh y \ . \tag{36}$$

From the Hamiltonian formalism the y-derivatives can be expressed in terms of the potential V(y) and the total energy of the system $\mathcal{E}$ as

$$\begin{aligned} y'' &= -\frac{\partial V}{\partial y} \\ y' &= 2(\mathcal{E} - V) \ , \end{aligned} \tag{37}$$

yielding

$$\frac{\delta n_e}{n_0} = 2\mathcal{E} \cosh y + 2\lambda_c^2 C_1 \sinh^2 y \cosh y + \sinh^2 y + 2\cosh^2 y \ . \tag{38}$$

The critical case, corresponding to a total depletion of the electrons, is given by $\frac{\delta n_e}{n_0} = -1$. This case is specified by the electron depletion curve given by Eq. (38) evaluated at $\frac{\delta n_e}{n_0} = -1$:

$$\mathcal{E}(C_1, y) = -\frac{3}{2} \cosh y - \lambda_c^2 C_1 \sinh^2 y \ . \tag{39}$$

The physically meaningful solutions lie above the electron depletion curve corresponding to positive values of the electron density. The critical case for the homoclinic orbit corresponds to the situation when the intersection of the depletion curve and the potential takes place at the maximum amplitude of the homoclinic orbit. Setting the potential given by Eq. (25) equal to the depletion curve given by Eq. (39) we obtain

$$\cosh y_* + \lambda_c^2 C_1 \sinh^2 y_* = 0 \ . \tag{40}$$

For the homoclinic orbit the energy is, accoring to Eq. (28), $\mathcal{E} = -\frac{1}{\lambda_c^2}$. Equation (39) thus yields

$$\cosh y_* = 2 \; . \tag{41}$$

Substituting this value for y_* back to Eq. (40) we obtain

$$\lambda_c^2 C_1 = -2/3 \, , \quad \text{or equivalently} \quad \kappa^2 < \frac{1}{3} \, . \tag{42}$$

For values satisfying the condition (42), the potential lies above the depletion curve corresponding to physically meaningful values of electron density. Therefore, within the framework of the model for electron density as given by Eq. (9) the self-consistent solitary profiles are given by Eq. (32) while the width of the profile is restricted by

$$0 < \kappa^2 < \frac{1}{3} \, . \tag{43}$$

Nonrelativistic Solutions

It is interesting to compare the results yielded by the present approach to work done earlier on the subject.[9,10] Schmidt and Horton[10] studied purely relativistic self-focusing for non-relativistic field amplitudes, $a^2 < 1$. Expanding the left-hand side of Eq. (29) for $a^2 < 1$, keeping the electron density constant, $N_e \equiv 1$, yields

$$a'^2 = \left(C_1 + \frac{1}{\lambda_c^2} \right) a^2 - \frac{1}{4} \frac{1}{\lambda_c^2} a^4 + O(a^6) \quad . \tag{44}$$

Recalling the shorthand notation $\kappa^2 \equiv \lambda_c^2 C_1 + 1$ Eq. (44) reduces to

$$\frac{d}{d\xi} a = \pm \sqrt{\kappa^2 a^2 - \frac{1}{4} a^4} \quad , \tag{45}$$

which yields the following expression for the intensity:

$$I(x) = -16\kappa^2 \frac{C_6 e^{\pm 2\kappa\xi}}{\left(1 - C_6 e^{\pm 2\kappa\xi}\right)^2} \quad . \tag{46}$$

For the solution to remain finite for all values of ξ, $C_6 \equiv -C_7 < 0$. The profile is found to have one (and only one) extremum when $C_7 > 0$, a condition that coincides with the one already established above for the finiteness of the solution. The extremum, x_m, is given by

$$x_m = -\frac{\ln C_7}{2\kappa} \quad , \tag{47}$$

and the intensity profile centered at $\xi = 0$ is

$$I(\xi) = 4\kappa^2 \frac{1}{\cosh^2(\kappa\xi)} \quad . \tag{48}$$

The complete solution obtained by Schmidt and Horton[10] is

$$\begin{aligned} I(\xi) \equiv A_n^2(\xi) = I_0 \frac{1}{\cosh^2(\sqrt{\alpha_{SH}}\xi)} \quad , \qquad \alpha_{SH} = \frac{1}{2} I_0 \\ \varphi = \frac{1}{c}\sqrt{\omega^2 + \omega_{p0}^2(\frac{1}{2}I_0 - 1)}\; z - \omega t \quad . \end{aligned} \tag{49}$$

while the complete solution obtained by us is

$$\begin{aligned} I(\xi) = I_0 \frac{1}{\cosh^2(\kappa\xi)} \quad , \qquad \kappa^2 = \frac{1}{4} I_0 \\ \varphi(\xi) = \frac{1}{c}\sqrt{\omega_0^2 + \omega_{p0}^2(\frac{1}{4}I_0 - 1)}\; z - \omega_0 t \quad . \end{aligned} \tag{50}$$

For a given frequency $\omega = \omega_0$, our solution coincides (within a factor of two) with the one obtained by Schmidt and Horton. The factor of two difference arises from the different choice for the polarization of the laser field: Schmidt and Horton assumed a linearly polarized wave for which the time averaging process produces a factor of $\frac{1}{2}$, whereas in this work we have used circular polarization for which the time averaging does not introduce any numerical factors. Taking this difference into account, it is seen that the profiles are consistent.

IV. NUMERICAL RESULTS

In this section we compare the theoretical results derived in the preceding sections with a recently developed computer simulation.[11] The code used is a time averaged particle simulation code developed for modeling transport of optical beams in plasmas.[11] The code operates on the same principles as any standard electromagnetic particle simulation code, except that it uses the wave equation which is phase averaged over the rapid laser oscillations. Similarly, the equation of motion for the relativistic electrons is averaged, thus yielding the laser contribution in the form of the ponderomotive force. We have here used a version of the code that is formally one-dimensional but which in fact potrays a two-dimensional situation — the code 'time' representing the the direction of propagation for a stationary state. The code uses periodic boundary conditions, the width of the simulation box is chosen to be $51.2\lambda_c$ and there are 100 electrons per grid cell. The number of grid points for the simulations discussed below was 256 and the time step was chosen at dt $= 0.1\omega_{pe}^{-1}$.

In the first case we set up a Gaussian intensity profile with normalized intensity (quivering velocity) $I = 0.16$ and beam waist $w_b = 8\lambda_c$, and the laser frequency was chosen to be $\omega_0 = 5\omega_p$. For these parameters the beam should self-focus (see Refs. 6 and 7), and it is indeed observed to focus as indicated by the increasing peak amplitude in Fig. 2(a). We then replace the Gaussian intensity profile by the sech^2 -profile derived above for the asymptotic profile. The profile should then remain practically unaltered while propagating in plasma, as it is a steady state solution. This turns out to be the case, as evidenced in Fig. 2(a), which shows that the fluctuations in the peak amplitude do remain within 1%. Also, the form the beam retains its sech^2 -profile, whereas in the case of the Gaussian profile quite strong deviation from the original form is observed (see Figs. 2(b) and 2(c)). Thus we conclude that the sech^2 -profile is a realistic physical candidate for the asymptotic shape of a self-focused laser beam.

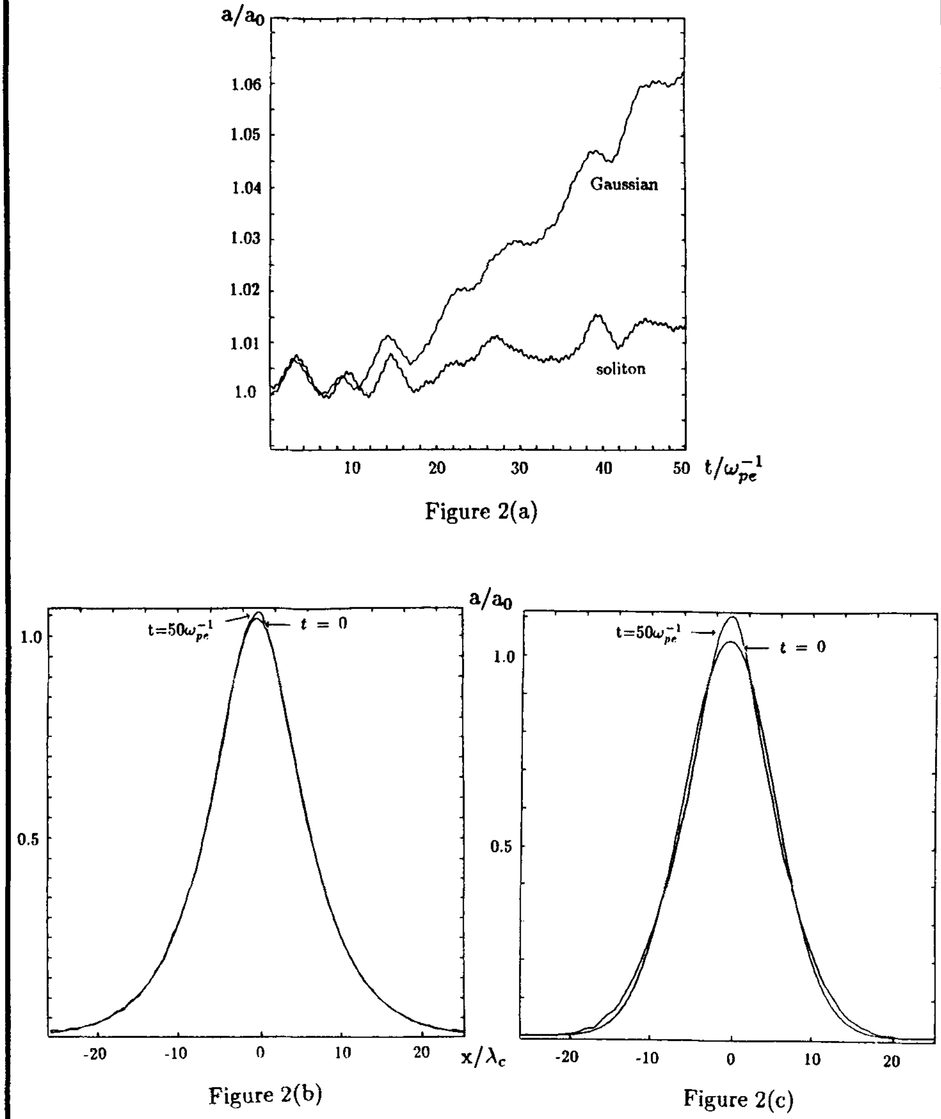

Fig. 2 - Gaussian profile vs. solitary profile - 2(a) The peak amplitude of the different profiles as a function of time. Compared to the self-focusing Gaussian profile, the solitary beam appears to propagate without secular changes. 2(b) The beam profile of an originally Gaussian beam plotted at t=0 and t=50 ω_p^{-1}. 2(c) The beam profile of a soliton-like beam plotted at $t = 0$ and $t = 50\ \omega_p^{-1}$.

V. DISCUSSION

We have addressed here the question of a possible asymptotic transverse profile for a short, self-focused laser pulse propagating in plasma. Using a stationary state model for the electron density, we arrived at a solitary wave shape for the asymptotic profile. In the nonrelativistic limit for the field intensity, keeping only relativistic electron effects, the result reduces to the one obtained earlier by Schmidt and Horton.[10] The general profile (including ponderomotive effects) agrees well with computation proving to be stable and stationary in the numerical particle simulation experiment.

The existence of a stable asymptotic profile of a self-focused laser beam may have important applications in e.g. laser fusion as well as in plasma based accelerators, where it is necessary to have the laser beam traverse considerable distances without significant depletion.

Acknowledgements

Two of us (T.K.S and T.T) would like to thank Professor Huson for his encouragement. This work was supported by the U.S. Department of Energy contracts DE-AC02-87ER-40338 and DE-FG05-80ET-53088, National Science Foundation grant ATM-85-06646, and HARC grant 88-T55-1.

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