

E & M - Basic Physical Concepts

Electric force and electric field

Electric force between 2 point charges:

$$|F| = k \frac{|q_1||q_2|}{r^2}$$

$$k = 8.987551787 \times 10^9 \text{ N m}^2/\text{C}^2$$

$$\epsilon_0 = \frac{1}{4\pi k} = 8.854187817 \times 10^{-12} \text{ C}^2/\text{N m}^2$$

$$q_p = -q_e = 1.60217733(49) \times 10^{-19} \text{ C}$$

$$m_p = 1.672623(10) \times 10^{-27} \text{ kg}$$

$$m_e = 9.1093897(54) \times 10^{-31} \text{ kg}$$

Electric field: $\vec{E} = \frac{\vec{F}}{q}$

Point charge: $|E| = k \frac{|Q|}{r^2}$, $\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots$

Field patterns: point charge, dipole, || plates, rod, spheres, cylinders, ...

Charge distributions:

$$\text{Linear charge density: } \lambda = \frac{\Delta Q}{\Delta x}$$

$$\text{Area charge density: } \sigma_A = \frac{\Delta Q}{\Delta A}$$

$$\text{Surface charge density: } \sigma_{surf} = \frac{\Delta Q_{surf}}{\Delta A}$$

$$\text{Volume charge density: } \rho = \frac{\Delta Q}{\Delta V}$$

Electric flux and Gauss' law

Flux: $\Delta\Phi = E \Delta A_{\perp} = \vec{E} \cdot \hat{n} \Delta A$

Gauss law: Outgoing Flux from S, $\Phi_S = \frac{Q_{enclosed}}{\epsilon_0}$

Steps: to obtain electric field

-Inspect \vec{E} pattern and construct S

-Find $\Phi_S = \oint_{surface} \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$, solve for \vec{E}

Spherical: $\Phi_S = 4\pi r^2 E$

Cylindrical: $\Phi_S = 2\pi r l E$

Pill box: $\Phi_S = E \Delta A$, 1 side; $= 2 E \Delta A$, 2 sides

Conductor: $\vec{E}_{in} = 0$, $E_{surf}^{\parallel} = 0$, $E_{surf}^{\perp} = \frac{\sigma_{surf}}{\epsilon_0}$

Potential

Potential energy: $\Delta U = q \Delta V$ 1 eV $\approx 1.6 \times 10^{-19}$ J

Positive charge moves from high V to low V

Point charge: $V = \frac{kQ}{r}$ $V = V_1 + V_2 = \dots$

Energy of a charge-pair: $U = \frac{kq_1q_2}{r_{12}}$

Potential difference: $|\Delta V| = |E \Delta s|$,

$$\Delta V = -\vec{E} \cdot \Delta \vec{s}, \quad V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

$$E = -\frac{dV}{dr}, \quad E_x = -\frac{\Delta V}{\Delta x} \Big|_{fix y,z} = -\frac{\partial V}{\partial x}, \text{ etc.}$$

Capacitances $Q = CV$

Series: $V = \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \dots$, $Q = Q_i$

Parallel: $Q = C_{eq} V = C_1 V + C_2 V + \dots$, $V = V_i$

Parallel plate-capacitor: $C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{\epsilon_0 A}{d}$

Energy: $U = \int_0^Q V dq = \frac{1}{2} \frac{Q^2}{C}$, $u = \frac{1}{2} \epsilon_0 E^2$

Dielectrics: $C = \kappa C_0$, $U_{\kappa} = \frac{1}{2\kappa} \frac{Q^2}{C_0}$, $u_{\kappa} = \frac{1}{2} \epsilon_0 \kappa E_{\kappa}^2$

Spherical capacitor: $V = \frac{Q}{4\pi\epsilon_0 r_1} - \frac{Q}{4\pi\epsilon_0 r_2}$

Potential energy: $U = -\vec{p} \cdot \vec{E}$

Current and resistance

Current: $I = \frac{dQ}{dt} = nqv_d A$

Ohm's law: $V = IR$, $E = \rho J$

$$E = \frac{V}{l}, \quad J = \frac{I}{A}, \quad R = \frac{\rho l}{A}$$

Power: $P = IV = \frac{V^2}{R} = I^2 R$

Thermal coefficient of ρ : $\alpha = \frac{\Delta\rho}{\rho_0 \Delta T}$

Motion of free electrons in an ideal conductor:

$$a\tau = v_d \rightarrow \frac{qE}{m} \tau = \frac{J}{nq} \rightarrow \rho = \frac{m}{nq^2 \tau}$$

Direct current circuits $V = IR$

Series: $V = I R_{eq} = I R_1 + I R_2 + I R_3 + \dots$, $I = I_i$

Parallel: $I = \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \dots$, $V = V_i$

Steps: in application of Kirchhoff's Rules

-Label currents: i_1, i_2, i_3, \dots

-Node equations: $\sum i_{in} = \sum i_{out}$

-Loop equations: " $\sum (\pm \mathcal{E}) + \sum (\mp iR) = 0$ "

-Natural: "+" for loop-arrow entering - terminal
"- " for loop-arrow-parallel to current flow

RC circuit: if $\frac{dy}{dt} + \frac{1}{RC} y = 0$, $y = y_0 \exp(-\frac{t}{RC})$

Charging: $\mathcal{E} - V_c - Ri = 0$, $\frac{1}{C} \frac{dq}{dt} + R \frac{di}{dt} = \frac{i}{C} + R \frac{di}{dt} = 0$

Discharge: $0 = V_c - Ri = \frac{q}{C} + R \frac{dq}{dt}$, $\frac{i}{C} + R \frac{di}{dt} = 0$

Magnetic field and magnetic force

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$$

Wire: $B = \frac{\mu_0 i}{2\pi r}$ Axis of loop: $B = \frac{\mu_0 a^2 i}{2(a^2 + x^2)^{3/2}}$

Magnetic force: $\vec{F}_M = i \vec{l} \times \vec{B} \rightarrow q \vec{v} \times \vec{B}$

Loop-magnet ID: $\vec{\tau} = i \vec{A} \times \vec{B}$, $\vec{\mu} = i A \hat{n}$

Circular motion: $F = \frac{mv^2}{r} = qvB$, $T = \frac{1}{f} = \frac{2\pi r}{v}$

Lorentz force: $\vec{F} = q \vec{E} + q \vec{v} \times \vec{B}$

Hall effect: $V_H = \frac{F_M d}{q}$, $U = -\vec{\mu} \cdot \vec{B}$

Sources of \vec{B} and magnetism of matter

Biot-Savart Law: $\Delta \vec{B} = \frac{\mu_0}{4\pi} \frac{i \Delta \vec{l} \times \hat{r}}{r^2}$, $B = \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \hat{r}}{r^2}$

$$\Delta B = \frac{\mu_0}{4\pi} \frac{i \Delta y}{r^2} \sin \theta, \quad \sin \theta = \frac{a}{r}, \quad \Delta y = \frac{r^2 \Delta \theta}{a}$$

Ampere's law: $\mathcal{M} = \oint \vec{B}_L \cdot d\vec{s} = \mu_0 I_{encircled}$

Steps: to obtain magnetic field

-Inspect \vec{B} pattern and construct loop L

-Find \mathcal{M} and I_{encl} , and solve for \vec{B} .

Displ. current: $I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \frac{d(EA)}{dt} = \frac{dQ_A}{dt}$

Magnetism in atom:

Orbital motion: $\mu = i A = \frac{e}{2m} L$

$L = mvr = n\hbar$, $\hbar = \frac{h}{2\pi} = 1.06 \times 10^{-34} \text{ J s}$

$\mu_{orbit} = n\mu_B$, $\mu_B = \frac{e\hbar}{2m} = 9.27 \times 10^{-24} \text{ J/T}$

Spin: $S = \frac{\hbar}{2}$, $\mu_{spin} = \mu_B$

Magnetism in matter:

$B = B_0 + B_M = (1 + \chi) B_0 = (1 + \chi) \mu_0 \frac{B_0}{\mu_0} = \kappa_m H$

Ferromagnetic: $\chi \gg 1$

Paramagnetic: $0 < \chi \ll 1$, $M = \frac{C}{T} B$

Diamagnetic: $-1 \ll \chi < 0$

Faraday's law

$$\mathcal{E} = -N \frac{d\phi_B}{dt}, \quad \phi_B = \int \vec{B} \cdot d\vec{A},$$
$$\mathcal{E} = \int \vec{E} \cdot d\vec{s}, \quad \vec{E} = \frac{\vec{F}_M}{q}$$

Lenz law: Induced \vec{B} opposes change of Φ_B

$$\frac{d\phi_B}{dt} = \frac{d(B A_{\perp})}{dt} = \frac{dB}{dt} A_{\perp} + B \frac{dA_{\perp}}{dt}$$

Moving rods: $\frac{dA}{dt} = \ell v$, $\frac{dA}{dt} = \frac{d}{dt} \left(\frac{1}{2} R \cdot R \theta \right)$

Rotating loop: $\frac{dA_{\perp}}{dt} = \frac{d}{dt} (A \cos \omega t)$

Cutting B lines \rightarrow change $\phi_B \rightarrow E_{ind} \rightarrow \mathcal{E}_{ind}$

Maxwell equations:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}, \quad \oint \vec{B} \cdot d\vec{A} = 0,$$
$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\phi_B}{dt}, \quad \oint \vec{B} \cdot d\vec{s} = \mu_0 [I + \epsilon_0 \frac{d\phi_E}{dt}]$$

Inductance

Mutual: $\mathcal{E}_2 = -M_{21} \frac{di_1}{dt}$, $M_{21} = M_{12} = \frac{N_2 \phi_{21}}{i_1}$

Self: $\mathcal{E} = -L \frac{di}{dt}$, $L = \frac{N\phi}{i}$, $V_L = L \frac{di}{dt}$

Long solenoid: $L = \frac{NB^2 A}{i}$, $B = \mu_0 n i$

Energies: $U_L = \frac{1}{2} L i^2$, $u_B = \frac{1}{2\mu_0} B^2$

$$U_C = \frac{1}{2C} q^2, \quad u_E = \frac{1}{2} \epsilon_0 E^2$$

LC: $V_L + V_C = 0 \Rightarrow L \frac{di}{dt} = -\frac{q}{C}$ $q = q_0 \cos(\omega t + \delta)$,

$$\omega = \sqrt{\frac{1}{LC}}, \quad U_C + U_L = U_C \max = U_L \max = U_0$$

Decay Equations: $\frac{dy}{dt} = -ay$, $y = y_0 \exp(-at)$

LR: $\mathcal{E} = V_L + R i$, $\frac{dV_L}{dt} + \frac{RV_L}{L} = 0$,

$$V_L = \mathcal{E} \exp\left(-\frac{Rt}{L}\right), \quad i = \frac{\mathcal{E}}{R} \left[1 - \exp\left(-\frac{Rt}{L}\right)\right]$$

LR C:

$$Q \approx Q_0 e^{-\frac{R}{2L}t} \cos \omega_d t, \quad \omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

Underdamped, critically damped & overdamped

A C Circuits

Mean value: $\bar{f}(t) = \frac{1}{T} \int_0^T f(t) dt$

$$[\sin \omega t]_{rms} = \frac{1}{\sqrt{2}} \int_0^T \sin^2 \omega t dt = \frac{1}{\sqrt{2}} \int_0^T \frac{1 - \cos 2\omega t}{2} dt = \frac{1}{\sqrt{2}}$$

Electromagnetic waves

Properties of em waves:

$$E = E_m \cos(kz - \omega t), \quad B = \frac{E}{c}$$
$$v = \frac{dz}{dt} = \frac{\omega}{k} = \lambda f = \frac{\lambda}{T}, \quad n = \frac{c}{v}$$
$$\text{speed of light: } c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8 \text{ m/s}$$

$\vec{B} \perp \vec{E}$, propagating along: $\vec{E} \times \vec{B}$

$$u = u_E + u_B, \quad u_E = u_B$$

Poynting vector: $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$, $\vec{S} = \vec{I} = \frac{E_{rms} B_{rms}}{\mu_0}$

$$\text{Intensity: } I = \frac{P}{A} = \frac{\Delta U}{A \Delta z} \frac{dz}{dt} = u c$$

Energy conservation: $\int \vec{S} \cdot d\vec{A} = \frac{dU}{dt} + P_R$

Complete absorption: Momentum $p = \frac{U}{c}$

Pressure: $P = \frac{F}{A} = \frac{\Delta p}{\Delta t} \frac{1}{A} = \frac{\Delta U}{c \Delta t} \frac{1}{A} = u = \frac{S}{c}$

Complete reflection: $P = \frac{2U}{c}$, $P = \frac{2S}{c}$

General case: $P = \left[(1 - \eta) \frac{S}{c} + \eta \frac{2S}{c} \right] \cos \theta$

Reflection and Refraction

Index of refraction: $\frac{n_1}{n_2} = \frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1}$

Snell's law: $n_1 \sin \theta_1 = n_2 \sin \theta_2$

Critical angle: $n_2 > n_1$, $n_2 \sin \theta_c = n_1 \sin 90^\circ$

Total reflection: $\theta > \theta_c$

Mirrors and lenses

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Ray tracing rules:

Mirror: At symm pt S , reflected symmetrically through center of sphere, undeflected. Parallel to axis, converges toward F (or diverges away from F), $f = \frac{R}{2}$.

Lens: Through center of lens, undeflected. Parallel to axis, converges toward F (or diverges away from F)

Image: $q > 0$ (real), $q < 0$ (virtual)

Focal point F : at $p = \infty$, $q = f$

$f = \pm |f|$, "+" convergent, "-" divergent

Magnification: $M = \frac{h'}{h} = -\frac{q}{p}$

Refraction at spherical surface: $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$

R is coordinate of center with origin at S , with

S the symmetry point of surface on the axis

Lens maker: $\frac{1}{f} = \left(\frac{n_2}{n_1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

Two media: $M = \frac{h'}{h} = -\frac{q}{p} \frac{n_1}{n_2}$

Huygen's principles:

Points in wave front are sources of next wavelets

Forward tangent surface is next wave front

Interference

Maxima $\phi = 0, 2\pi, 4\pi, \dots$; Minima $\phi = \pi, 3\pi, 5\pi, \dots$

Double slits: $I_{average} = I_0 \cos^2 \left(\frac{\phi}{2} \right)$, $\phi = k \Delta$.

$\sin \theta = \frac{\Delta}{\lambda}$, $\tan \theta = \frac{y}{L}$, for small θ , $\theta \approx \sin \theta \approx \tan \theta$

Phasor diagram: $\vec{A} = \vec{A}_1 + \vec{A}_2 + \vec{A}_3 + \dots$

$$A_x = A_{1x} + A_{2x} + A_{3x} + \dots, \quad A_y = A_{1y} + A_{2y} + \dots$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

First minimum for N slits: $\phi = \frac{2\pi}{N}$

Thin film: $\phi = k \Delta + |\phi_{1reflected} - \phi_{2reflected}|$, $\Delta = 2t$

$\phi_{reflected} = \pi$ (denser medium); $=0$ (lighter medium)

Diffraction

Single slit: $I = I_0 \left[\frac{\sin \frac{\beta}{2}}{\frac{\beta}{2}} \right]^2$, $\beta = k \Delta$, $\Delta = a \sin \theta$

Resolution criterion: $\theta_{criterion} = 1.22 \frac{\lambda}{D}$

Grating: Principle maxima $\Delta = m \lambda$

Polarization

Brewster ($n_1 < n_2$): $n_1 \sin \theta_{br} = n_2 \sin \left(\frac{\pi}{2} - \theta_{br} \right)$

Polarizer: $E_{transmit} = E_0 \cos \theta$, $I = I_0 \cos^2 \theta$

Unpolarized light: $\frac{\Delta I}{\Delta \theta} = \frac{I_0}{2\pi}$

Transmitted Intensity: $\Delta I' = \Delta I \cos^2 \theta$

$$I' = \frac{I_0}{2\pi} \int_0^{2\pi} \cos^2 \theta d\theta = \frac{I_0}{2}$$