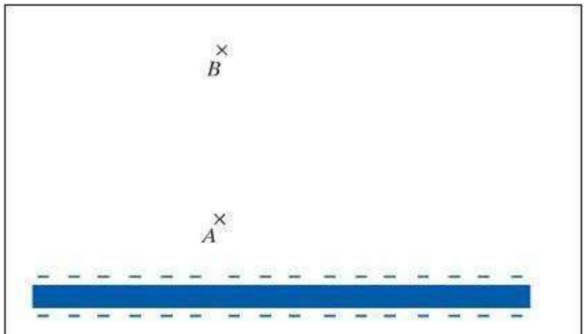
This print-out should have 45 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

The long negatively charged rod
001 10.0 points



Figure

Figure above shows a portion of long, negatively charged rod. You need to determine the potential difference $V_A - V_B$ due to the charged rod. Use the convention that up is along the +y direction.

Consider the following statements:

Ia. Direction of the path is along the +y direction

Ib. Direction of the path is along the -y direction

IIa. The sign of $V_A - V_B$ is positive

IIb. The sign of $V_A - V_B$ is negative

Now bring a test charge q from A to B. Consider the following statements:

IIIa. The sign of the potential difference $V_A - V_B$ due to the rod depends on the sign of the test charge q.

IIIb. The sign of the potential difference $V_A - V_B$ due to the rod does not depend on the sign of the test charge q.

Choose the correct choice:

1. Ib, IIb, IIIa

2. Ib, IIa, IIIa

3. Ib, IIb, IIIb **correct**

4. Ia, IIb, IIIb

5. Ia, IIb, IIIa

6. Ia, IIa, IIIa

7. Ib, IIa, IIIb

8. Ia, IIa, IIIb

Explanation:

Ib is correct. Since we want to determine $V_A - V_B$, A is the final point and B is the initial point, so the path is from B to A.

IIb is correct. Notice that with the source charge on the rod being negative, it generates a downward electric field. By inspection the vector E is parallel to the path, so $\Delta V = V_A - V_B < 0$.

IIIb is correct. The potential difference is a property of E generated by the source charges. It is independent of the sign or magnitude of the test charge q.

Flat Parallel Conductors
002 10.0 points

Two flat conductors are placed with their inner faces separated by 2 mm.

If the surface charge density on inner face A is 97 pC/m^2 and on inner face B is -97 pC/m^2, calculate the electric potential difference $\Delta V = V_A - V_B$. Use $\epsilon_0 = 8.85419 \times 10^{-12}$ C^2/Nm^2.

Correct answer: 0.0219105 V.

Explanation:

Let:

$$\epsilon_0 = 8.85419 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2}\,,$$
$$\sigma = 97\ \mathrm{pC/m^2} = 9.7 \times 10^{-11}\ \mathrm{C/m^2}\,, \quad \text{and}$$
$$d = 2\ \mathrm{mm} = 0.002\ \mathrm{m}\,.$$

The electric field between two flat conductors is

$$E = \frac{\sigma}{\epsilon_0}\,.$$

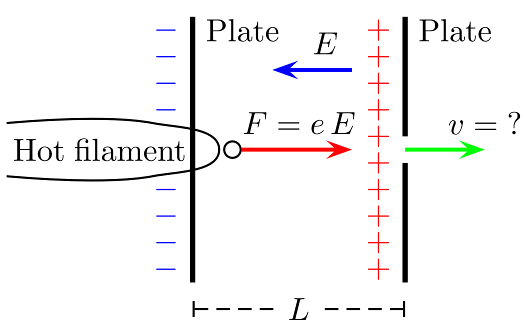
Plate A is positively charged and therefore at a higher potential than the negatively charged plate B, so we know the answer must be positive. The magnitude of ΔV is given by

$$\begin{aligned}\Delta V &= E\,d = \frac{\sigma\,d}{\epsilon_0}\\ &= \frac{\left(9.7\times10^{-11}\ \text{C/m}^2\right)(0.002\ \text{m})}{8.85419\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}\\ &= \boxed{0.0219105\ \text{V}}\,.\end{aligned}$$

Intuitive reasoning: Let us place a positive charge in between the two plates and release it. It will be repelled by the positive plate and attracted to the negative plate. The natural tendency for a positive charge is to move from high potential to low potential. In other words, the positively charged plate is at a higher potential. This agrees with the math results where $V_f - V_i = -\vec{E}\bullet\Delta\vec{\ell}$.

TVTubeMI17p045
003 10.0 points

In a television picture tube, electrons are boiled out of a very hot metal filament placed near a negative metal plate. These electrons start out nearly at rest and are accelerated toward a positive metal plate. They pass through a hole in the positive plate on their way toward the picture screen, as shown in the diagram.



The high-voltage supply in the television set maintains a potential difference of 145 V between the two plates, what speed do the electrons reach? Use $m_e = 9.11\times10^{-31}$ kg and $q_e = 1.6\times10^{-19}$ C and assume that this is not relativistic.

Correct answer: 7.13674×10^{6} m/s.

Explanation:

The net energy remains constant throughout the whole process. We can use the following train of logic:

$$\begin{aligned}\Delta E &= 0\\ \Delta U + \Delta K &= 0\\ q\,\Delta V + \Delta K &= 0\\ (-e)\Delta V + \Delta K &= 0\\ \Rightarrow \Delta K &= e\,\Delta V\\ &= (1.6\times10^{-19}\ \text{C})(145\ \text{V})\\ &= 2.32\times10^{-17}\ \text{J}\,.\end{aligned}$$

Then

$$\begin{aligned}\Delta K &= K_f - K_i\\ &= \frac{1}{2}\,m\,v_f^2 - 0\\ \Rightarrow v_f &= \sqrt{\frac{2\Delta K}{m}}\\ &= \sqrt{\frac{2(2.32\times10^{-17}\ \text{J})}{9.11\times10^{-31}\ \text{kg}}}\\ &= \boxed{7.13674\times10^{6}\ \text{m/s}}\,.\end{aligned}$$

Interestingly, a more realistic accelerating voltage for a "modern" picture tube television is in the range of 17kV–25kV. In this range, the electrons reach relativistic speeds and cannot be treated classically.

Three parallel plate capacitors
004 10.0 points

Given three parallel conducting plates which are aligned perpendicular to the x-axis. They are labeled, from left to right as plate 1, 2 and 3 respectively. The total charges on the corresponding plates are $Q_1 = -5q$, $Q_2 = 3q$ and $Q_3 = 2q$. The width of the gap between 1 and 2 is d which is the same as the width across 2 and 3.

Determine the $\Delta V = V_3 - V_1$. That is going from the leftmost plate to the rightmost plate.

1. $\dfrac{2(q/A)d}{\epsilon_0}$

2. $\dfrac{4(q/A)d}{\epsilon_0}$

3. $\dfrac{3(q/A)d}{\epsilon_0}$

4. $-\dfrac{9(q/A)d}{2\epsilon_0}$

5. $-\dfrac{7(q/A)d}{\epsilon_0}$

6. $\dfrac{6(q/A)d}{\epsilon_0}$

7. $-\dfrac{3(q/A)d}{\epsilon_0}$

8. $-\dfrac{5(q/A)d}{\epsilon_0}$

9. $\dfrac{7(q/A)d}{\epsilon_0}$ **correct**

Explanation:

One may regard the 3-plate system as a composite system which involves two capacitor systems with the 12-capacitor followed by the 23-capacitor.

The 12-capacitor has charges Q_1 and $Q_2 + Q_3$, i.e charges of $-4q$ and $+4q$ respectively.

The 23-capacitor has charges $Q_1 + Q_2$ and Q_3, i.e charges of $-q$ and $+q$ respectively.

The potential difference is

$$V_3 - V_1 = E_{gap,12}\, d + E_{gap,23}\, d$$

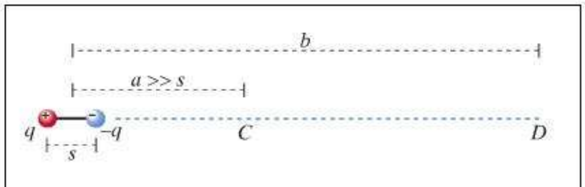
$$V_3 - V_1 = \frac{5(q/A)d}{\epsilon_0} + \frac{2(q/A)d}{\epsilon_0}$$

$$V_3 - V_1 = \frac{7(q/A)d}{\epsilon_0}$$

Digression: Notice that E is pointing to the left. This implies that the "potential hill" has an upward slope to the right.

Going from plate 1 to plate 3, corresponds to moving to the right, which is climbing the potential hill. This implies $V_3 - V_1 > 0$.

The change in potential energy
005 10.0 points



Figure

Consider the setup in Figure above. What is the change in potential energy $\Delta U = U_C - U_D$, in moving an electron from D to C?

1. $2k\,e\,q\,s\left(\dfrac{1}{a^2} - \dfrac{1}{b^2}\right)$

2. $k\,e\,q\,s\left(\dfrac{1}{a} - \dfrac{1}{b}\right)$

3. -$2k\,e\,q\,s\left(\dfrac{1}{a^2} - \dfrac{1}{b^2}\right)$

4. -$2k\,e\,q\,s\left(\dfrac{1}{a} - \dfrac{1}{b}\right)$

5. -$k\,e\,q\,s\left(\dfrac{1}{a^2} - \dfrac{1}{b^2}\right)$

6. $k\,e\,q\,s\left(\dfrac{1}{a^2} - \dfrac{1}{b^2}\right)$ **correct**

7. -$k\,e\,q\,s\left(\dfrac{1}{a} - \dfrac{1}{b}\right)$

8. $k\,q\,s\left(\dfrac{1}{a} - \dfrac{1}{b}\right)$

9. $2k\,e\,q\,s\left(\dfrac{1}{a} - \dfrac{1}{b}\right)$

10. $k\,q\,s\left(\dfrac{1}{a^2} - \dfrac{1}{b^2}\right)$

Explanation:

$$\begin{aligned} V_C - V_D &= -\int \left(-\frac{2\,k\,q\,s}{x^3}\right) dx \\ &= 2\,k\,q\,s \int d\left(\frac{-1}{2\,x^2}\right) \\ &= 2\,k\,q\,s\left(-\frac{1}{2\,a^2} + \frac{1}{2\,b^2}\right). \qquad (1) \end{aligned}$$

Multiplying eq(1) by the electronic charge -e, we arrive at the potential energy difference from D to C is given by

$$U_C - U_D = -e(V(a) - V(b)) = k\,e\,q\,s\left(\frac{1}{a^2} - \frac{1}{b^2}\right).$$

Intuitive reasoning on the sign of ΔU: Natural tendency of the motion is from high potential energy to lower potential energy. Since when the electron is released it should move from C to D, so $U_C > U_D$.

Alternative explanation:

Let the center of the dipole be at the origin. At a distance x along the $+\hat{x}$ direction

$$V_{dipole}(x) = V_q\left(x + \frac{s}{2}\right) + V_{-q}\left(x - \frac{s}{2}\right)$$

$$V_{dipole}(x) = \frac{kq}{x + s/2} + \frac{k(-q)}{x - s/2}$$

$$V_{dipole}(x) = \frac{(x - s/2) - (x + s/2)}{x^2 - (s/2)^2} \approx -\frac{kqs}{x^2}$$

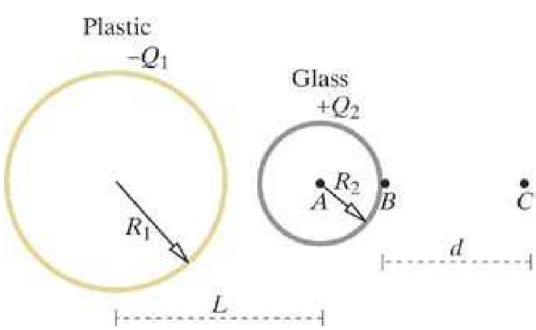
So, we have

$$V_C(x) - V_D(x) = -k\,q\,s\left(\frac{1}{a^2} - \frac{1}{b^2}\right)$$

Check: E is along the -$\hat{x}$ direction, V_C is expected to be lower than V_D.

17P70MI

006 (part 1 of 3) 10.0 points



A thin spherical shell of radius R_1 made of plastic carries a uniformly distributed negative charge $-Q_1$. A thin spherical shell of radius R_2 made of glass carries a uniformly distributed positive charge $+Q_2$. The distance between centers is L, as shown in the above figure.

Find the potential difference $V_B - V_A$. Location A is at the center of the glass sphere, and location B is just outside the glass sphere.

1. $-\frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L} - \frac{1}{L - R_2}\right)$

2. $-\frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L} - \frac{1}{R_2}\right)$

3. $\frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L} - \frac{1}{R_2}\right)$

4. $\frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L} - \frac{1}{L - R_2}\right)$

5. $-\frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L} - \frac{1}{L + R_2}\right)$

6. $\frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L} - \frac{1}{L + R_2}\right)$ **correct**

Explanation:

We may write down the potential difference as

$$\Delta V_{A,B} = \Delta V_{A,B,plastic} + \Delta V_{A,B,glass}$$

V_{glass} is constant in the region between A and B, and hence $\Delta V_{A,B,glass} = 0$.

$$\Delta V_{A,B} = \Delta V_{A,B,plastic}$$

$$V_B - V_A = \frac{1}{4\pi\epsilon_0}\frac{(-Q_1)}{r_B} - \frac{1}{4\pi\epsilon_0}\frac{(-Q_1)}{r_A}$$

Now, we substitute $r_A = L$ and $r_B = L + R_2$.

$$V_B - V_A = \frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L} - \frac{1}{L + R_2}\right)$$

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007 (part 2 of 3) 10.0 points

Find the potential difference $V_C - V_B$. Location B is just outside the glass sphere, and location C is a distance d to the right of B.

1. $-\frac{Q_2}{4\pi\epsilon_0}\left(\frac{1}{L + d} - \frac{1}{L}\right) + \frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L + R_2} - \frac{1}{L + R_2 + d}\right)$

2. $-\frac{Q_2}{4\pi\epsilon_0}\left(\frac{1}{R_2 + d} - \frac{1}{R_2}\right) + \frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L} - \frac{1}{L + d}\right)$

3. $\frac{Q_2}{4\pi\epsilon_0}\left(\frac{1}{L + d} - \frac{1}{L}\right) + \frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L + R_2} - \frac{1}{L + R_2 + d}\right)$

4. $\frac{Q_2}{4\pi\epsilon_0}\left(\frac{1}{R_2+d}-\frac{1}{R_2}\right)+\frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L}-\frac{1}{L+d}\right)$

5. $\frac{Q_2}{4\pi\epsilon_0}\left(\frac{1}{R_2+d}-\frac{1}{R_2}\right)-\frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L}-\frac{1}{L+d}\right)$

6. $\frac{Q_2}{4\pi\epsilon_0}\left(\frac{1}{R_2+d}-\frac{1}{R_2}\right)-\frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L+R_2}-\frac{1}{L+R_2+d}\right)$

7. $\frac{Q_2}{4\pi\epsilon_0}\left(\frac{1}{L+d}-\frac{1}{L}\right)-\frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L+R_2}-\frac{1}{L+R_2+d}\right)$

8. $\frac{Q_2}{4\pi\epsilon_0}\left(\frac{1}{R_2+d}-\frac{1}{R_2}\right)+\frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L+R_2}-\frac{1}{L+R_2+d}\right)$ **correct**

9. $-\frac{Q_2}{4\pi\epsilon_0}\left(\frac{1}{R_2+d}-\frac{1}{R_2}\right)+\frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L+R_2}-\frac{1}{L+R_2+d}\right)$

Explanation:

The potential difference can be expressed as

$$\begin{aligned}\Delta V_{B,C} &= \Delta V_{B,C,glass}+\Delta V_{B,C,plastic}\\ V_C-V_B &= \frac{Q_2}{4\pi\epsilon_0}\left(\frac{1}{r_{C,glass}}-\frac{1}{r_{B,glass}}\right)\\ &\quad+\frac{(-Q_1)}{4\pi\epsilon_0}\left(\frac{1}{r_{C,plastic}}-\frac{1}{r_{B,plastic}}\right)\end{aligned}$$

where the different distances are:

$$r_{B,glass}=R_2$$

$$r_{C,glass}=R_2+d$$

$$r_{B,plastic}=R_2+L$$

$$r_{C,plastic}=R_2+L+d$$

$$\begin{aligned}V_C-V_B &= \frac{Q_2}{4\pi\epsilon_0}\left(\frac{1}{R_2+d}-\frac{1}{R_2}\right)\\ &\quad+\frac{Q_1}{4\pi\epsilon_0}\left(\frac{1}{L+R_2}-\frac{1}{L+R_2+d}\right)\end{aligned}$$

008 (part 3 of 3) 10.0 points

Suppose the glass shell is replaced by a solid metal sphere with radius R_2 carrying charge $+Q_2$. Which of the following statements concerning the new potential difference $V_B - V_A$ is true?

1. The new potential difference is equal to the old potential difference (from part 1). **correct**

2. The new potential difference is greater than the old potential difference (from part 1).

3. The new potential difference is less than the old potential difference (from part 1).

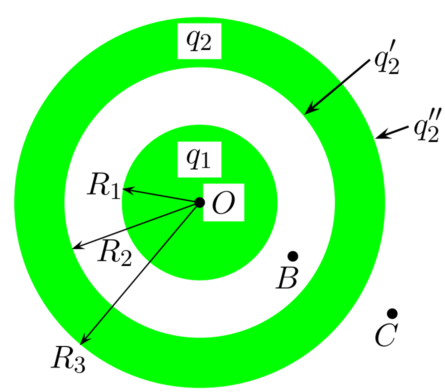
Explanation:

Since $E = 0$ everywhere inside the metal sphere, $\Delta V_{AB,metal} = 0$, the same as for the glass shell. Since the plastic shell is unchanged, the potential difference $V_B - V_A$ must be the same.

Shell Game 06

009 (part 1 of 2) 10.0 points

Consider a system of a metallic ball with net charge q_1 and radius R_1 enclosed by a spherically symmetric metallic shell with net charge q_2, inner radius R_2 and outer radius R_3. If q_2'' is the charge on the outside surface of the shell and q_2' the charge on its inside surface, then $q_2'' + q_2' = q_2$.



Find the potential at C. $\overline{OB} = b$ and $\overline{OC} = c$.

1. $V_C = k\,\dfrac{q_1+q_2}{R_3} - k\,\dfrac{q_1}{R_2} + k\,\dfrac{q_1}{R_1}$

2. $V_C = k\,\dfrac{q_1}{c}$

3. $V_C = k\,\dfrac{q_1+q_2}{c} - k\,\dfrac{q_1}{R_2} + k\,\dfrac{q_1}{b}$

4. $V_C = 2\,k\,\dfrac{q_1}{c}$

5. $V_C = k\,\dfrac{q_2}{c}$

6. $V_C = k\,\dfrac{q_1+q_2}{r_3} - k\,\dfrac{q_1}{b} + k\,\dfrac{q_1}{R_1}$

7. $V_C = k\,\dfrac{q_1-q_2}{\sqrt{2}\,c}$

8. $V_C = \sqrt{2}\,k\,\dfrac{q_2}{c}$

9. $V_C = k\,\dfrac{q_1+q_2}{R_3} - k\,\dfrac{q_1}{R_2} + k\,\dfrac{q_1}{b}$

10. $V_C = k\,\dfrac{q_1+q_2}{c}$ **correct**

Explanation:

C is outside of the entire charge distribution a distance c from the center, so the enclosed charge $Q_{encl} = q_1 + q_2$ can be treated as a point charge, and

$$V_C = k\,\frac{q_1+q_2}{c}\,.$$

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010 (part 2 of 2) 10.0 points

Determine the potential at B.

1. $V_B = k\,\dfrac{q_1+q_2}{R_3} - k\,\dfrac{q_1}{b} + k\,\dfrac{q_1}{R_1}$

2. $V_B = k\,\dfrac{q_2}{b}$

3. $V_B = k\,\dfrac{q_1+q_2}{R_3} - k\,\dfrac{q_1}{R_2} + k\,\dfrac{q_1}{R_1}$

4. $V_B = \sqrt{2}\,k\,\dfrac{q_2}{c}$

5. $V_B = k\,\dfrac{q_1}{b}$

6. $V_B = k\,\dfrac{q_1+q_2}{c} - k\,\dfrac{q_1}{R_2} + k\,\dfrac{q_1}{b}$

7. $V_B = k\,\dfrac{q_1-q_2}{\sqrt{2}\,c}$

8. $V_B = k\,\dfrac{q_1+q_2}{b}$

9. $V_B = 2\,k\,\dfrac{q_1}{b}$

10. $V_B = k\,\dfrac{q_1+q_2}{R_3} - k\,\dfrac{q_1}{R_2} + k\,\dfrac{q_1}{b}$ **correct**

Explanation:

B is between the shell and the sphere. Consider a Gaussian surface through B concentric to the system. Let us start from the inside and use superposition to add contributions as we go outward.

We are outside of the sphere, so we can treat its charge q_1 as a point charge, and its potential is

$$V_1 = k\,\frac{q_1}{b}\,.$$

The inner surface of the shell carries an induced charge of $q_2' = -q_1$ so its potential is

$$V_2 = k\,\frac{q_2'}{R_2} = -k\,\frac{q_1}{R_2}\,.$$

The outer surface of the shell carries a charge of $q_2'' = q_1 + q_2$, so its potential is

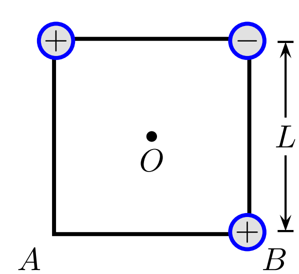
$$V_3 = k\,\frac{q_2''}{R_3} = k\,\frac{q_1+q_2}{R_3}\,.$$

Thus the total potential is

$$\begin{aligned} V_B &= V_1 + V_2 + V_3 \\ &= k\,\frac{q_1}{b} - k\,\frac{q_1}{R_2} + k\,\frac{q_1+q_2}{R_3}\,. \end{aligned}$$

Four Charges in a Square 03
011 10.0 points

Three point charges, each of magnitude q, are placed at 3 corners of a square with sides of length L. The charge farthest from the empty corner is negative $(-q)$ and the other two charges are positive $(+q)$.



What is the potential at point A?

1. $V = 2\sqrt{2}\,\dfrac{k\,q}{L^2}$

2. $V = 2\,\dfrac{k\,q^2}{L}$

3. $V = \left(2 - \dfrac{1}{\sqrt{2}}\right)\dfrac{k\,q^2}{L}$

4. $V = 2\sqrt{2}\,\dfrac{k\,q^2}{L}$

5. $V = \left(2 - \dfrac{1}{\sqrt{2}}\right)\dfrac{k\,q}{L}$ **correct**

6. $V = \left(2 - \dfrac{1}{\sqrt{2}}\right)\dfrac{k\,q}{L^2}$

7. $V = 2\sqrt{2}\,\dfrac{k\,q}{L}$

8. $V = 2\,\dfrac{k\,q}{L^2}$

9. $V = \dfrac{k\,q}{L}$

10. $V = 2\,\dfrac{k\,q}{L}$

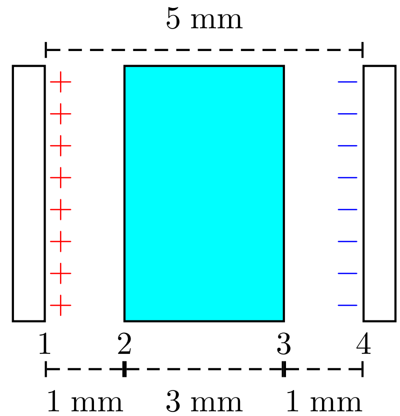
Explanation:

The length of the diagonal is $\sqrt{2}\,L$, so

$$V = \sum_i V_i = \frac{k\,q}{L} + \frac{k\,-q}{\sqrt{2}\,L} + \frac{k\,q}{L} = \frac{k\,q}{L}\left(2 - \frac{1}{\sqrt{2}}\right).$$

CapacitorMI17p103 v2
012 10.0 points

An isolated large-plate capacitor (not connected to anything) originally has a potential difference of 800 V with an air gap of 5 mm. Then a plastic slab 3 mm thick, with dielectric constant 6, is inserted into the middle of the air gap as shown in the figure below.



Calculate $V_1 - V_4$.

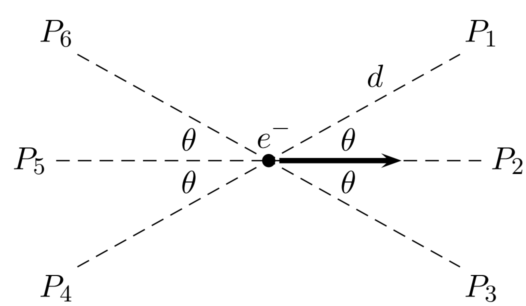
Correct answer: 400 V.

Explanation:

Here we simply add the potential differences we've already found:

$$\begin{aligned}\Delta V_{14} &= \Delta V_{12} + \Delta V_{23} + \Delta V_{34} \\ &= (160\text{ V}) + (80\text{ V}) + (160\text{ V}) \\ &= \boxed{400\text{ V}}.\end{aligned}$$

The magnetic field of moving electron
013 10.0 points



An electron is moving horizontally to the right with speed 5×10^6 m/s. Each location is $d = 5$ cm from the electron, and the angle $\theta = 31\,^\circ$. Give the magnetic field at P_3 using the convention that out of the page is positive.

1. 1.1×10^{-17} T

2. 3.3×10^{-17} T

3. -2.47×10^{-17} T

4. 2.47×10^{-17} T

5. -1.65×10^{-17} T

6. -1.1×10^{-17} T

7. 1.65×10^{-17} T **correct**

8. -3.3×10^{-17} T

Explanation:

$$\text{Let}: \quad v = 5 \times 10^6 \text{ m/s}, \quad q = 1.6 \times 10^{-19} \text{ C}, \quad d = 5 \text{ cm} = 0.05 \text{ m}, \quad \text{and} \quad \theta = 31^\circ .$$

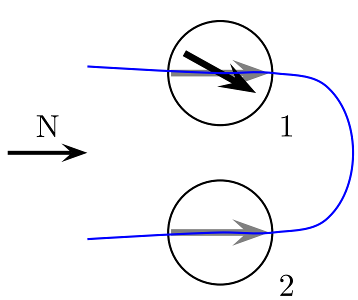
The magnitude of the field at P_3 is

$$B = \left| \frac{\mu_0}{4\pi} \frac{q\, v\, sin(\theta)}{d^2} \right| = (1 \times 10^{-7} \text{ T} \cdot \text{m/A})(1.6 \times 10^{-19} \text{ C}) \times \frac{(5 \times 10^6 \text{ m/s}) sin(31^\circ)}{(0.05 \text{ m})^2} = 1.65 \times 10^{-17} \text{ T}.$$

Using the right hand rule (and remembering that q is negative), the magnetic field is out of the page.

BDirTwoCompMI18x040
014 10.0 points

Consider the following diagram.



The wire rests on top of two compasses. When no current is running, both compasses point North (the direction shown by the gray arrows). When the current runs in the circuit, the needle of compass 1 deflects as shown. In what direction will the needle of compass 2 point?

1. West

2. North

3. South

4. Southeast

5. Southwest

6. East

7. Northeast

8. Northwest **correct**

Explanation:
At location 1, current flows to the left (South) to make the compass deflect Northeast. Thus, at location 2, current flows to the right (North). $\vec{B}$ due to the current, at location 2 beneath the wire, is upward toward the top of the page (West). Thus the net magnetic field due to the Earth and the current carrying wire is Northwest, so the compass needle will point Northwest. The actual angle will depend on the value of the current.

GS6HWMI
015 (part 1 of 2) 10.0 points

An electron is moving through space with a non-relativistic velocity $\vec{v} = \langle v_0, 0, 0 \rangle$ and passes through the origin at time $t = 0$. A magnetic field detector is located at $\vec{s} = \langle a, b, 0 \rangle$.

Use the Biot-Savart law for a moving charge to calculate $\vec{B}(\vec{s})$, the field measured at the detector due to the electron passing through the origin. Use the RHR to verify the vector direction of your answer.

1. $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{e v_0 b}{(a^2 + b^2)} (-\hat{z})$

2. $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{e v_0 b}{(a^2+b^2)} \hat{z}$

3. $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{e v_0 b}{(a^2+b^2)} (-\hat{y})$

4. $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{e v_0 b}{(a^2+b^2)^{3/2}} (-\hat{y})$

5. $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{e v_0 b}{(a^2+b^2)^{3/2}} (-\hat{z})$ **correct**

6. $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{e v_0 b}{(a^2+b^2)^{3/2}} \hat{z}$

7. $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{e v_0 b}{(a^2+b^2)^{3/2}} \hat{y}$

8. $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{e v_0 b}{(a^2+b^2)} \hat{y}$

Explanation:

We are given everything needed for the calculation except for $\hat{r}$, which is given by

$$\hat{r} = \frac{\vec{s}}{|\vec{s}|} = \frac{\langle a, b, 0\rangle}{\sqrt{a^2+b^2}}$$

Substituting this into the B-S equation, and note that $r^2 = a^2 + b^2$

$$\vec{B} = \frac{\mu_0}{4\pi} (-e) \frac{\langle v_0, 0, 0\rangle}{a^2+b^2} \times \frac{\langle a, b, 0\rangle}{\sqrt{a^2+b^2}}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{e v_0 b}{(a^2+b^2)^{3/2}} (-\hat{z})$$

---

016 (part 2 of 2) 10.0 points

Now calculate $\vec{B}(\vec{q})$, where the detector is now located at $\vec{q} = \langle a, 0, b\rangle$.

1. $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{e v_0 b}{(a^2+b^2)^{3/2}} (-\hat{y})$

2. $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{e v_0 b}{(a^2+b^2)^{3/2}} \hat{z}$

3. $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{e v_0 b}{(a^2+b^2)} (-\hat{y})$

4. $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{e v_0 b}{(a^2+b^2)} (-\hat{z})$

5. $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{e v_0 b}{(a^2+b^2)^{3/2}} (-\hat{z})$

6. $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{e v_0 b}{(a^2+b^2)^{3/2}} \hat{y}$ **correct**

7. $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{e v_0 b}{(a^2+b^2)} \hat{y}$

8. $\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{e v_0 b}{(a^2+b^2)} \hat{z}$

Explanation:

Note that there will be no change in the magnitude of $\vec{B}$. The only change that will occur is in the direction. To find the new direction, evaluating the following cross-product yields

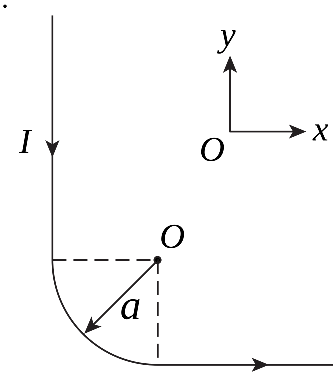
$$\langle v_0, 0, 0\rangle \times \langle a, 0, b\rangle = v_0 b(-\hat{y})$$

The B-S law is applied to an electron with charge $-e$, so an extra negative sign comes into the equation. The new direction will be $-(-\hat{y}) = \hat{y}$. Hence, the answer is

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{e v_0 b}{(a^2+b^2)^{3/2}} (\hat{y})$$

Curved Wire Segment
017 (part 1 of 2) 10.0 points

Consider a current configuration shown below. A long (effectively infinite) wire segment is connected to a quarter of a circular arc with radius a. The other end of the arc is connected to another long horizontal wire segment. The current is flowing from the top coming down vertically and flows to the right along the positive x-axis.



What is the direction of the magnetic field at O due to this current configuration?

1. along the positive y-axis

2. 135° counterclockwise from the $+x$-axis

3. 45° counterclockwise from the $+x$-axis

4. 225° counterclockwise from the $+x$-axis

5. along the negative y-axis

6. 315° counterclockwise from the $+x$-axis

7. perpendicular to and into the page

8. along the negative x-axis

9. perpendicular to and out of the page **correct**

10. along the positive $+x$-axis

Explanation:

From the Biot-Savart law we know that

$$d\vec{B} \propto I\, d\vec{l} \times \hat{r}\,.$$

One should first verify that the magnetic field at O contributed by any infinitesimal current element along the current configuration given is perpendicular to the page, which is coming out of the page. Therefore the resulting magnetic field must also be pointing out of the page.

---

018 (part 2 of 2) 10.0 points

Let $I = 1.6$ A and $a = 0.94$ m.

What is the magnitude of the magnetic field at O due to the current configuration?

Correct answer: 6.07795×10^{-7} T.

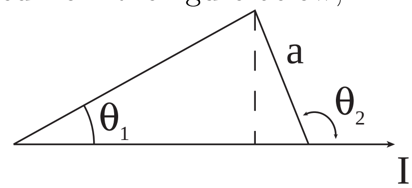
Explanation:

$$\begin{aligned} \text{Let}: \quad & I = 1.6 \text{ A}\,, \quad \text{and} \\ & a = 0.94 \text{ m}\,. \end{aligned}$$

Consider first the one-quarter of a circular arc. Since each current element is perpendicular to the unit vector pointing to O, we can write

$$\begin{aligned} B_{\text{arc}} &= \frac{\mu_0}{4\pi} I \int_0^{\pi/2} \frac{a\, d\theta}{a^2} \\ &= \frac{\mu_0}{4\pi} I \frac{\pi}{2\,a} \\ &= \frac{\mu_0\, I}{8\,a}\,. \end{aligned}$$

For the straight sections, we apply the formulas derived from the figure below,



$$\begin{aligned} \vec{B} &= \hat{z}\, \frac{\mu_0\, I}{4\,\pi\, a} \int_{\theta_1}^{\theta_2} d\theta \sin\theta \\ &= \hat{z}\, \frac{\mu_0\, I}{4\,\pi\, a} (\cos\theta_1 - \cos\theta_2) \end{aligned}$$

where $\hat{z}$ is the unit vector perpendicular to the plane of the paper that points to the reader.

For the downward y-directed current, $\theta_2 = \dfrac{\pi}{2}$ and $\theta_1 = 0$. For the horizontal x-directed current $\theta_2 = \pi$ and $\theta_1 = \dfrac{\pi}{2}$.

Thus we find the contribution at the point O for the magnetic field from the long vertical wire is same as in the long horizontal wire, and the sum is equal to

$$\begin{aligned} \vec{B} &= \hat{z}\, \frac{\mu_0\, I}{4\,\pi\, a} (1 + 1) \\ &= \hat{z}\, \frac{\mu_0\, I}{2\,\pi\, a}\,, \end{aligned}$$

/noindent the same as in a long straight wire.

Adding the contributions from the straight sections and the arc,

$$\begin{aligned} B_{tot} &= \frac{\mu_0\, I}{8\,a} + \left(\frac{\mu_0\, I}{2\,\pi\, a} \right) \\ &= \frac{\mu_0\, I}{2a} \left(\frac{1}{4} + \frac{1}{\pi} \right) \\ &= \frac{(1.25664 \times 10^{-6} \text{ T} \cdot \text{m/A})\,(1.6 \text{ A})}{2\,(0.94 \text{ m})} \\ &\quad \times \left(\frac{1}{4} + \frac{1}{\pi} \right) \\ &= \boxed{6.07795 \times 10^{-7} \text{ T}}\,. \end{aligned}$$

Conceptual 17 Q01

019 10.0 points

The figure represents two long, straight, parallel wires extending in a direction perpendicular to the page. The current in the right wire runs into the page and the current in the left runs out of the page.

What is the direction of the magnetic field created by these wires at location a, b and c? (b is midway between the wires.)

1. up, zero, down

2. down, down, up

3. down, zero, up

4. down, up, down **correct**

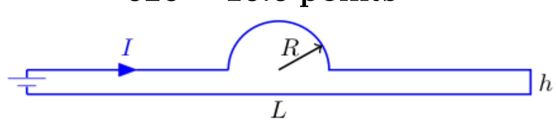
5. up, down, up

6. up, up, down

Explanation:

By the right-hand rule the right wire has a clockwise field and the left wire a counter-clockwise field.

B of Asymmetric Loop 01
020 10.0 points



The circuit shown above consists of a battery and a Nichrome wire and has a conventional current I. At the center of the semicircle, what is the magnitude of the magnetic field? You may assume $L \gg h$.

1. $\dfrac{\mu_0}{2\pi}\dfrac{I}{R}(2+\pi)$

2. $\dfrac{\mu_0}{4\pi}\dfrac{I}{R}(1+\pi)$

3. $\dfrac{\mu_0}{4\pi}\dfrac{I}{R}(2+\pi)$

4. $\dfrac{\mu_0 I}{4\pi}\left(\dfrac{\pi}{R}+\dfrac{2}{h}\right)$ **correct**

5. $\dfrac{\mu_0}{2\pi}\dfrac{2I}{R}(1+\pi)$

6. $\dfrac{\mu_0 I}{2\pi}\left(\dfrac{\pi}{R}+\dfrac{2}{h}\right)$

7. $\dfrac{\mu_0 I}{4\pi}\left(\dfrac{2\pi}{R}+\dfrac{2}{h}\right)$

8. $\dfrac{\mu_0 I}{2\pi}\left(\dfrac{2\pi}{R}+\dfrac{2}{h}\right)$

Explanation:

Examining the figure, we see that the semicircular section and the lower straight wire contribute to $|B|$. The upper straight wire portions have $d\vec{l} \parallel \hat{r}$, so do not contribute. Likewise, the short segment of length $h \ll L$ may be neglected. Consequently, we have a superposition of the magnetic fields of a half-loop and a straight wire of length L,

$$\begin{aligned} |\vec{B}| &= \frac{1}{2}\frac{\mu_0}{4\pi}\frac{2\pi I}{R} + \frac{\mu_0}{4\pi}\frac{2I}{h} \\ &= \frac{\mu_0 I}{4\pi}\left(\frac{\pi}{R}+\frac{2}{h}\right) \end{aligned}$$

AlnicoMagnetMI18x072
021 (part 1 of 2) 10.0 points

A particular alnico (aluminum, cobalt, nickel, and iron) bar magnet (magnet A) has a mass of 10 g. It produces a magnetic field of magnitude 6×10^{-5} T at a location 0.2 m from the center of the magnet, on the axis of the magnet.

Approximately what is the magnitude of the magnetic field of magnet A a distance of 0.4 m from the center of the magnet, along the same axis? (You may assume that the physical dimensions of the magnet are much smaller than the distances involved in the problem.) Also, use

$$\frac{\mu_0}{4\pi} = 1 \times 10^{-7}\ \text{T}\cdot\text{m/A}.$$

Correct answer: 7.5×10^{-6} T.

Explanation:

The expression for the magnitude of the field of a magnetic dipole is

$$\left|B_{\text{dipole}}\right| = \frac{\mu_0}{4\pi}\frac{2\,\mu}{r^3}.$$

One way to solve this is to find μ at $r = 0.2$ m and then solve for B at $r = 0.4$ m. Alternately, note that $B \propto 1/r^3$, so $B\,r^3$ is a constant:

$$\begin{aligned} B_1\,r_1^3 &= B_2\,r_2^3 \\ (6\times 10^{-5}\text{ T})(0.2\text{ m})^3 &= (B)(0.4\text{ m})^3 \\ \Rightarrow B &= \boxed{7.5\times 10^{-6}\text{ T}}. \end{aligned}$$

Note that doubling the distance causes B to change by a factor of $(1/2)^3 = 1/8$.

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022 (part 2 of 2) 10.0 points

If you removed the original magnet and replaced it with a magnet made of the same material but with a mass of 30 g (magnet B), approximately what would be the magnetic field at a location 0.2 m from the center of the magnet, on the axis of the magnet?

Correct answer: 0.00018 T.

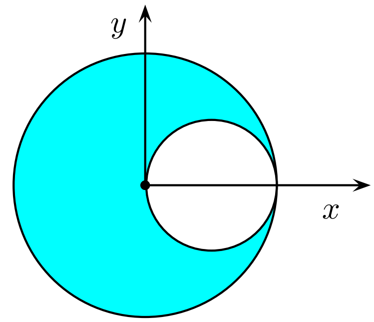
Explanation:

Increasing the mass by a factor of 3 causes the dipole moment to increase by a factor of 3. This is like placing 3 identical magnets end to end. As a result, B at $r = 0.2$ m increases by a factor of 3, since $B \propto \mu$. Thus,

$$B = (3)(6\times 10^{-5}\text{ T}) = \boxed{0.00018\text{ T}}.$$

Off Centered Hole
023 10.0 points

A total current of 52 mA flows through an infinitely long cylinderical conductor of radius 3 cm which has an infinitely long cylindrical hole through it of diameter r centered at $\dfrac{r}{2}$ along the x-axis as shown.



What is the magnitude of the magnetic field at a distance of 11 cm along the positive x-axis? The permeability of free space is $4\,\pi\times 10^{-7}\,\text{T}\cdot\text{m/A}$. Assume the current density is constant throughout the conductor.

Correct answer: 8.95694×10^{-8} T.

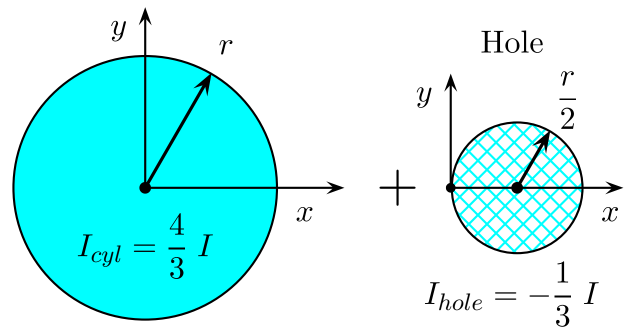
Explanation:

Basic Concepts: Magnetic Field due to a Long Cylinder

$$B = \frac{\mu_0\,I}{2\,\pi\,r}.$$

Principle of Superposition.

Our goal is to model the given situation, which is complex and lacks symmetry, by adding together the fields from combinations of simpler current configurations which together match the given current distribution. The combination of the currents in Fig. 2 will do so if we choose I_{cyl} and I_{hole} correctly.



Since the current is uniform, the current density $J = \dfrac{I}{A}$ is constant. Then

$$J = I_{cyl}\,A_{cyl} = -I_{hole}\,A_{hole}$$

Clearly, $A_{cyl} = \pi\,r^2$, and $A_{hole} = \dfrac{\pi\,r^2}{4}$, so $I_{hole} = -\dfrac{I_{cyl}}{4}$.

Note: The minus sign means I_{hole} is flowing in the direction opposite I_{cyl} and I, as it must if it is going to cancel with I_{cyl} to model the hole.

We also require $I = I_{cyl} + I_{hole}$. We then have $I_{cyl} = \frac{4}{3} I$, and $I_{hole} = -\frac{1}{3} I$. With these currents, the combination of the two cylinders in figure 2 gives the same net current and current distribution as the conductor in our problem.

The magnetic fields are

$$B_{cyl} = \frac{\mu_0 \left(\frac{4}{3} I\right)}{2\pi x}$$
$$B_{hole} = \frac{\mu_0 \left(-\frac{1}{3} I\right)}{2\pi (x - r/2)},$$

so the total magnetic field is

$$\begin{aligned} B_{total} &= B_{cyl} + B_{hole} \\ &= \frac{\mu_0 I}{6\pi} \left(\frac{4}{x} - \frac{1}{x - \frac{r}{2}} \right) \\ &= \frac{\mu_0 I}{6\pi} \left[\frac{3x - 2r}{x\left(x - \frac{r}{2}\right)} \right] \\ &= \frac{(4\pi \times 10^{-7}\ \mathrm{T\,m/A})\,(52\ \mathrm{mA})}{6\pi} \\ &\quad \times \left[\frac{3\,(11\ \mathrm{cm}) - 2\,(3\ \mathrm{cm})}{(11\ \mathrm{cm})\left(11\ \mathrm{cm} - \frac{3\ \mathrm{cm}}{2}\right)} \right] \\ &= \boxed{8.95694 \times 10^{-8}\ \mathrm{T}}. \end{aligned}$$

keywords:

BreakingcoilMI
024 10.0 points

A current-carrying solenoidal coil of length L is uniformly wound with N turns. Suppose the coil is now cut into half, and the original current is run through each of the resulting new solenoids with half the number of turns $(N/2)$ as the original coil and half the length $L/2$ as the original coil. We are now left with:

(Ia) One coil with only a north pole and the other with only a south pole.

(Ib) Two smaller coils, each with a North end and a South end.

(Ic) Two coils that don't make any magnetic field when a current runs through them.

If the magnetic field created inside of one of the new coils (far from the ends) is B', and that created by the original coil is B (all other parameters being the same), then which of the following relations is true?

(IIa) $B' = B$

(IIb) $B' = \frac{B}{2}$

(IIc) $B' = \frac{B}{8}$

(IId) $B' = 2B$

1. Ia, IIa

2. Ic, IIc

3. Ib, IIb

4. Ic, IIa

5. Ic, IId

6. Ia, IIb

7. Ia, IId

8. Ic, IIb

9. Ib, IIc

10. Ib, IIa **correct**

Explanation:

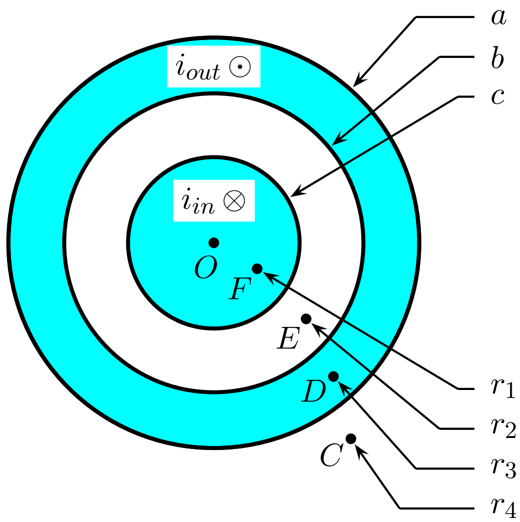
Magnetic strength of a coil is proportional to N/L, where N is the total number of turns and L is the length of the coil. When carrying a current, each smaller coil still acts like a magnetic dipole, so must have a North and a South pole. Hence, Ib is correct.

Since magnetic field strength is proportional to N/L and this ratio does not change when the coil is divided, we still have $B' = B$. Hence, IIa is correct.

Cable 01

025 (part 1 of 2) 10.0 points

The figure below shows a straight cylindrical coaxial cable of radii a, b, and c in which equal, uniformly distributed, but antiparallel currents i exist in the two conductors.



Which expression gives the magnitude of the magnetic field in the region $r_1 < c$ (at F)?

1. $B(r_1) = \dfrac{\mu_0\, i}{2\,\pi\, r_1}$

2. $B(r_1) = \dfrac{\mu_0\, i\, r_1}{2\,\pi\, c^2}$ **correct**

3. $B(r_1) = \dfrac{\mu_0\, i\, r_1}{2\,\pi\, b^2}$

4. $B(r_1) = \dfrac{\mu_0\, i\,(r_1^2 - b^2)}{2\,\pi\, r_1\,(a^2 - b^2)}$

5. $B(r_1) = \dfrac{\mu_0\, i\,(a^2 - r_1^2)}{2\,\pi\, r_1\,(a^2 - b^2)}$

6. $B(r_1) = \dfrac{\mu_0\, i\,(a^2 - b^2)}{2\,\pi\, r_1\,(r_1^2 - b^2)}$

7. $B(r_1) = \dfrac{\mu_0\, i}{\pi\, r_1}$

8. $B(r_1) = \dfrac{\mu_0\, i\, r_1}{2\,\pi\, a^2}$

9. $B(r_1) = 0$

10. $B(r_1) = \dfrac{\mu_0\, i\,(a^2 + r_1^2 - 2\,b^2)}{2\,\pi\, r_1\,(a^2 - b^2)}$

Explanation:

Ampere's Law states that the line integral $\oint \vec{B} \cdot d\vec{\ell}$ around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path.

Considering the symmetry of this problem, we choose a circular path, so **Ampere's Law** simplifies to

$$B\,(2\,\pi\, r_1) = \mu_0\, I_{en}\,,$$

where r_1 is the radius of the circle and I_{en} is the current enclosed.

For $r_1 < c$,

$$\begin{aligned} B &= \frac{\mu_0\, I_{en}}{2\,\pi\, r_1} \\ &= \frac{\mu_0 \left(i\,\dfrac{\pi\, r_1^2}{\pi\, c^2}\right)}{2\,\pi\, r_1} \\ &= \frac{\mu_0\, i \left(\dfrac{r_1^2}{c^2}\right)}{2\,\pi\, r_1} \\ &= \boxed{\frac{\mu_0\, i\, r_1}{2\,\pi\, c^2}}\,. \end{aligned}$$

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026 (part 2 of 2) 10.0 points

Which expression gives the magnitude of the magnetic field in the region $c < r_2 < b$ (at E)?

1. $B(r_2) = \dfrac{\mu_0\, i\,(r_2^2 - b^2)}{2\,\pi\, r_2\,(a^2 - b^2)}$

2. $B(r_2) = \dfrac{\mu_0\, i}{2\,\pi\, r_2}$ **correct**

3. $B(r_2) = \dfrac{\mu_0\, i\,(a^2 + r_2^2 - 2\,b^2)}{2\,\pi\, r_2\,(a^2 - b^2)}$

4. $B(r_2) = \dfrac{\mu_0\, i\, r_2}{2\,\pi\, c^2}$

5. $B(r_2) = 0$

6. $B(r_2) = \dfrac{\mu_0\, i\, r_2}{2\,\pi\, b^2}$

7. $B(r_2) = \dfrac{\mu_0\, i\,(a^2 - b^2)}{2\,\pi\, r_2\,(r_2^2 - b^2)}$

8. $B(r_2) = \dfrac{\mu_0\, i\, (a^2 - r_2^2)}{2\,\pi\, r_2\, (a^2 - b^2)}$

9. $B(r_2) = \dfrac{\mu_0\, i}{\pi\, r_2}$

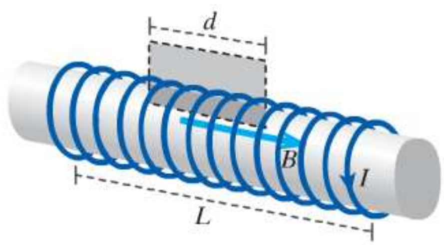
10. $B(r_2) = \dfrac{\mu_0\, i\, r_2}{2\,\pi\, a^2}$

Explanation:

For $c < r_2 < b$,

$$\begin{aligned} B &= \frac{\mu_0\, I_{en}}{2\,\pi\, r_2} \\ &= \frac{\mu_0\ (i)}{2\,\pi\, r_2} \\ &= \boxed{\frac{\mu_0\, i}{2\,\pi\, r_2}}. \end{aligned}$$

Ampere's law and a long solenoid
027 10.0 points



Consider a solenoid with the setup shown in the figure. The total number of turns is N within a total length L. Apply Ampere's law for the path along the boundary of the rectangle shown, where the number of turns within the gray area is given by ΔN. Choose the correct pair of statements below.

Ia. $B\, d = \mu_0\, N\, I$.
Ib. $2\, B\, d = \mu_0\, N\, I$.
Ic. $B\, d = \mu_0\, \Delta N\, I$.
Id. $2\, B\, d = \mu_0\, \Delta N\, I$.
Ie: $B\, d = \mu_0\, I$.

IIa. B outside the solenoid has the same magnitude as B inside and is opposite in direction.

IIb. B outside the solenoid is negligible compared to B inside.

1. Ia, IIb

2. Ic, IIa

3. Ib, IIa

4. Id, IIa

5. Ic, IIb **correct**

6. Ie, IIb

7. Ie, IIa

8. Ib, IIb

9. Id, IIb

10. Ia, IIa

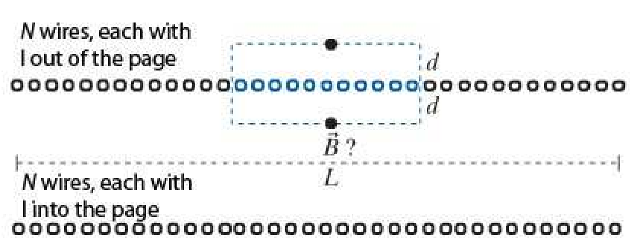
Explanation:

By inspection the correct Ampere's law expression along the path specified is given by: $B\, d = \mu_0\, \Delta N\, I$, where the condition that B outside is negligible was used. So the correct answer is the pair: Ic and IIb.

This leads to $B = \dfrac{\mu_0\, \Delta N\, I}{d} = \mu_0 \left(\dfrac{N}{L}\right) I$, which is the expected answer.

Exam3 22.P.31
028 10.0 points

The figure shows a large number N of closely packed wires, each carring a current I out of the page. The width of this shweet of wires is L. Find B in the region between two parallel current sheets with equal currents running in opposite directions.



1. $B = \dfrac{\mu_0\, N\, I}{2d}$

2. $B = \dfrac{\mu_0\, d\, I}{2L}$

3. $B = \dfrac{2\mu_0\, N\, I}{d}$

4. $B = \dfrac{\mu_0\, d\, I}{L}$

5. $B = \dfrac{\mu_0\, N\, I}{2L}$

6. $B = \dfrac{2\mu_0\, d\, I}{L}$

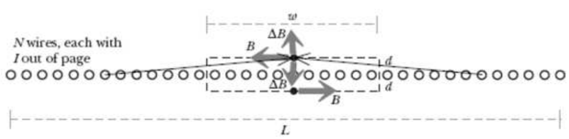
7. $B = \dfrac{\mu_0\, N\, I}{d}$

8. $B = \dfrac{2\mu_0\, N\, I}{L}$

9. $B = \dfrac{\mu_0\, N\, I}{L}$ **correct**

Explanation:

From the diagram it is clear that there is cancellation of the vertical components of magnetic field contributed by two wires to the left and the right of the observagtion location. Therefore the direction of the magnetic field must be to the left above the current sheet and to the right below :



We first work out the contribution to the magnetic field due to the top wire sheet alone. Use Ampere's law, and go counterclockwise around the closed rectangular path.

Along the sides of the path,

$$\int \vec{B} \cdot d\vec{l} = 0$$

, since $\vec{B}$ is perpendicular to $d\vec{l}$.

Along the upper part of the path,

$$\int \vec{B} \cdot d\vec{l} = B_{top} w\,.$$

Along the lower part of the path,

$$\int \vec{B} \cdot d\vec{l} = B_{top} w\,.$$

Therefore,

$$\oint \vec{B} \cdot d\vec{l} = 2B_{top} w = \mu_0\, I_{inside\ path} = \mu_0 \left(\frac{N}{L}\right) w\, I$$

, since there are N/L current-carrying wires per meter, and a width w of the enclosing path. So the top sheet contribution along is given by

$$B_{top} = \frac{\mu_0\, N\, I}{2L}\,.$$

Based on the superposition principle, the contributions between the two sheets are pointing in the same direction, they should add. So we have the resultant magnetic field

$$B = 2B_{top} = \frac{\mu_0\, N\, I}{L}\,.$$

Moving a Charge
029 10.0 points

It takes 143 J of work to move 2.2 C of charge from the negative plate to the positive plate of a parallel plate capacitor.

What voltage difference exists between the plates?

Correct answer: 65 V.

Explanation:

$$\text{Let :} \quad W = 143 \text{ J} \quad \text{and} \quad q = 2.2 \text{ C}\,.$$

The voltage difference is

$$V = \frac{W}{q} = \frac{143 \text{ J}}{2.2 \text{ C}} = \boxed{65 \text{ V}}\,.$$

Delta V 01
030 10.0 points

You move from location i at $\langle 5, 5, 2\rangle$ m to location f at $\langle 5, 4, 9\rangle$ m. All along this path is a uniform electric field whose value is $\vec{E} = \langle 900, 100, -700\rangle$ N/C. Calculate $\Delta V = V_f - V_i$.

Correct answer: 5000 V.

Explanation:

Recalling that

$$\Delta V = -\int_i^f \vec{E} \bullet d\vec{l},$$

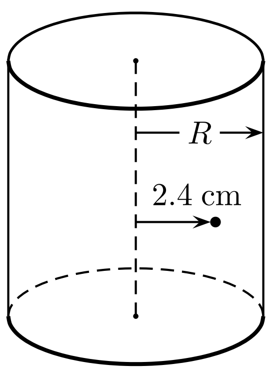
and noting that $\vec{E}$ is uniform so that it may come outside the integral, ΔV may be calculated simply from $-\vec{E} \bullet \Delta \vec{l}$.

$$\begin{aligned}\Delta V &= -\vec{E} \bullet \Delta \vec{l} \\ &= -[E_x(f_x - i_x) + E_y(f_y - i_y) + E_z(f_z - i_z)] \\ &= 5000 \text{ V}\end{aligned}$$

Long Cylindrical Insulator

031 (part 1 of 3) 10.0 points

Consider a long, uniformly charged, cylindrical insulator of radius R and charge density $1.5\ \mu\text{C/m}^3$.



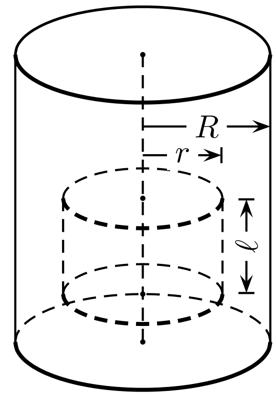
What is the electric field inside the insulator at a distance 2.4 cm $< R$ from the axis? The volume of a cylinder with radius r and length ℓ is $V = \pi r^2 \ell$. The value of the permittivity of free space is $8.85419 \times 10^{-12}\ \text{C}^2/\text{N/m}^2$.

Correct answer: 2032.94 N/C.

Explanation:

$$\begin{aligned}\text{Let :}\quad \rho &= 1.5\ \mu\text{C/m}^3 = 1.5 \times 10^{-6}\ \text{C/m}^3, \\ r &= 2.4\ \text{cm} = 0.024\ \text{m}, \quad \text{and} \\ \epsilon_0 &= 8.85419 \times 10^{-12}\ \text{C}^2/\text{N/m}^2.\end{aligned}$$

Consider a cylindrical Gaussian surface of radius r and length ℓ much less than the length of the insulator so that the component of the electric field parallel to the axis is negligible.



The flux leaving the ends of the Gaussian cylinder is negligible, and the only contribution to the flux is from the side of the cylinder. Since the field is perpendicular to this surface, the flux is $\Phi_s = 2\pi r \ell E$ and the charge enclosed by the surface is $Q_{enc} = \pi r^2 \ell \rho$.

Using Gauss' law,

$$\begin{aligned}\Phi_s &= \frac{Q_{enc}}{\epsilon_0} \\ 2\pi r \ell E &= \frac{\pi r^2 \ell \rho}{\epsilon_0} \\ E &= \frac{\rho r}{2\epsilon_0} \\ &= \frac{(1.5 \times 10^{-6}\ \text{C/m}^3)(0.024\ \text{m})}{2(8.85419 \times 10^{-12}\ \text{C}^2/\text{N/m}^2)} \\ &= \boxed{2032.94\ \text{N/C}}.\end{aligned}$$

---

032 (part 2 of 3) 10.0 points

Determine the absolute value of the potential difference between r_1 and R, where $r_1 < R$. For $r < R$ the electric field takes the form $E = C\,r$, where C is positive.

1. $|V| = \dfrac{1}{2} C (R - r_1) r_1$

2. $|V| = C (R - r_1) r_1$

3. $|V| = C \left(\dfrac{1}{r_1^2} - \dfrac{1}{R^2}\right)$

4. $|V| = C \left(R^2 - r_1^2\right)$

5. $|V| = C (R - r_1)$

6. $|V| = \frac{1}{2} C \left(R^2 - r_1^2\right)$ **correct**

7. $|V| = C \frac{r_1}{2}$

8. $|V| = C\, r_1$

9. $|V| = C \sqrt{R^2 - r_1^2}$

10. $|V| = C \left(\frac{1}{r_1} - \frac{1}{R}\right)$

Explanation:

The potential difference between a point A inside the cylinder a distance r_1 from the axis to a point B a distance R from the axis is

$$\Delta V = -\int_A^B \vec{E} \cdot \vec{ds} = -\int_{r_1}^R E\, dr$$

since E is radial, so

$$\begin{aligned} \Delta V &= -\int_{r_1}^R C\, r\, dr = -C \left.\frac{r^2}{2}\right|_{r_1}^R \\ &= -C \left(\frac{R^2}{2} - \frac{r_1^2}{2}\right) \\ |\Delta V| &= C \left(\frac{R^2}{2} - \frac{r_1^2}{2}\right) = \frac{1}{2} C \left(R^2 - r_1^2\right) . \end{aligned}$$

---

033 (part 3 of 3) 10.0 points

What is the relationship between the potentials V_{r_1} and V_R?

1. $V_{r_1} = V_R$

2. $V_{r_1} > V_R$ **correct**

3. None of these

4. $V_{r_1} < V_R$

Explanation:

Since $C > 0$ and $R > r_1$,

$$V_B - V_A = \Delta V = -\frac{1}{2} C \left(R^2 - r_1^2\right) < 0 .$$

Thus $V_B < V_A$ and the potential is higher at point A where $r = r_1$ than at point B, where $r = R$.

Intuitive Reasoning: The natural tendency for a positive charge is to move from A to B, so A has a higher potential.

MI17p099a

034 (part 1 of 2) 10.0 points

An isolated parallel plate capacitor of area A_1 with an air gap of s_1 is charged up to a potential difference of V_{1i}. A second parallel plate capacitor is initially uncharged, has an area A_2 and a gap of length s_2 filled with plastic whose dielectric constant is κ. Connect a wire from the positive plate of the first capacitor to one of the plates of the second capacitor and connect another one from the negative plate of the first capacitor to the other plate of the second capacitor.

The initial voltage of the first capacitor before connection is given by V_{1i}. Denote the final potential differences of the first and the second capacitors to be ΔV_{1f} and ΔV_{2f} respectively and the final field in the gap the second capacitor in presence of the dielectric be E_{2f}. After the connection, let the charge across the second capacitor be denoted by Q_2.

Consider the following statements:

Ia. $\Delta V_{1f} = \Delta V_{2f}$
Ib. $\Delta V_{1f} > \Delta V_{2f}$

IIa. $E_{2f} = \frac{1}{\epsilon_0 \kappa} \frac{Q_2}{A_2}$
IIb. $E_{2f} = \frac{1}{2\epsilon_0 \kappa} \frac{Q_2}{A_2}$

IIIa. $\Delta V_{2f} = 2 E_{2f}\, s_2$
IIIb. $\Delta V_{2f} = E_{2f}\, s_2$

1. Ib, IIa, IIIb

2. Ib, IIb, IIIb

3. Ia, IIb, IIIa

4. Ib, IIb, IIIa

5. Ia, IIa, IIIa

6. Ib, IIa, IIIa

7. Ia, IIb, IIIb

8. Ia, IIa, IIIb **correct**

Explanation:

For the first capacitor alone, from Gauss law it can be shown that

$$E = \frac{1}{\epsilon_0}\frac{Q}{A_1} = \frac{V_{1i}}{s_1}$$

Solving for Q leads to:

$$Q = \left(\frac{\epsilon_0 A_1}{s_1}\right) V_{1i}$$

After the connection, say that the top plate of capacitor 1 is connected to the top plate of capacitor 2, and bottom plate of 1 to that of the bottom plate of 2. By definition: $\Delta V_{1f} = \Delta V_{2f}$ and the correct answer is Ia.

Denote the plate charges to be Q_1 and Q_2. Analogous to solving for Q, we can similarly solve for the second capacitor with plate charge Q_2 with the presence of the dielectric K,

$$E_{2f} = \frac{1}{\epsilon_0 \kappa}\frac{Q_2}{A_2}$$

Hence, the choice IIa is correct.

The correct definition for the potential difference is given by $\Delta V_{2f} = E_{2f}\, s_2$. Hence, IIIb is the correct answer.

---

035 (part 2 of 2) 10.0 points

For this part of the problem, we will simplify the problem assuming that the two capacitors have identical geometry, i.e. $A_1 = A_2 = A$ and $s_1 = s_2 = s$. There is still the dielectric slab with dielectric constant κ in capacitor 2.

Before the connection let the initial voltage of the first capacitor by V_{1i}. Denote the final potential differences of the first and the second capacitor to be ΔV_{1f} and ΔV_{2f}. After the connection, let the charge across the second capacitor be denoted by Q_2.

Consider the following statements:

Ia. $\Delta V_{1f} = \Delta V_{2f}$

Ib. $\Delta V_{1f} > \Delta V_{2f}$

IIa. $E_{2f} = \dfrac{1}{\epsilon_0 \kappa}\dfrac{Q_2}{A}$

IIb. $E_{2f} = \dfrac{1}{2\epsilon_0 \kappa}\dfrac{Q_2}{A}$

IIIa. $\Delta V_{1f} = \dfrac{\kappa}{1+\kappa} V_{1i}$

IIIb. $\Delta V_{2f} = \dfrac{1}{1+\kappa} V_{1i}$

1. Ib, IIb, IIIa

2. Ia, IIa, IIIa

3. Ia, IIb, IIIa

4. Ib, IIa, IIIb

5. Ib, IIb, IIIb

6. Ib, IIa, IIIa

7. Ia, IIa, IIIb **correct**

8. Ia, IIb, IIIb

Explanation:

The explanation for the choices of Ia and IIa is the same as that given in part 1.

Since the two potential differences are the same, we will refer to this common potential difference as V_f. In terms of this common potential difference V_f the total charge Q can be written as

$$Q = Q_1 + Q_2 = \left(\frac{\epsilon_0 A}{s}\right) V_f + \kappa\left(\frac{\epsilon_0 A}{s}\right) V_f$$

Expressing Q in terms of V_{1i} we obtain

$$\left(\frac{\epsilon_0 A}{s}\right) V_{1i} = \left(\frac{\epsilon_0 A}{s}\right) V_f + \kappa\left(\frac{\epsilon_0 A}{s}\right) V_f$$

On dividing through with the factor of $\dfrac{\epsilon_0 A}{s}$ we have

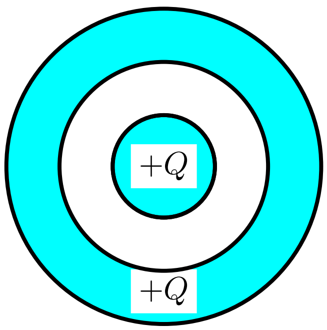
$$V_{1i} = V_f + \kappa V_f = (1+\kappa)\, V_f$$

$$V_f = \left(\frac{1}{1+\kappa}\right) V_{1i}$$

By definition, $\Delta V_{1f} = V_f$. So, IIIb is the correct answer.

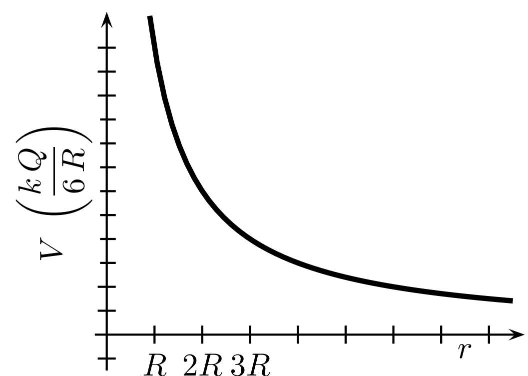
Potential Diagrams 01
036 10.0 points

Consider a conducting sphere with radius R and charge $+Q$, surrounded by a conducting spherical shell with inner radius $2\,R$, outer radius $3\,R$ and net charge $+Q$.

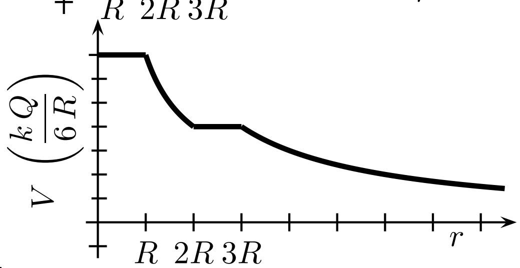


What potential *vs* radial distance diagram describes this situation?

1.

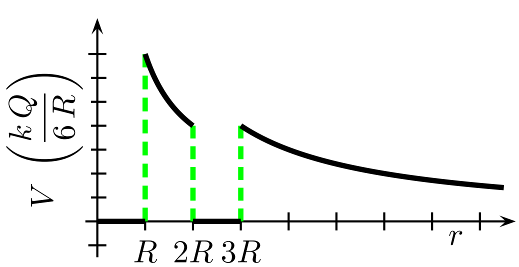


2.

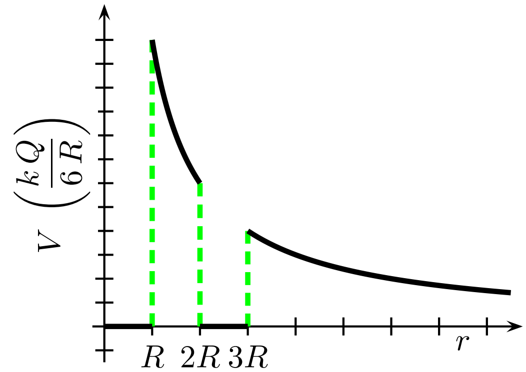


correct

3.



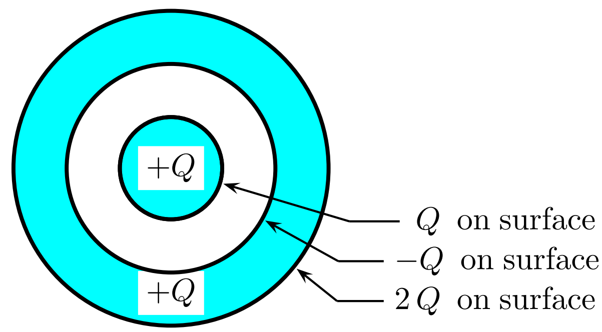
4.



Explanation:

The charge on the inner sphere is $+Q$, concentrated on its surface. The induced charge on the inner surface of the spherical shell is $-Q$, so the charge on the outer surface of the spherical shell is

$$Q_{net} - Q_{inner} = +Q - (-Q) = +2\,Q\,.$$



The potential within a conductor is constant and the electric field within a conductor is zero.

The potential for $3\,R < R < \infty$ (outside the conductors) is

$$V_r = k\left[\frac{+Q + (+Q)}{r}\right] = k\left(\frac{2\,Q}{r}\right)\,.$$

For $2\,R \le r \le 3\,R$ (inside the conducting shell),

$$V_{3R} = V_r = V_{2R} = k\left(\frac{2\,Q}{3\,R}\right)\,.$$

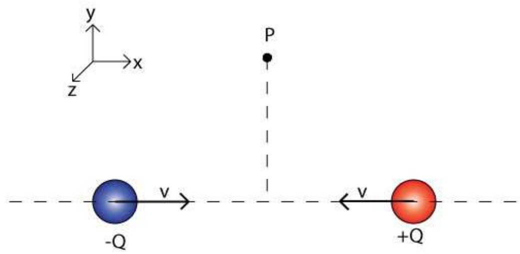
For $R < r < 2\,R$ (between the conductors),

$$V_r = k\left[\frac{+Q}{r} + \frac{-Q}{2\,R} + \frac{2\,Q}{3\,R}\right]$$
$$= k\,Q\left(\frac{1}{r} + \frac{1}{6\,R}\right)\,.$$

For $0 < r \leq R$ (inside the conducting sphere),

$$V_R = V_r = V_0 = k\,Q\left[\frac{1}{R} + \frac{1}{6\,R}\right] = 7\,\frac{k\,Q}{6\,R}\,.$$

Mag. field of two moving charges
037 10.0 points



Two equal and opposite charges are equidistant from point P and are moving towards each other with the same speed v. The charges and point P are all in the xy plane.

The E-field at point P

Ia) Is **not** the zero vector
Ib) Is the zero vector

The B-field at point P

IIa) Is in the $+\hat{k}$ direction
IIb) Is in the $-\hat{k}$ direction
IIc) Is in the $+\hat{\jmath}$ direction
IId) Is in the $-\hat{\jmath}$ direction
IIe) Is the zero vector

1. Ia, IIe

2. Ia, IIb **correct**

3. Ib, IIb

4. Ib, IIc

5. Ia, IIa

6. Ib, IIe

7. Ia, IIc

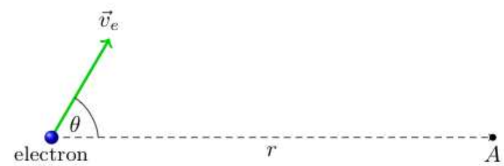
8. Ib, IIa

9. Ia, IId

10. Ib, IId

Explanation:
The E-field at P points in the $-\hat{\imath}$ direction and is nonzero. Using the Biot-Savart law, we know that the B-field is given by $\vec{B} = q\,\vec{v}\times\hat{r}$. Thus, the B-field is in the $-\hat{k}$ direction for both the charges.

B of Moving Charge 01
038 10.0 points



The electron in the figure is located at the origin and traveling with velocity $\vec{v} = \langle 1.2\times10^6, 2.1\times10^6, 0\rangle$ m/s. What is B_z at point $A = (1\times10^{-10}, 0, 0)$ m?

Correct answer: 3.36 T.

Explanation:
This is a straightforward application of the Biot-Savart Law. For B_z, we have

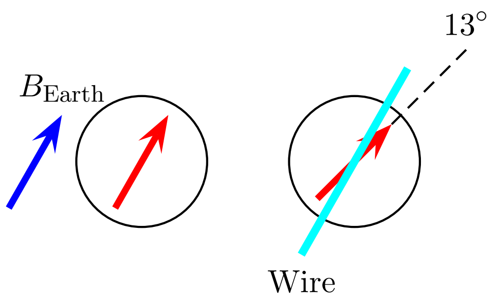
$$B_z = \frac{\mu_0}{4\pi}\frac{q}{r^2}(v_x|\hat{r}_y| - v_y|\hat{r}_x|)\,.$$

Inserting the values of the problem, this becomes

$$\begin{aligned} B_z &= \frac{\mu_0}{4\pi}\frac{-e}{r_x^2}(-v_y) \\ &= \frac{\mu_0}{4\pi}\frac{e v_y}{r_x^2} \\ &= 3.36 \text{ T} \end{aligned}$$

WireAndCompassMI18p057
039 (part 1 of 2) 10.0 points

When you bring a current-carrying wire down onto the top of a compass, aligned with the original direction of the needle and 7 mm above the needle, the needle deflects by 13 degrees, as in the figure below.



Assuming the compass needle was originally pointing toward the north, what direction is the conventional current traveling in the wire, and what is the direction of the force on the compass needle due to the magnetic field caused by the current in the wire?

1. North, West

2. North, East

3. South, East **correct**

4. South, West

Explanation:

We just need to think about how the compass needle responds to the presence of the current-carrying wire to answer this question. Since the compass needle deflects to the east, that is the direction of the magnetic force on the needle due to the current in the wire. To decide which way current is traveling in the wire, we use the right hand rule. Your fingers should point toward the east, so in order for your fingers to curl around the wire and point toward the east, your thumb must point toward the south. So the current is traveling toward the south.

---

040 (part 2 of 2) 10.0 points

Calculate the amount of current flowing in the wire. The measurement was made at a location where the horizontal component of the Earth's magnetic field is

$$B_{\text{Earth}} = 2 \times 10^{-5}\ \text{T}.$$

Use

$$\frac{\mu_0}{4\pi} = 1 \times 10^{-7}\ \text{Tm/A}.$$

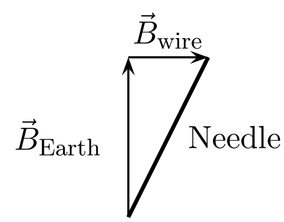
Correct answer: 0.161608.

Explanation:

Ultimately, we want to use the expression

$$B_{\text{wire}} = \frac{\mu_0}{4\pi}\frac{2\,I}{r}$$

to find the current in the wire. We know r, but not B_{wire} yet. To find B_{wire}, we can use what we know about the Earth's magnetic field and the deflection of the compass needle. Consider the following simple diagram:



From this drawing, we can write down

$$\begin{aligned}\tan\theta &= \frac{B_{\text{wire}}}{B_{\text{Earth}}}\\ \Rightarrow B_{\text{wire}} &= B_{\text{Earth}}\,\tan\theta\\ &\approx (2 \times 10^{-5}\ \text{T})\tan 13^\circ\\ &= 4.61736 \times 10^{-6}\ \text{T}\,.\end{aligned}$$

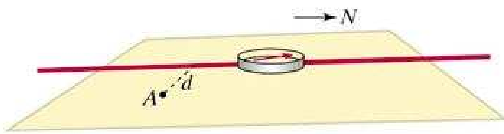
Now we simply rearrange the expression above to find the current:

$$\begin{aligned}I &= \frac{B_{\text{wire}}\,r}{2\left(\dfrac{\mu_0}{4\pi}\right)}\\ &= \frac{(4.61736 \times 10^{-6}\ \text{T})(7\ \text{mm})}{2(1 \times 10^{-7}\ \text{Tm/A})}\\ &= \boxed{0.161608\ \text{A}}\,.\end{aligned}$$

FieldOfLongWireMI18p062
041 10.0 points

A long current-carrying wire, oriented North-South, lies on a table (it is connected to batteries which are not shown). A compass lies on top of the wire, with the compass needle about 3 mm above the wire. With the current running, the compass deflects 9 ° to the

West. At this location, the horizontal component of the Earth's magnetic field is about 2×10^{-5} T.



What is the magnitude of the magnetic field at location A, on the table top, a distance 2.7 cm to the East of the wire, due only to the current in the wire?

Correct answer: 3.52×10^{-7} T.

Explanation:

$$\begin{aligned} \text{let :}\quad B_{earth} &= 2 \times 10^{-5}\ \text{T}\,, \\ r_{compass} &= 3\ \text{mm} = 0.003\ \text{m}\,, \\ r_A &= 2.7\ \text{cm} = 0.027\ \text{m}\,, \quad \text{and} \\ \theta &= 9\,^\circ\,. \end{aligned}$$

The magnetic field due to a wire is

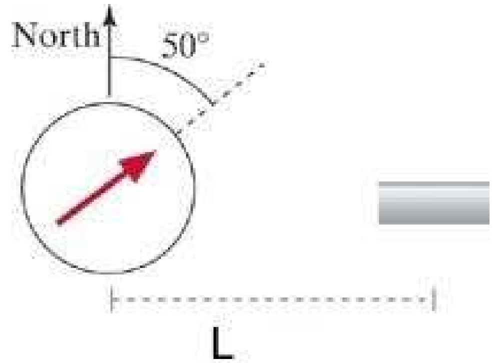
$$B = \frac{\mu_0}{4\,\pi}\,\frac{2\,I}{r}$$

Since $tan(\theta) = \dfrac{B_{compass}}{B_{earth}}$, $B_{compass} = B_{earth}\, tan(\theta)$. By writing the equation for the magnetic field of a wire for the compass and for point A and dividing the two equations, we obtain

$$\begin{aligned} B_A &= B_{compass}\,\frac{r_{compass}}{r_A} \\ &= B_{earth}\, tan(\theta)\,\frac{r_{compass}}{r_A} \\ &= (2 \times 10^{-5}\ \text{T})\, tan(9\,^\circ)\,\frac{0.003\ \text{m}}{0.027\ \text{m}} \\ &= 3.52 \times 10^{-7}\ \text{T}\,. \end{aligned}$$

---

number of atoms
042 10.0 points



A bar magnet is aligned east - west, with its center $L = 0.32$ m from the center of a compass as shown in the above figure. The compass is observed to deflect 50° away from north as shown, and the horizontal component of the Earth's magnetic field is known to be $2{\times}10^{-5}$ tesla.

Approximately how many atoms are in the bar magnet, assuming that one atom has a magnetic dipole moment of 9.268×10^{-24} Am^2.

Correct answer: 4.213×10^{23}.

Explanation:

The magnitude of the B-field is given by

$$|\vec{B}| = |\vec{B}_E|\, \tan\theta \approx (2{\times}10^{-5}) \tan 50^\circ$$

$$B = |\vec{B}| \approx 2.3835{\times}10^{-5}\, T$$

The magnetic dipole moment is given by

$$|\vec{\mu}| = \frac{|\vec{B}|\,|\vec{r}|^3}{2\left(\dfrac{\mu_0}{4\pi}\right)} = \frac{B\,L^3}{2\left(\dfrac{\mu_0}{4\pi}\right)}$$

$$|\vec{\mu}| = \frac{(2.3835 \times 10^{-5})(0.32\ \text{m})^3}{2(1 \times 10^{-7})} = 3.905\ \text{Am}^2$$

The no. of atoms in the bar magnet is given by

$$N = \frac{\mu}{\mu_{atom}} = \frac{3.905\ \text{Am}^2}{9.268 \times 10^{-24}\ \text{Am}^2} = 4.213 \times 10^{23}$$

Conceptual 24 Q08
043 10.0 points

A normal piece of iron produces no external magnetic field. Suppose a piece of iron consisted of one very large domain instead of many small ferromagnetic domains.

Would this piece of iron produce an external magnetic field?

1. No. It does not create magnetic field at all.

2. Yes. It creates magnetic field. **correct**

3. No. It does not create magnetic field because the net magnetic field is zero.

Explanation:

In normal iron, each domain acts like a small bar magnet. However, the random orientation of the domains causes cancellation and there would be no cancellation and it would create magnetic field.

WireSheetDensityMI
044 10.0 points

In the figure below, a large number of closely-spaced wires parallel to the z-axis form a 'sheet of current' in the xz plane, with each wire carrying a current I in the $+z$ direction (out of the plane of the figure). It is known that the magnitude of magnetic field at point P shown in figure is B. What is the algebraic expression for the number density of the sheet (number of wires per unit distance along the x-axis)? You can assume the dimensions of the sheet to be infinite in both x and z directions (implying that there are a large number of these closely-packed wires and they are all very long). In the figure, $+x$ points to the right, $+y$ points upwards and $+z$ points out of the plane of the figure, towards you.

closely spaced wires (cross section)

1. $\dfrac{2B}{\mu_0 I}$ **correct**

2. $\dfrac{B}{\mu_0 I}$

3. $\dfrac{B}{2\mu_0 I}$

4. $\dfrac{1}{2d}$

5. $\dfrac{B}{4\mu_0 I}$

6. $\dfrac{1}{d}$

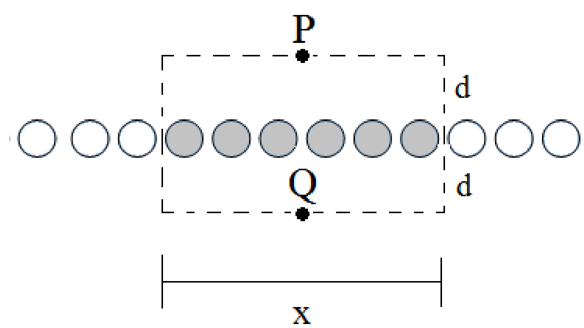
7. $\dfrac{2}{d}$

8. $\dfrac{4B}{\mu_0 I}$

Explanation:

Using Ampere's law, it is easy to obtain the magnetic field due to the sheet of wire at a point above (or below) the sheet, in terms of the number density n of the wires. On inverting this algebraic expression, we can get the number density in terms of the magnetic field. We proceed in the following manner.

First, we shall choose a rectangular closed path as shown in the figure.



Using right hand rule and symmetry, it is straightforward to show that the magnetic field due to the sheet points to left at P and right at Q and has the same magnitude B at any point on the top and bottom sides of the rectangle. Now applying Ampere's law, we get B in the following manner.

$$\int \vec{B}.\vec{dl} = \mu_0 I_{enclosed}$$

$$Bx + 0 + Bx + 0 = \mu_0 I\, n\, x$$

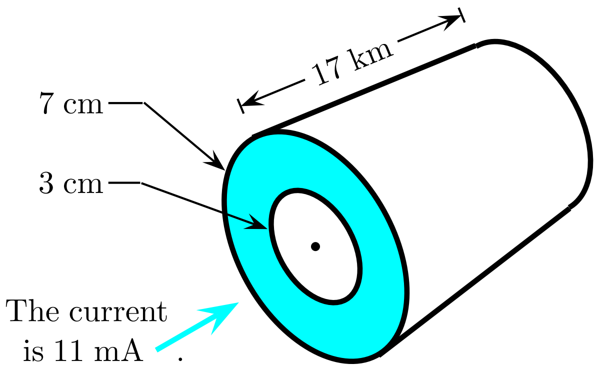
$$B = \frac{\mu_0\, nI}{2}$$

From this, the number density can be obtained as $n = \dfrac{2B}{\mu_0 I}$.

Cylindrical Shell of Current
045 10.0 points

A long cylindrical shell has a uniform current density. The total current flowing through the shell is 11 mA.

The permeability of free space is 1.25664×10^{-6} T · m/A .

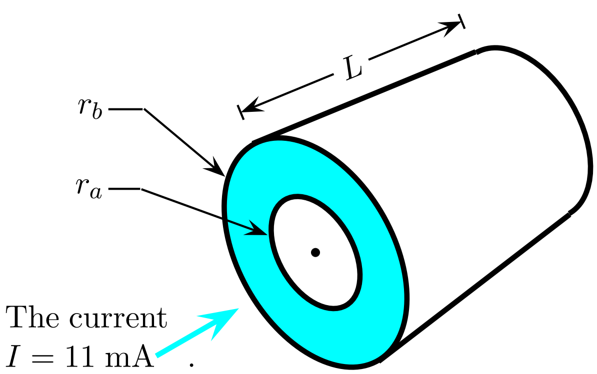


Find the magnitude of the magnetic field at a point $r_1 = 4.1$ cm from the cylindrical axis.

Correct answer: 10.4768 nT.

Explanation:

$$\begin{aligned}\text{Let :} \quad L &= 17 \text{ km}\,, \\ r_a &= 3 \text{ cm} = 0.03 \text{ m}\,, \\ r_b &= 7 \text{ cm} = 0.07 \text{ m}\,, \\ r_1 &= 4.1 \text{ cm} = 0.041 \text{ m}\,, \\ I &= 11 \text{ mA}\,, \quad \text{and} \\ \mu_b &= 1.25664 \times 10^{-6} \text{ T} \cdot \text{m/A}\,.\end{aligned}$$



Since the cylindrical shell is infinitely long, and has cylindrical symmetry, Ampere's Law gives the easiest solution. Consider a circle of radius r_1 centered around the center of the shell. To use Ampere's law we need the amount of current that cuts through this circle of radius r_1. To get this, we first need to compute the current density, for the current flowing through the shell.

$$\begin{aligned}J &= \frac{I}{A} \\ &= \frac{I}{\pi\, r_b^2 - \pi\, r_a^2} \\ &= \frac{(11 \text{ mA})}{\pi\,[(0.07 \text{ m})^2 - (0.03 \text{ m})^2]} \\ &= 0.875352 \text{ A/m}^2\,.\end{aligned}$$

The current enclosed within the circle is

$$\begin{aligned}I_{\text{enc}} &= \pi\,[r_1^2 - r_a^2] \cdot J \\ &= \pi\,[(0.041 \text{ m})^2 - (0.03 \text{ m})^2] \\ &\quad \times (0.875352 \text{ A/m}^2) \\ &= 0.00214775 \text{ A}\,.\end{aligned}$$

Ampere's Law,

$$\begin{aligned}\oint \vec{B} \cdot d\vec{s} &= \mu_0\, I_{\text{enc}} \\ B\, 2\,\pi\, r_1 &= \mu_0\, I_{\text{enc}} \\ B &= \frac{\mu_0\, I_{\text{enc}}}{2\,\pi\, r_1} \\ &= \frac{1.25664 \times 10^{-6} \text{ T} \cdot \text{m/A}}{2\,\pi\,(0.041 \text{ m})} \\ &\quad \times (0.00214775 \text{ A}) \\ &= \boxed{10.4768 \text{ nT}}\,.\end{aligned}$$