PHY 392T PRESENTATION

QUANTUM CONTROL

Swaroop Ganguly

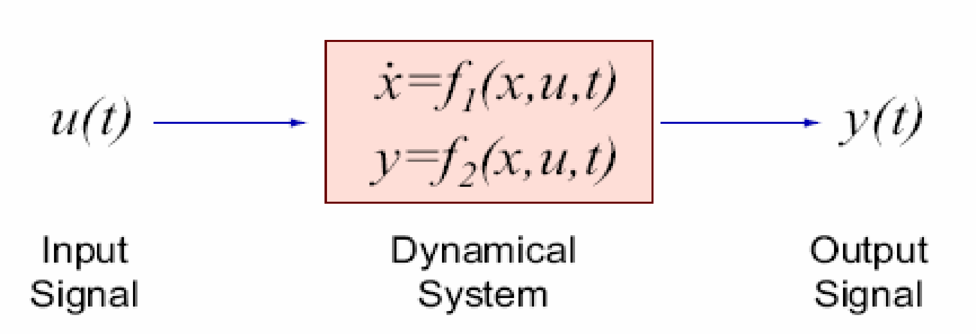
Microelectronics Research Center,
The University of Texas at Austin.

OUTLINE OF TALK

- Introduction to classical control
- Theory of quantum or coherent control
- Applications of quantum control:
  - ➢ chemical reaction rates
  - ➢ carrier dynamics in NANOSTRUCTURES

CLASSICAL CONTROL THEORY

- What is control? Why do we need it?



$u(t)$ → $\dot{x}=f_1(x,u,t)$, $y=f_2(x,u,t)$ → $y(t)$

Input Signal

Dynamical System

Output Signal

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{a}\left(\mathbf{x}_{[0,t)}, t\right) + \mathbf{b}\left(\mathbf{x}_{[0,t)}, \mathbf{u}_{[0,t)}, t\right)$$

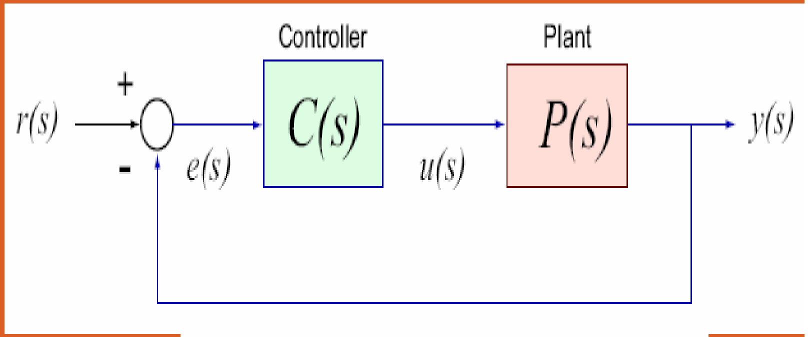
$$\mathbf{y}(t) = \mathbf{c}\left(\mathbf{x}(t), \mathbf{u}(t), t\right)$$

- What about quantum control??

CONTROL SYSTEMS ENGINEERING

- Performance
- Stability
- Robustness

- Open loop
- Closed loop (feedback) →



$$u(t) = \int_0^t F(t-\tau)y(\tau)\,d\tau$$

- Learning

FEEDBACK CONTROL OF OPEN QUANTUM SYSTEMS

- Measurements
    - the good – *required* for feedback
    - the ugly – projective measurement
    - the not so bad – ‘weak’ measurement

- Back-action noise

CLASSICAL → QUANTUM

- State space → Hilbert space
- State vector → Density operator
- System matrix $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}$ → Liouvillian $d\,\hat{\rho}(t) = \mathcal{L}_0\hat{\rho}(t)\,dt$

  (Hamiltonian evolution: $\frac{d}{dt}\hat{\rho} = -i\hbar[\hat{H}_0, \hat{\rho}]$)
- Forcing term $\mathbf{B}\mathbf{u}(t)$ → Interaction Hamiltonian $u(t)\hat{H}_{\mathrm{fb}}$

  (control implementation: $d\hat{\rho}(t) = \mathcal{L}_0\hat{\rho}(t)\,dt - i\hbar u(t)[\hat{H}_{\mathrm{fb}}, \hat{\rho}]$)
- Observation process → Conditional evolution

CONTINUOUS OBSERVATION

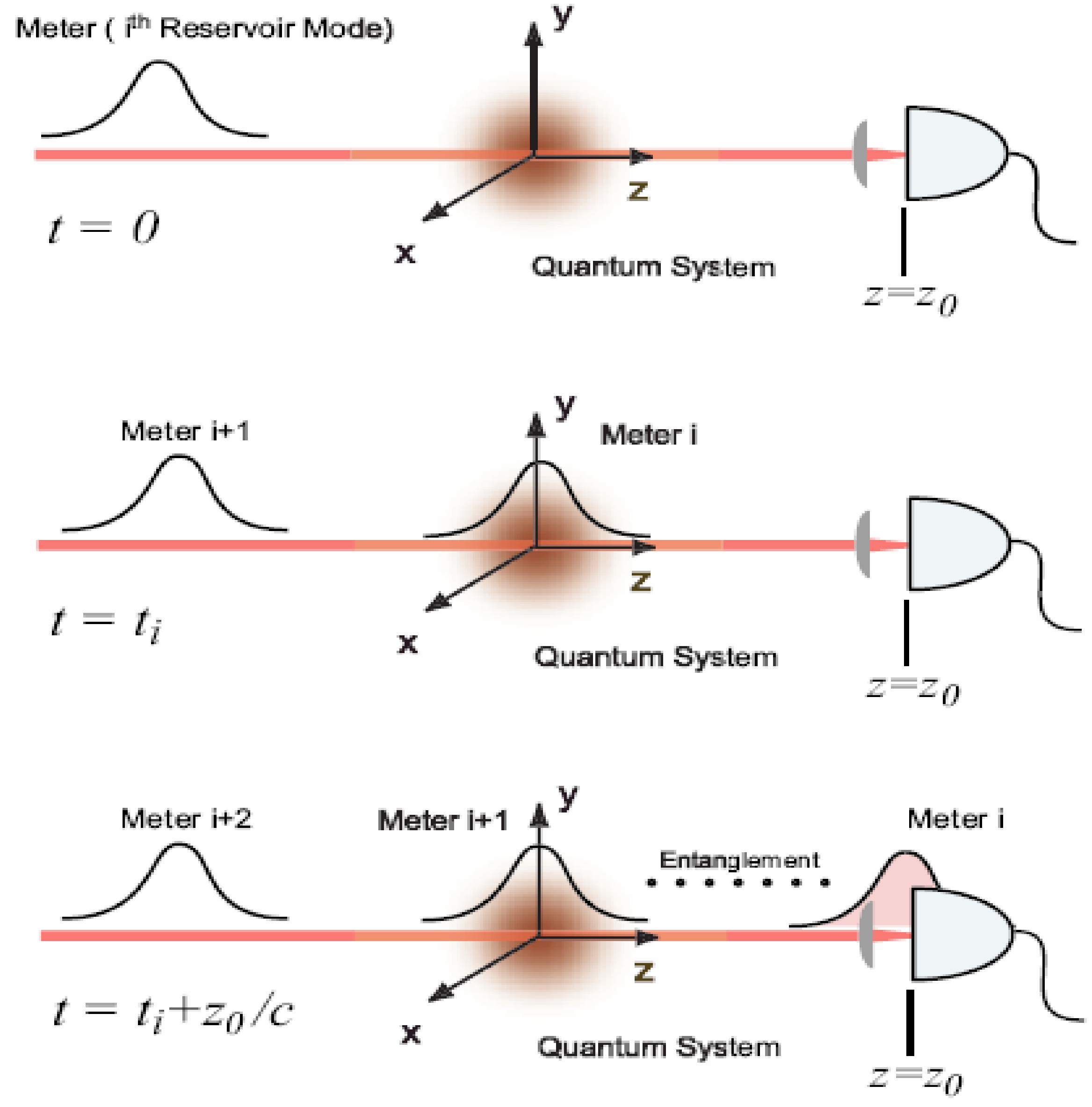
- Initial state: $|\psi\rangle(t) = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$, control target $\langle \hat{j}_z \rangle = 0$

- Stern-Gerlach type observation disastrous!

- Desired observation scheme: $|\psi\rangle(t + \Delta t) = \sqrt{\frac{1}{2} + \epsilon}|\uparrow\rangle + \sqrt{\frac{1}{2} - \epsilon}|\downarrow\rangle$

  ε random → measurement noise in EOM

WEAK MEASUREMENT SCHEME



WEAK MEASUREMENT SCHEME

- Initial density operator of combined system:

$\hat{\chi}(0) = \hat{\rho}_S(0) \otimes \hat{\sigma}_i(0)$, in general $\hat{\chi} \neq \hat{\rho} \otimes \hat{\sigma}_i$

- Measurement operator: $\hat{C} = \mathbb{1}_S \otimes \hat{C}_R$

- ‘Conditioned’ system density operator:

$\hat{\rho}_c(t_i + z_0/c) = \mathrm{tr}_R[\hat{C}\chi(t_i + z_0/c)]$ ➔ ‘quantum trajectory’

CONDITIONAL EVOLUTION

- Interaction Hamiltonian: $\hat{H}_{\mathrm{SR}} = \sqrt{M}(sr^{\dagger} + s^{\dagger}r)$
- Expectation of measured ‘photocurrent’ output:

$y(t) = \sqrt{M}\,\langle \hat{s}(t) \rangle$

- Conditional evolution → ‘conditional innovation’

$y(t)dt = 2\sqrt{M}\,\langle \hat{s} \rangle_c(t)dt + dW$ ‘Itó increment’ $dW(t)$: $E[dW(t)] = 0$ $E[dW^2] = dt$

(detection noise)

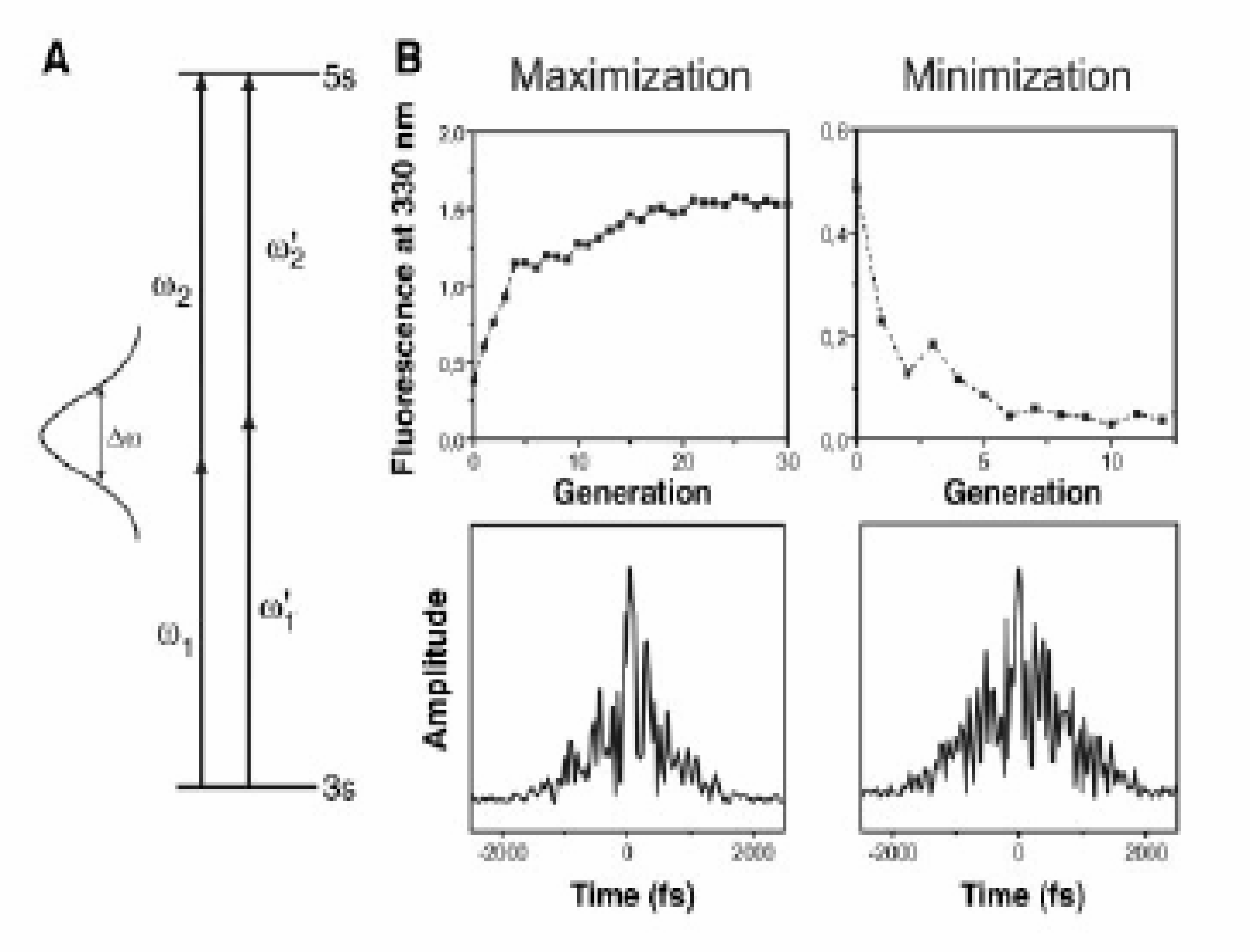
QUANTUM CONTROL

- Continuous information about $\langle\hat{s}\rangle_c(t)$ may be used to control $\langle\hat{s}\rangle$ by driving the system with a signal computed from the ‘photocurrent’
- Feedback strength: $u(t) = \int_0^t F(t-\tau) I_c(\tau)\, d\tau$
- Realization: high-speed digital / quantum filter

APPLICATION: CHEMICAL KINETICS

- Quantum control ≡ chemists' dream?!
- Coherent manipulation of molecular quantum interferences driven by ultrafast laser pulses
- Example: 3s → 5s transition in Na (by learning)
  - ➢ two different 'slits' ($\omega_1+\omega_2$) and ($\omega_1'+\omega_2'$)
  - ➢phase modulation by laser pulse shaping

APPLICATION: CHEMICAL KINETICS



APPLICATION: NANOSTRUCTURES

- Ex. 1: turning (II,Mn)VI’s ferromagnetic
    - ➢ laser induced VB, CB coherence
    - ➢ optical exchange ➔ ferromagnetism
- Ex. 2: exciton-mediated impurity spin interaction
    - ➢ tune laser to bonding-antibonding gap
    - ➢ increased coupling ➔ F-AF transition

CONCLUSIONS

- Quantum control is an exciting new field with great potential for nanoscience/nanotechnology

- The chemists have raced ahead of the solid state folks! What are YOU gonna do about it???