Persistent Currents in Mesoscopic Normal Rings: A Persistent Controversy

Ion Garate

May 9, 2005

Abstract

The topic of persistent currents in normal mesoscopic rings has been a focus of research for more than two decades now. Yet, only a handful of experiments have been carried out successfully, and theorists are still immersed in a sea of controversy. In this paper, we report on the current status of the field.

1 Introduction

Ever-speeding advances in nanotechnology and cryogenics have led to the proliferation of devices that preserve quantum coherence, thus allowing scientists to open the Pandora box of the Quantum Kingdom. One of the many fascinating phenomena thus encountered is the appearance of persistent currents (PC) when a magnetic field is applied through a mesoscopic *non-superconducting* ring.

The theoretical conception of PC in mesoscopic rings was established several years ago, when an analogy was noted[1] between such problem and the motion of an electron in a 1D periodic potential.[1] The application of standard solid sate techniques showed that the energy levels (and thereby the free energy F) of the conduction electrons in the ring are periodic in the applied flux, with a fundamental period of $\Phi_0 \equiv \frac{hc}{e}$. As a consequence, a periodic *equilibrium* (persistent) current $I = -\frac{\partial F}{\partial \Phi}$ flows along the bulk. Due to time-reversal invariance, $I(\Phi) = -I(-\Phi)$, and thus there is no current in absence of external fields.[2]. A crucial assumption in the foregoing discussion is that the dynamics of electrons be described by the Schrodinger equation, which is strictly applicable only for *closed* systems. In practice, this sets the limits for the energy and

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[1] the circumference of the ring corresponds to the “Brillouin zone”, whereas the Aharonov-Bohm flux plays the role of the crystal momentum.

[2] in contrast, superconducting rings may show *surface* persistent currents even in zero field, due to the Meissner effect

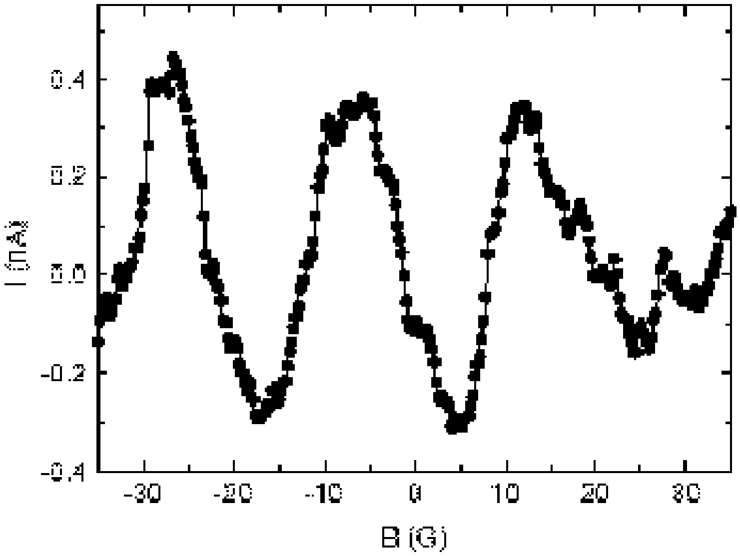

Figure 1: Periodic PC as a function of the applied magnetic field in Au rings with strong spin-orbit coupling (from Ref[10])

lengthscales within which we can observe PC[3].

On the experimental front[2], PC are detected by measuring the magnetization of the ring. Due to the small amplitude of the currents, achieving a good signal-to-noise ratio in single rings is challenging, and hence most experiments are carried out on *arrays* of rings. It must be noted that such experiments measure the *ensemble averaged* PC, where the fundamental mode is washed out because of the sample-dependent disorder configuration. In contrast, the first harmonic (with period $\frac{hc}{2e}$) survives thanks to the constructive interference between time-reversed paths.

The quantitative understanding of the experimental results requires a microscopic theory for the electrons in the ring. The simple picture of independent electrons moving in a random potential due to impurities [3]turns out to be satisfactory for *ballistic* rings[4], but not for *diffusive*[5]ones because

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[3] the "bandwidth", the "band-gap" and the Thouless energy (the inverse of diffusing time) must be larger than the *inelastic* scattering rate. Also, the circumference L of the rings must be smaller than the dephasing length $L_\phi \simeq 10\mu$m

[4] where $\lambda_e \equiv$ (electron wavelength) $\ll$ L $\equiv$ (circumference of the ring) $\leq l_e \equiv$ (elastic mean free path of e-s)

[5] where $\lambda_e \ll l_e \leq L$

(i) the observed amplitudes overwhelm the theoretical expectations[6] by a factor of 100, and
(ii) the *sign* is opposite to the expected.
Including electron-electron interactions yields a sizable contribution to the PC amplitude[4], but still falls short by a factor of 5. Moreover, these calculations too claim the wrong sign[7] for the PC.

Remarkably, the above puzzles persist at present[5].In the following, we will introduce the latest theoretical scenarios and experimental results aimed at appeasing the persistent confusion.

2 Phase Breaking Mechanisms as Sources of Persistent Currents

At first glance, the concepts of quantum decoherence and persistent currents seem antagonistic. However, several new ideas[5] suggest that decoherence mechanisms may indeed contribute to the observed PC.

2.1 Magnetic Impurities

On one hand, it is well-known that magnetic impurities suppress the quantum interference, thus damping the persistent currents. On the other hand, it is often overlooked that the coupling between the magnetic impurities and conduction electrons creates an effective spin-dependent e-e interaction, which may *enhance* the PC[6]. The winner of these competing trends depends on the concentration of the magnetic impurities.*A priori*, this idea could be readily checked by designing experiments that control the amount of magnetic impurities. Unfortunately, the predicted temperature dependence differs from the one observed in experiments, and thus the role of magnetic impurities in our story is no longer considered to be primordial[7].

2.2 Impurity-mediated interactions

The ordinary picture of impurities consists of point particles interacting with electrons via Coulomb interaction. If in addition the impurities had internal degrees of freedom (not necessarily magnetic), they would renormalize the Coulomb interaction and end up contributing significantly to the PC amplitude[7] Moreover, this hypothesis can explain the observed diamagnetic response . However, the existing theory is inconclusive, for it relies on a large number of phenomenological parameters.

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[6] theoretical expectation: $I_{typical} \sim \frac{e}{\text{diffusion time}} \sim \frac{ev_F}{L}\frac{l_e}{L}$; experimental finding: $I_{typical} \sim \frac{ev_F}{L} \simeq$ 0.01 nanoamperes

[7] when the PC reinforces (opposes) the external flux, it is called *paramagnetic* (*diamagnetic*)

2.3 Non-Equilibrium Electromagnetic Noise

Several recent experiments [8] point at an unexpectedly large decoherence rate in mesoscopic rings, even in the zero-temperature limit. This seemingly unrelated observation motivates the striking idea that perhaps *part* of the observed PC is a *non-equilibrium* (steady-state) current[9]. The electromagnetic field created by the moving electrons is blamed for *both* the large decoherence and the large PC. Detailed calculations show that this mechanism yields the correct amplitude for the PC. Furthermore, it is foreseen that the sign of the current should change from diamagnetic to paramagnetic depending on the strength of spin-orbit scattering. Regrettably, this prediction has been disproved by the latest experiments[10], which detect a diamagnetic response in the strong spin-orbit regime.

2.4 Phonons

In the same spirit as above, it has been suggested[11] that when a mesoscopic ring is coupled to a source of phonons, the magnitude of PC may be *controlled* by choosing the appropriate frequency and intensity of the acoustic radiation. This idea is yet to be tested by experiments.

3 Contribution from non-local Coulomb interactions

Soon after e-e interactions were incorporated into the initial theories, it was understood that repulsive (attractive) interactions give paramagnetic (diamagnetic) PC. Therefore the diamagnetic response observed in experiments hinted at the possibility of having weak superconductivity in mesoscopic rings. However, the initial calculations failed to obtain the correct PC amplitudes. It is now believed that a possible shortcoming of the early scenario was to model both attractive and repulsive e-e interactions to be local in *real* space[8].

3.1 BCS-type model

A BCS-type interaction is local in *momentum* space. Consequently, its predictions depart sharply from the earlier scenario for temperatures above the “transition temperature”[12]. For instance, the BCS model yields a significantly larger magnetic response (and thereby PC) at low applied fields[9].

3.2 Landau-type model

An interaction dominated by small momentum transfers need not be of BCS-type. In fact, such kind of interaction lies at the heart of Landau’s Fermi-liquid theory. Calculations within this

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[8] due to a presumably efficient Coulomb screening in mesoscopic rings

[9] magnetic susceptibility$\sim \frac{\partial^2 F}{\partial \Phi^2} \sim \frac{\partial I}{\partial \Phi}$

framework[13] show that the forward-scattering dominates the magnetic response in the linear regime[10], while the interaction terms involving large momentum transfers dominate outside the linear regime.

3.3 Time-reversal symmetry-breaking

A more bizarre proposal[14] shows that the time-reversal symmetry of the hamiltonian is spontaneously broken as the coupling between the rings and the leads (if there are any) is increased. Hence $I(0) \neq 0$, which certainly affects the magnetic response in the linear regime.

4 Other recent ideas

4.1 Pure 1D effects

In current experiments[2][10], a uniform magnetic field is applied through the entire array of rings, and extra care is taken to ensure that the magnetic flux penetrating in the ring itself is negligible.[11]. But how close are these experiments from the "pure" 1D case? If this limit is reached, any arbitrarily small Coulomb interaction will produce a Luttinger liquid, which is *qualitatively* different from a 3D Fermi liquid with frozen transverse motion. Fortunately, there is a variety of powerful methods which allow to treat the e-e interactions in 1D. In particular, numerical calculations within the Hubbard-Anderson model[15] show that PC increase for weak disorder and moderate interaction strengths, whereas for stronger interactions it shows a dependence on the filling factor.

4.2 Effects of surface curvature

How important is the ring-like geometry for the observation of the PC? This is a germane question because the potential that confines the carriers within the mesoscopic device is not perfectly controlled in experiments. Arguably, so long as the ring is thin, the exact geometry does not make a qualitative difference[12]: the surface irregularities can be modeled as an additional scattering potential. Nevertheless, the quantitative details will differ. In particular, some recent calculations for ballistic 2D rings[16] show that the surface roughness may contribute coherently to the PC. *Au contraire*, a more recent publication[17] claims that the effect of surface curvature in 2D rings is to *reduce* both the amplitude and the period of the PC!

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[10] i.e., the weak-field regime

[11] the ring must be thin enough so that the electrons are kept "frozen" in the lowest subbands of the confining potential. Typical thickness: 10-100 nm

[12] in fact, many experiments are performed for mesoscopic *squares* rather than rings

4.3 Berry Phase contributions

Even in absence of electromagnetic fields, a quantum state undergoing an adiabatic evolution along a closed curve in *parameter* space develops a phase which depends only on the curve. This Berry phase is likely present in semiconducting (e.g. GaAs) mesoscopic rings with spin-orbit coupling[13]. Though probably not instrumental for the main features of the phenomenon (amplitude and sign of PC), the Berry phase leaves fingerprints in the current vs flux characteristics[18].

4.4 Non-classical Electromagnetic Fields

It is a widespread practice to assume that the applied magnetic flux is classical. Would anything change if the quantum mechanical nature of the EM fields were considered? Quite surprisingly[14], mesoscopic rings are very sensitive devices that can detect the non-classical fields[19]. It turns out that the non-classical light does not destroy the amplitude of the PC oscillations, but can change the magnetic response from paramagnetic to diamagnetic or vice versa.

5 Conclusions and Outlook

In this survey, we have reported on recent advances in the understanding of persistent currents in mesoscopic normal rings. We have shown that despite the intense theoretical effort, progress has been incomplete. In fact, the consensus is that there is *no* consensus on how the impurity effects combine with e-e interactions. In other words, "a better microscopic theory is needed"[13]

Nevertheless, far from drowning in pessimism, this field continues to inspire enticing research venues in different directions, some of which are
(i)Persistent currents in carbon nanotubes[20].
(ii)Effect of spin-orbit coupling on PC in mesoscopic *semiconductor* rings, and in particular the non-trivial interplay between the Coulomb repulsion and the SO interaction[15].
(iii)Persistent *spin* currents[21].
(iv)PC in *macroscopic* devices made out of *connected* mesoscopic rings[22]
(v)PC in *superconducting nano*rings[23]
(vi)Quantum computation with PC.
(vii)The effect of the *in-ring* magnetic in *2D*, and its interplay with the PC.
(viii)Neutral PC.

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[13] the SO interaction plays the role of an inhomogeneous magnetic field along the ring

[14] our estimations point out that only at exceedingly small applied fields the effects of quantization would be important

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