Sheaths and Langmuir Probes

References:

Chapter by F. F. Chen in Plasma Diagnostic Techniques, editors R. H. Huddlestone and S. L. Leonard, Academic Press (1965).

Principles of plasma diagnostics. I. H. Hutchinson Cambridge University Press (1987).

Debye shielding

Poisson's equation relating the electric potential to the density of ions (Z=1), electrons, and a test charge is

$$\nabla^2\varphi = -\frac{\rho}{\varepsilon_o} = \frac{e(n_e - n_i) - q_t\delta(r)}{\varepsilon_o}$$

In electrostatic equilibrium -- time scale of fast thermal motion, but not necessarily thermal equilibration between species,

$$n_e = n_o e^{e\varphi/kT_e} \quad \text{and} \quad n_i = n_o e^{-e\varphi/kT_i}$$

Note that each density becomes n_o at large distances from the test charge -- quasi-neutrality.

Assume $e\phi/T \ll 1$ and expand the exponent.

In spherical coordinates with spherical symmetry,

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\varphi}{dr}\right) = \frac{n_o e^2}{\varepsilon_o}\left(\frac{1}{T_e}+\frac{1}{T_i}\right)\varphi - \frac{q_t \delta(r)}{\varepsilon_o}$$

Define the Debye length $\lambda_{e,i} \equiv \left(\frac{\varepsilon_o T_{e,i}}{n_o e^2}\right)^{1/2}$ $\lambda_D^{-2} = \lambda_e^{-2} + \lambda_i^{-2}$

The equation for the potential becomes

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\varphi}{dr}\right) = \frac{\varphi}{\lambda_D^2} - \frac{q_t \delta(r)}{\varepsilon_o}$$

with the solution $\varphi = \dfrac{q_t}{4\pi\varepsilon_o r} e^{-r/\lambda_D}$

Debye shielding

Quite generally perturbing charge or boundary effects will penetrate into the plasma a distance of order the Debye length.

In a plasma, strict <u>charge neutrality</u> will obtain over <u>distances greater than the Debye length</u>.

For most laboratory plasmas, the Debye length is short.

1 eV plasma with density $10^{17}m^{-3}$ has a Debye length of 20 microns << typical probe dimensions

At a boundary, charge neutrality may not obtain over distances of a Debye length, the **sheath region**.

A sheath will respond to changes on the time scale of the electron plasma frequency.

Langmuir probes

From gas kinetic theory, the number of particles of a gas species crossing a unit area per unit time is

$$\Gamma \approx \frac{1}{4} n\bar{v}$$

where $\bar{v}$ is the rms thermal speed (3-D).

The current to a probe of collecting area A *which does not perturb the plasma* is dominated by electron current because of the higher velocity of electrons.

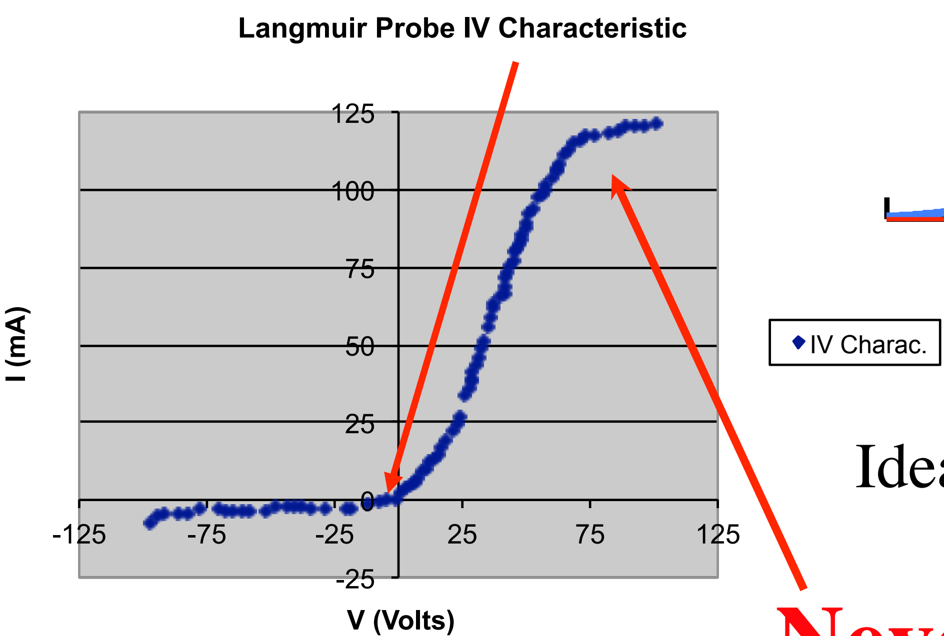
$$I = -eA\left(\frac{1}{4} n_i \bar{v}_i - \frac{1}{4} n_e \bar{v}_e\right) \approx \frac{1}{4} e n_e \bar{v}_e$$

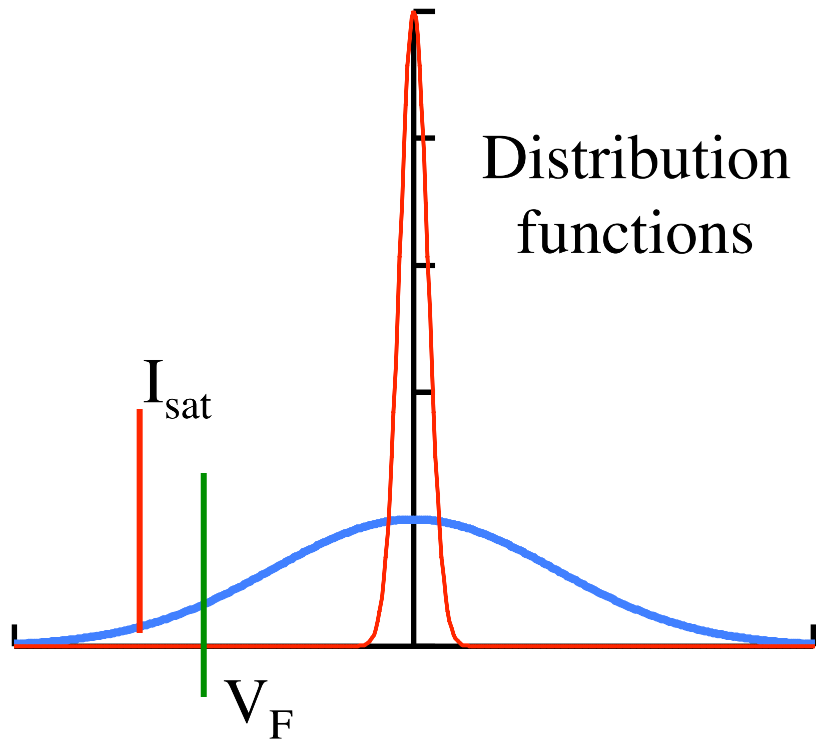
This is not observable in any real plasma.

(It would greatly perturb the plasma, generally violating charge neutrality)

Langmuir Probe, I-V characteristic

- Floating potential-no net current to the probe. This is the potential that an insulator would reach.

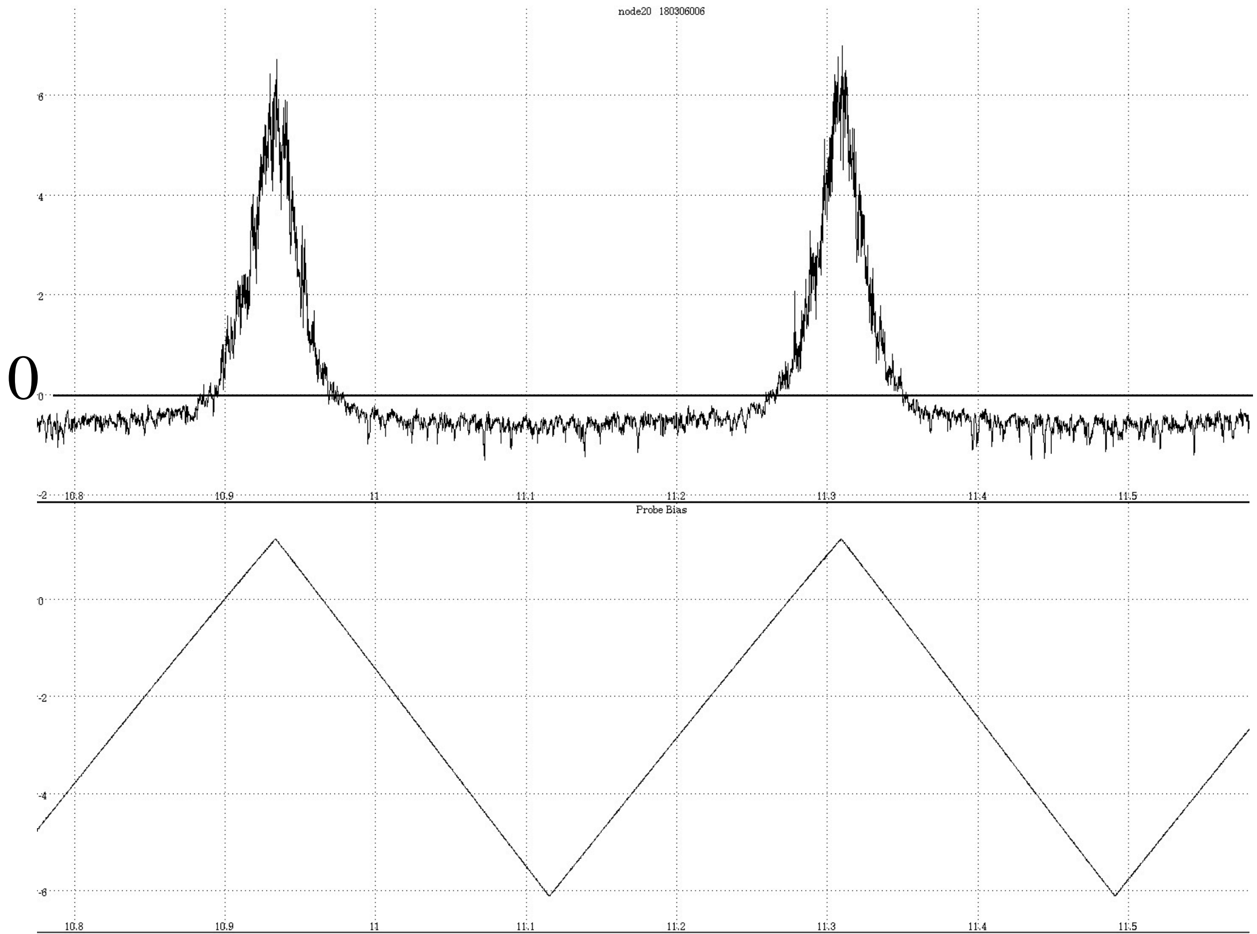




Ideal probe curve

Never seen

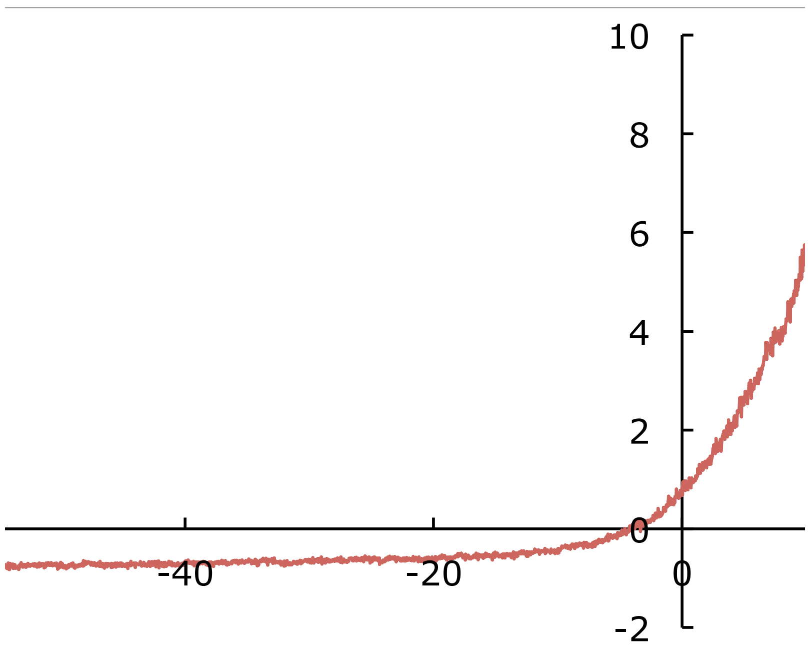
Measuring the I-V Curve



Measure current as applied voltage is swept – here a sawtooth from -60V to 10V with a 0.5 sec period.

What does the “noisy” current signal indicate?

Measuring the I-V Curve



Recorded data in volts. Y axis is $I_{probe} * R_{meter}$, here 5 kΩ

Averaging over multiple sweeps (here ~50) reduces the effect of fluctuations and gives an I-V curve suitable for analysis.

"True" Plasma Potential

- At plasma potential, the probe would be at the same potential as plasma. There are no electric fields from probe bias and charged particles move to probe because of their unperturbed thermal velocities.
- <u>Not directly measurable with probes</u>.
- Relation to floating potential

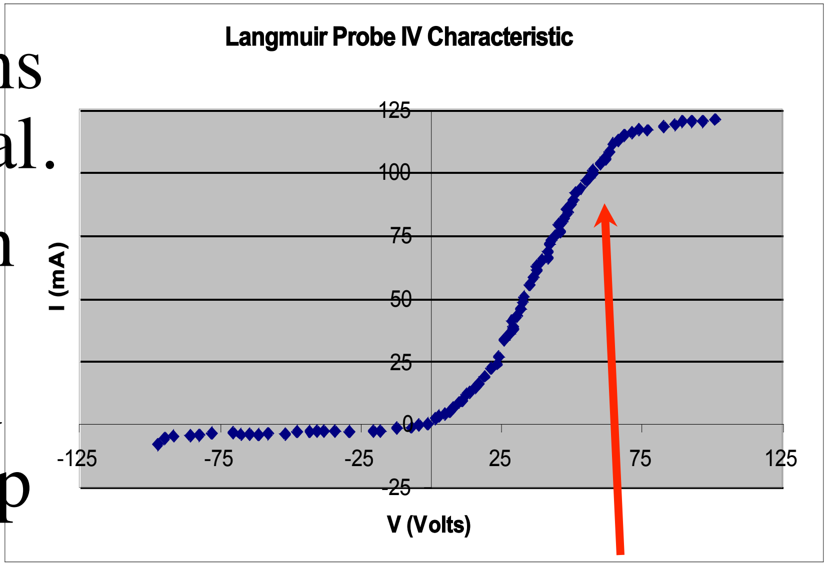
$$V_p = V_f + \frac{T_e}{2e}\left[\ln(2\pi m_e / m_i) - 1\right]$$

$$\approx V_f + 3.6 T_e(eV) \qquad \text{for hydrogen}$$

Difference is that required to repel all but a small fraction of the electrons from the tail to balance the ion current.

Ion saturation current

- Only ions are collected. Electrons are repelled by the probe potential.
- Ion saturation current depends on electron temperature-not ion temperature. The ions are pulled into the sheath by a potential drop of order kT_e (Bohm sheath).
- Ion current is not completely flat with respect to the bias voltage.
- J. E. Allen,Proc. Phys. Soc. (London) B70,297, (1957).
- Factor 0.61 (+200%, -60%) depending on configuration.



Never seen

$$I_s = \frac{1}{2} n_o A \left(\frac{kT_e}{M} \right)^{1/2}$$

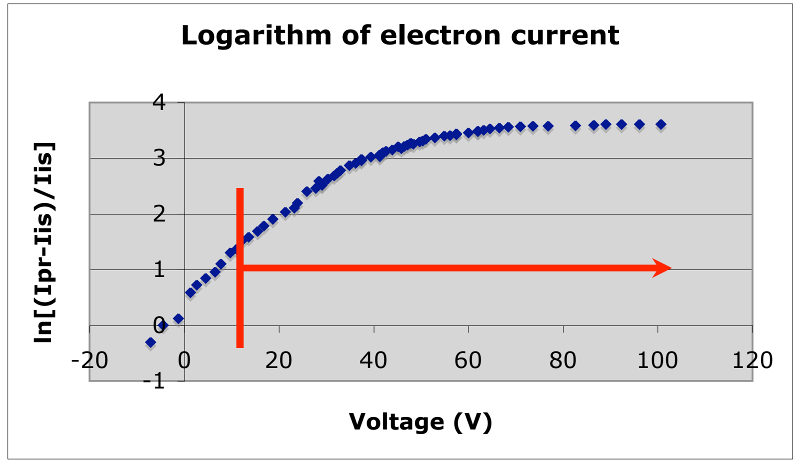
$$I_s = 0.61 n_o A \left(\frac{kT_e}{M} \right)^{1/2}$$

Transition region

- Assume electron density is in thermal equilibrium and follows Boltzmann Law. $$n = n_o e^{-eV/kT_e}$$

- Only the electron current is changed by potential; ion current always saturated.

- $I_e = Aj_e = An_o\left(\frac{kT_e}{2\pi m}\right)^{1/2} e^{-eV/kT_e}$

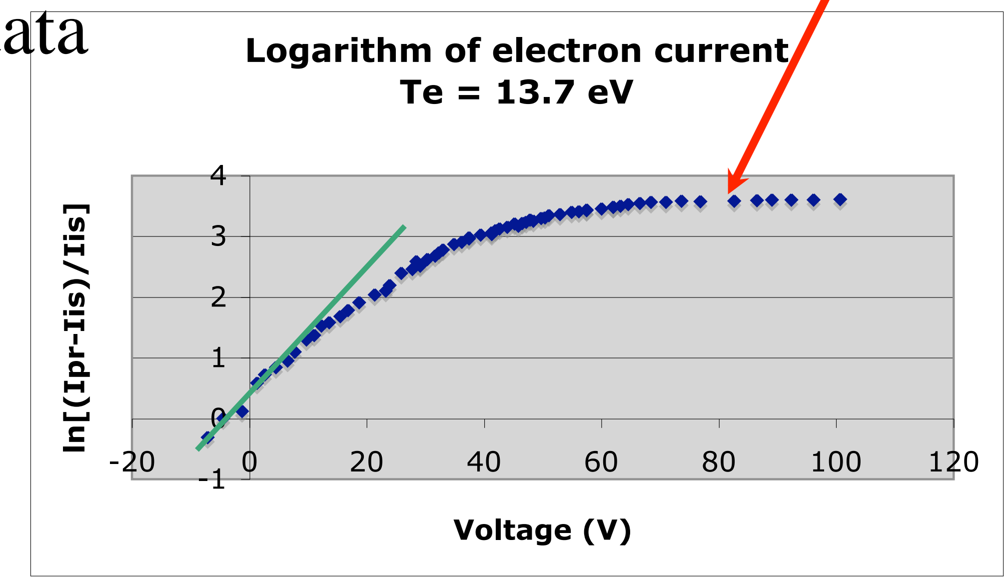


Assumptions invalid

Temperature measurement

- Measure temperature by plotting ln I_e versus bias potential.
- Slope is e/kT_e
- T_e = 13.7 eV for data shown

Never seen

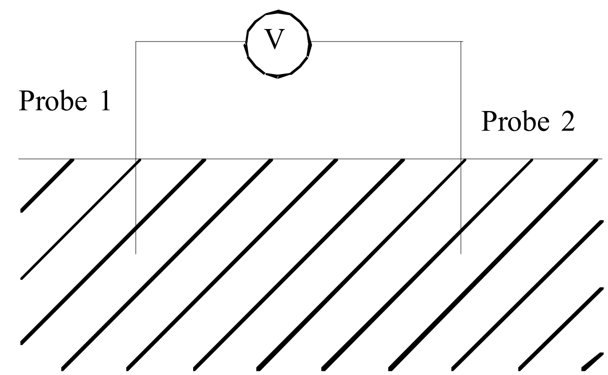


Electron saturation

- Do not use -- generally not observable
- Theory and experiment are sensitive to local conditions.
- In magnetized plasma, probes biased to collect electrons usually become part of a double probe system with current limited by ion saturation current of other “probe” at the other end of the magnetic field line.

Floating double probe

- Net current drawn from plasma must be zero. The entire system floats with plasma and follows changes in floating potential.
- Assume probes have equal area, and plasma is homogeneous.
- Probes are always negative enough to be collecting ion saturation current -- system floating potential adjusts to make net current to the probe apair zero.

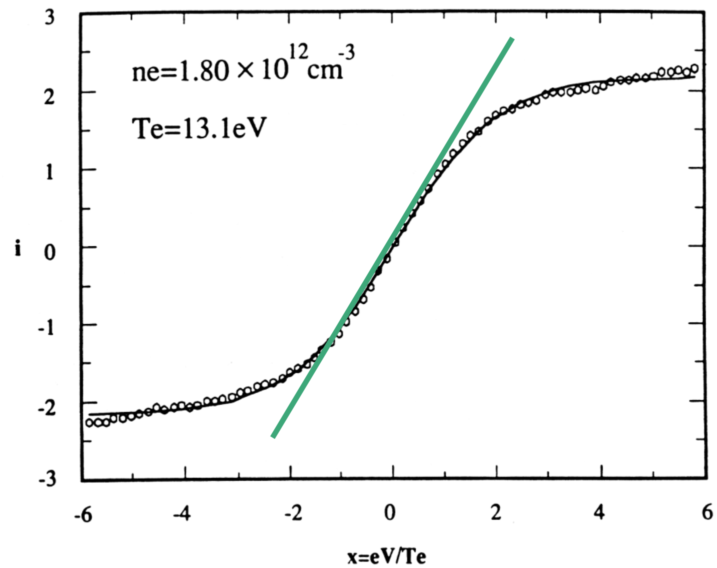


Floating double probe II

- I-V characteristic

$$I = I_o \tanh\left(eV/2kT_e\right)$$

I_o is ion saturation current



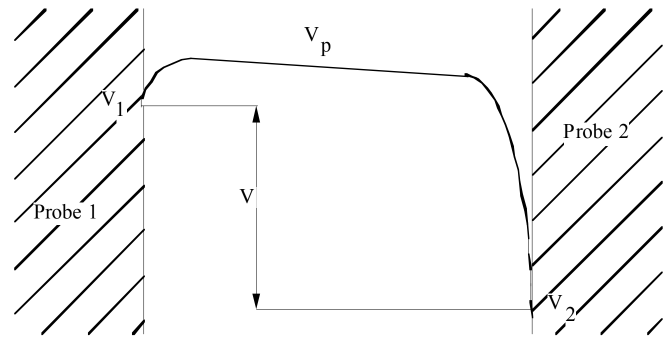
- To find temperature

$$\left.\frac{dI}{dV}\right|_0 = \frac{e}{2kT_e} I_o$$

Double probe III

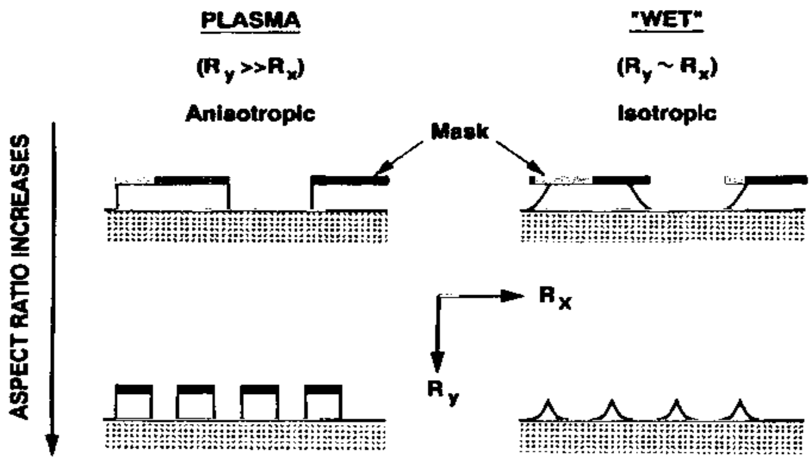
- Advantages of double probe
  - Less perturbation
  - Easier to analyze
  - No large bias voltages needed
- Disadvantages

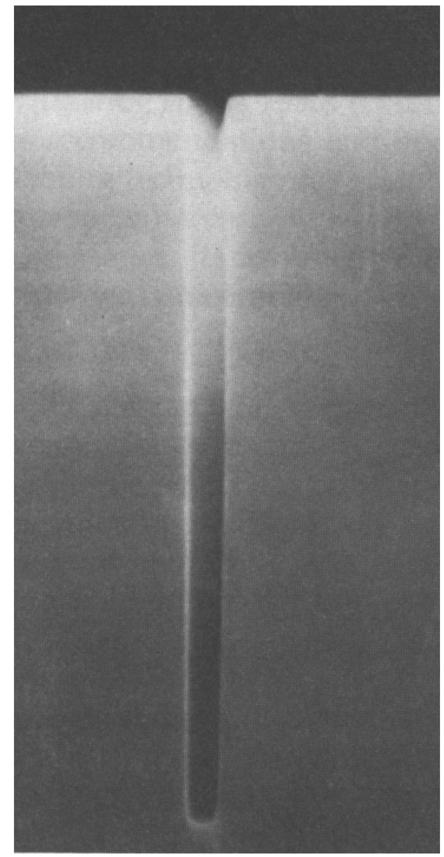
  -- Difficult to "float" – isolate all circuitry (both DC and AC)
  - Samples tail of velocity distribution
  - ALL probes have this "feature"



Schematic of potential distribution between the probes of a double floating double-probe

Plasma processing





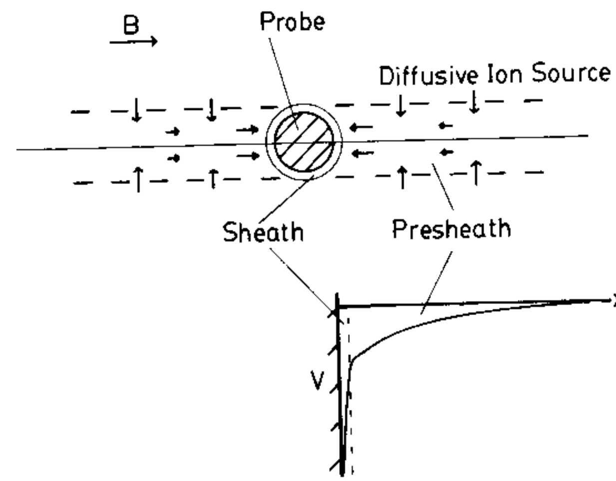
Illustrates sheath at surface that accelerates ions ⊥ to surface with energies several times T_e.

Mach probes

Objective is to measure flow of plasma [ion velocity].

- Collect ion saturation current on two or more probes that collect current only from one direction. If flow is present then one expects to measure larger ion current on the surfaces that face upstream than from probes that face downstream.
- Theories for the interpretation of I_{right}/I_{left} vary widely depending on probe configuration and plasma parameters. Inferred Mach numbers should not be considered quantitatively accurate.

Probes in a magnetized plasma



- Ions orbit the magnetic field with a Larmor radius

$$\rho = \frac{mv}{eB}$$

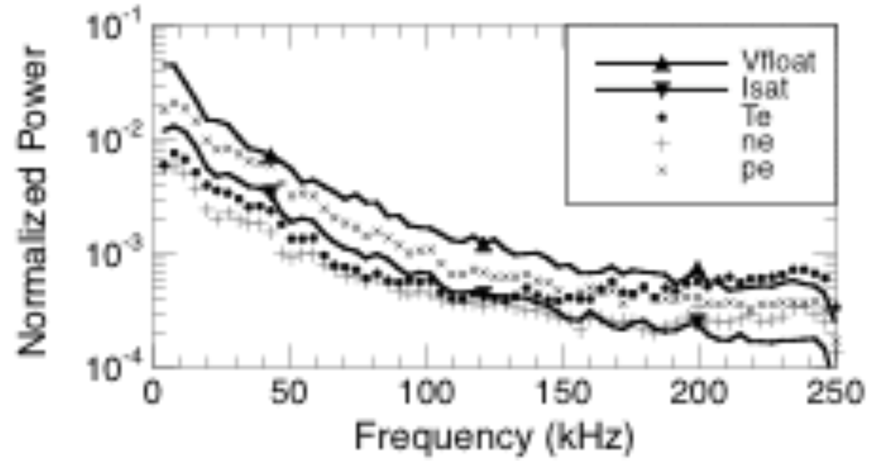
  but v may be v_{ith} or c_s (ion at T_e).
- Presheath is a long flux tube along B. The particles may move into flux tube from perpendicular electric fields.
- Effective collection area changes -- multiplicative factor between Isat and n may be different from simple geometry.

Probe Measurements

- Probe I-V characteristic gives good measurement of n, T_e, and V_f. Fluctuations $\Delta V_f(t)$ and $\Delta I_{sat}(t) \sim \Delta n(t)$ also well measured.
- A plethora of additional probe techniques have been devised and promoted with various claimed advantages and evidence, but <u>none</u> is well-established and <u>none</u> should be considered reliable.

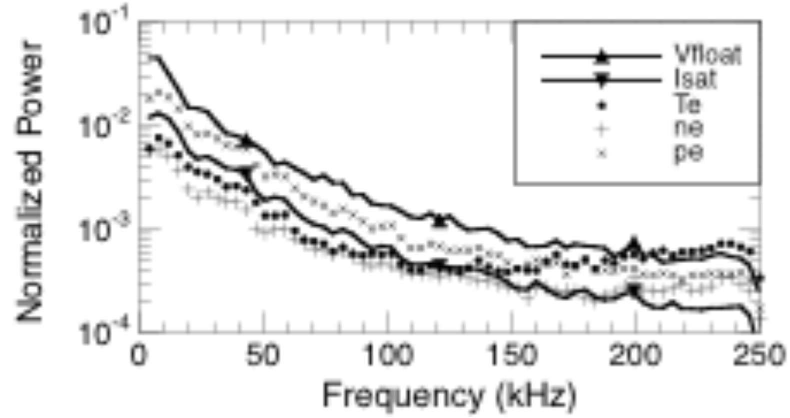
Probes to measure fluctuations

- Usual measurements are $V_f(t)$ and $I_{sat}(t)$.
- Common assumption of constant temperature: Interpret $I_{sat}(t)$ as n(t) -- roughly true -- and $V_f(t)$ as $V_{plasma}(t)$ -- an error.
- Motivation is direct measurement of particle transport.



Transport and fluctuations

- Objectives
  - Measure $n_e(t)$, $T_e(t)$, $V_p(t)$ at a single location in plasma
- Particle flux



$\Gamma = \langle n v_r \rangle$ = radial particle flux (Exact)
Take $v_r = E_\theta \times B_T/B^2$ (Correct in typical plasmas below Ω_{ci})

Measure E_θ using two probes separated in θ, but with universal, incorrect use of V_f for V_p -- renders all published inferences suspect.

We lack probe techniques for $V_p(t)$ and $T_e(t)$

Principles of Scientific Communication

I. The purpose of publication is <u>not</u> to advance science, but to advance the authors.

II. Publication is more a social than a scientific activity.

III. Almost all publications are insignificant -- irrelevant, trivial, repetitious, inconsequential, or insufficient to support conclusions.

IV. Refereeing serves only to prevent publication of absurdly wrong or broadly unpopular results.

V. A significant fraction of published conclusions are quite wrong -- often inconsistent with the data shown.

These observations are universal, applying to all journals.

This concludes the discussion of Langmuir Probe diagnostics.

The remaining lectures will treat various aspects of the interaction of plasmas with electromagnetic radiation, which encompass a broad and powerful spectrum of measurement techniques.