Scattering of Electromagnetic Radiation

References:

Plasma Diagnostics: Chapter by Kunze

Methods of experimental physics, **9a**, chapter by Alan Desilva and George Goldenbaum, Edited by Loveberg and Griem.

Plasma Scattering of electromagnetic radiation, by John Sheffield, Academic Press (1975)

Principles of plasma diagnostics,

by I. H. Hutchinson. Cambridge University Press (1987)

Preface

Scattering of EM radiation by a medium "is not possible." Insofar as the system can be described by bulk parameters, e.g an ε, an EM wave is either reflected or transmitted, never scattered. Scattering results **only** from a failure of this description, either single-particle effects or $\lambda d(\ln\varepsilon)/dx >> 1$, which invalidates the usual WKB treatment of the wave equation in an inhomogenous medium.

- Why is the sky blue?
- **Incoherent Thompson scattering**
- Coherent Thompson scattering
- Scattering of radiation by a bound electron -- also called Laser Induced Florescence (LIF)

Scattering of radiation by a single free electron

Assume v/c <<1.

The electric field of a plane wave incident on an electron is

$$\vec{E}(\vec{r},t) = \hat{e}\, E_o e^{i\omega_o t}$$

The electron oscillates with an acceleration

$$\frac{d\mathbf{v}}{dt} = \frac{e}{m}\mathbf{E_o} e^{i\omega_o t}$$

The radiation emitted by an accelerated particle is given by

$$\vec{E}(\vec{r},t) = \frac{1}{4\pi\varepsilon_o}\frac{e}{c^2 r}\left(\dot{\vec{v}} \times \hat{n}\right) \times \hat{n}$$

where $\hat{n}$ is the unit vector in the direction of propagation of the scattered radiation. The scattered radiation field is then

$$\vec{E}_s(\vec{r},t) = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{mc^2}\frac{1}{r} E_o e^{-i\omega_o t}\left(\hat{e} \times \hat{n}\right) \times \hat{n}$$

Scattering of radiation by a single electron II

The frequency of the scattered radiation is the same as that of the incident wave.

Introduce the classical radius of the electron

$$r_o = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{mc^2} \approx 2.8\times 10^{-15}\,m$$

and set $\left|(\hat{e}\times\hat{n})\times\hat{n}\right| = \sin\theta$ (Angle between E and k_{scat})

The amplitude of the scattered field can be written as

$$E_S = \frac{r_o}{r}E_o\sin\theta$$

To obtain the scattering cross section, divide the power scattered into a solid angle dΩ by an incident flux S_o.

$$d\sigma = \frac{I_s r^2 d\Omega}{S_o} \qquad \text{or} \qquad \frac{d\sigma}{d\Omega} = r_o^2\sin^2\theta$$

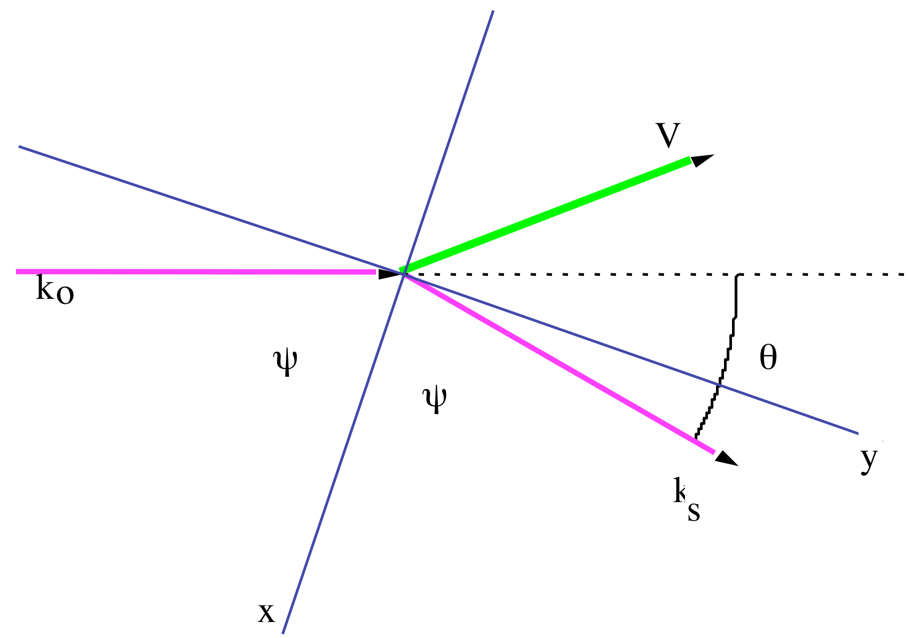
Note that scattering cross section is independent of frequency. Integrate over solid angle and get the Thomson cross section.

$$\sigma_{Th} = \frac{8\pi}{3}r_e^2 = 0.665\times 10^{-24}\,cm^2$$

Scattering from an electron with a velocity v

Note: Interactions with particles or a medium must <u>always</u> be analyzed in the rest frame of object, where all ω are the same. <u>No</u> relativistic theory of moving media.

Choose x,y axis such that x axis makes an angle $\Psi = 90° - \theta/2$ with incident and scattered waves.



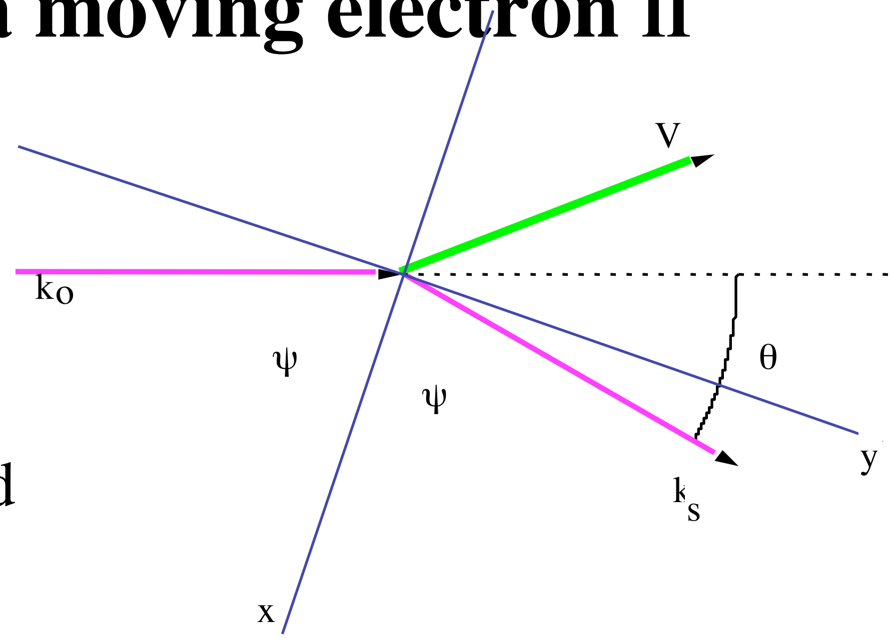
In the direction of the incoming EM wave, the velocity is given by

$$v_{k_o} = -v_x \cos\psi + v_y \sin\psi$$

This means that the electron sees an incoming wave of frequency

$$\omega' = \omega_o - \frac{\omega_o}{c}\left(-v_x \cos\psi + v_y \sin\psi\right)$$

Scattering from a moving electron II



The velocity of the electron in the direction of the scattering measurements k_s is

$$v_{k_s} = v_x \cos\psi + v_y \sin\psi$$

The shifted frequency of scattered radiation is given by

$$\omega_s = \omega' + \frac{\omega'}{c}\left(v_x \cos\psi + v_y \sin\psi\right)$$

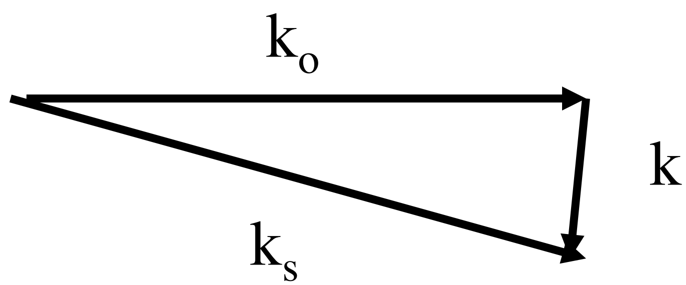
The Doppler shift of the scattered radiation compared to the initial radiation

$$\Delta\omega = \omega_s - \omega_o = \frac{\omega_o}{c} 2v_x \cos\psi = \frac{\omega_o}{c} 2v_x \sin(\theta/2)$$

For a given scattering angle and frequency, the final frequency is uniquely determined by the component v_x of the velocity.

Scattering from a moving electron III

Introduce a vector given by $\vec{k} = \vec{k}_s - \vec{k}_o$



For small Doppler shifts $k_o \sim k_s$

$$k \approx 2k_o \sin(\theta/2) = \frac{2\omega_o}{c} \sin(\theta/2)$$

The Doppler shift can be written as $\Delta\omega = \vec{k} \bullet \vec{v}$

The frequency shift is determined only by the velocity component in the direction k and not by the component in the direction of observation.

Scattering of radiation by a fully ionized plasma

For the common case of no correlations between scattering electrons:

For a Maxwellian distribution $dn_e \approx e^{-\frac{mv^2}{2T}} dv$

Integrating over all velocities gives the total density. Then a monochromatic beam will be scattered into a Gaussian scattered profile with a total half width in wavelength space.

$$\Delta\lambda_{1/2} = 4\lambda_o \sin\frac{\theta}{2}\sqrt{\frac{2T}{mc^2}\ln 2}$$

The width of the profile gives you the electron temperature. The total number of scattered photons gives the electron density.

Other Approaches

An alternate approach, but one which ignores thermal motion, uses a statistical description of the particles and computes the scattered radiation as an S(**k**, ω) -- the Fourier transform -- which depends on the transform of the density autocorrelation function.

The preceding analysis calculated scattering from uncorrelated electrons. For correlated electrons, think of the scattering in the same sense as Bragg scattering from a (moving)crystal with $\mathbf{k}_p$, ω_p.

The parameter that separates correlated from uncorrelated scattering is

$$\alpha = \frac{1}{k\lambda_{Debye}}$$

The correlated (collective) modes must have wavelengths **longer** than the Debye length.

Coherent/noncoherent scattering

For noninteracting particle spectra $\alpha \ll 1$. The scattered spectra reduces to expressions given earlier.

For $\alpha > 1$, the scattered light will be characteristic of highly correlated electron motion. The scattered signal depends on the amplitude of the density fluctuations and on their $\mathbf{k}$ and ω spectra. The electromagnetic wave interacts with electrons on a spatial scale larger than the Debye length

WHY WORRY ABOUT CORRELATED ELECTRON MOTION?
Correlated electron motion dominates transport of energy and particles. Plasmas are far from thermodynamic equilibrium, and the free energy drives large, coherent turbulent fluctuations.

A scattering system

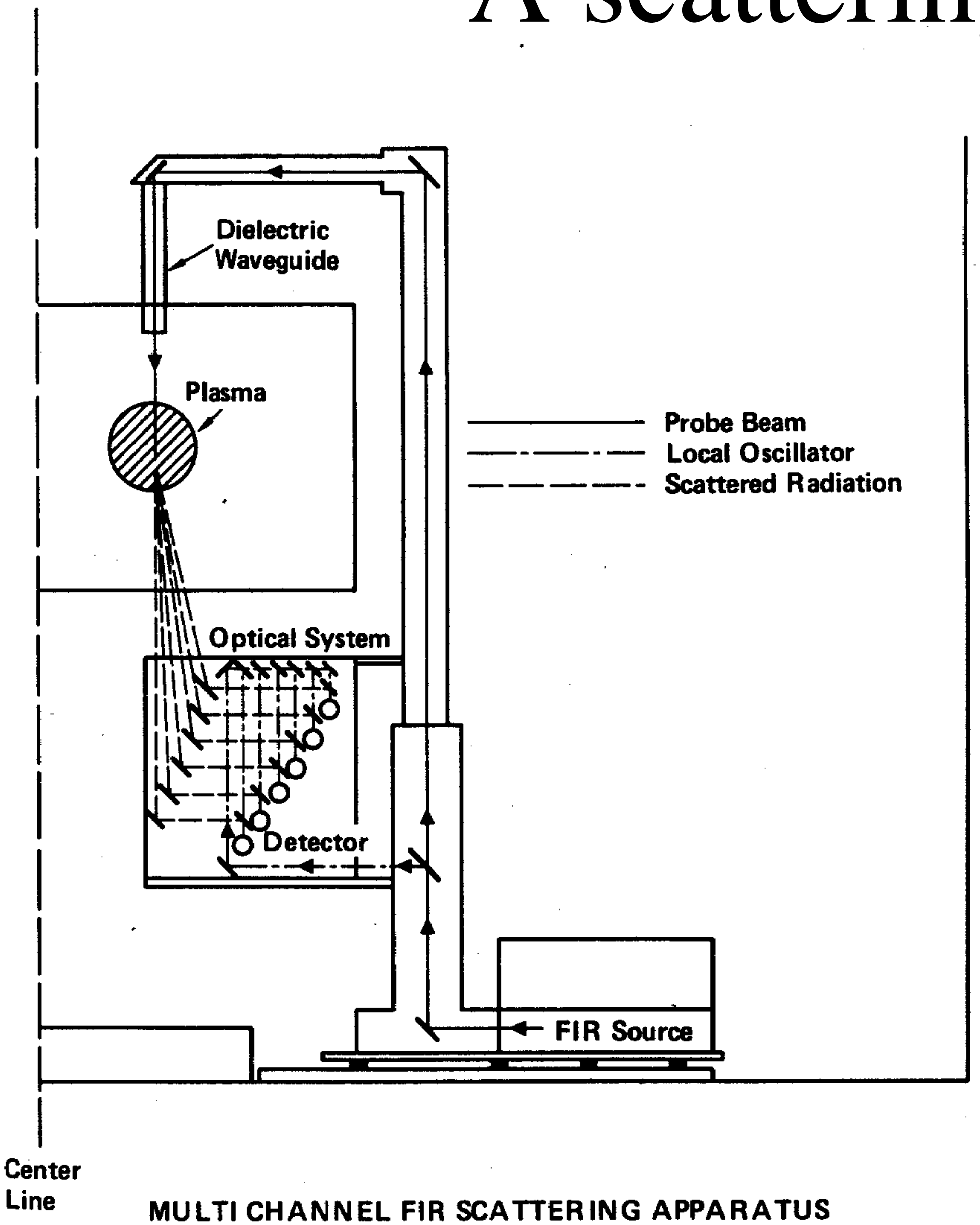


FIG. 16 Schematic of multichannel FIR scattering system on the TEXT tokamak.

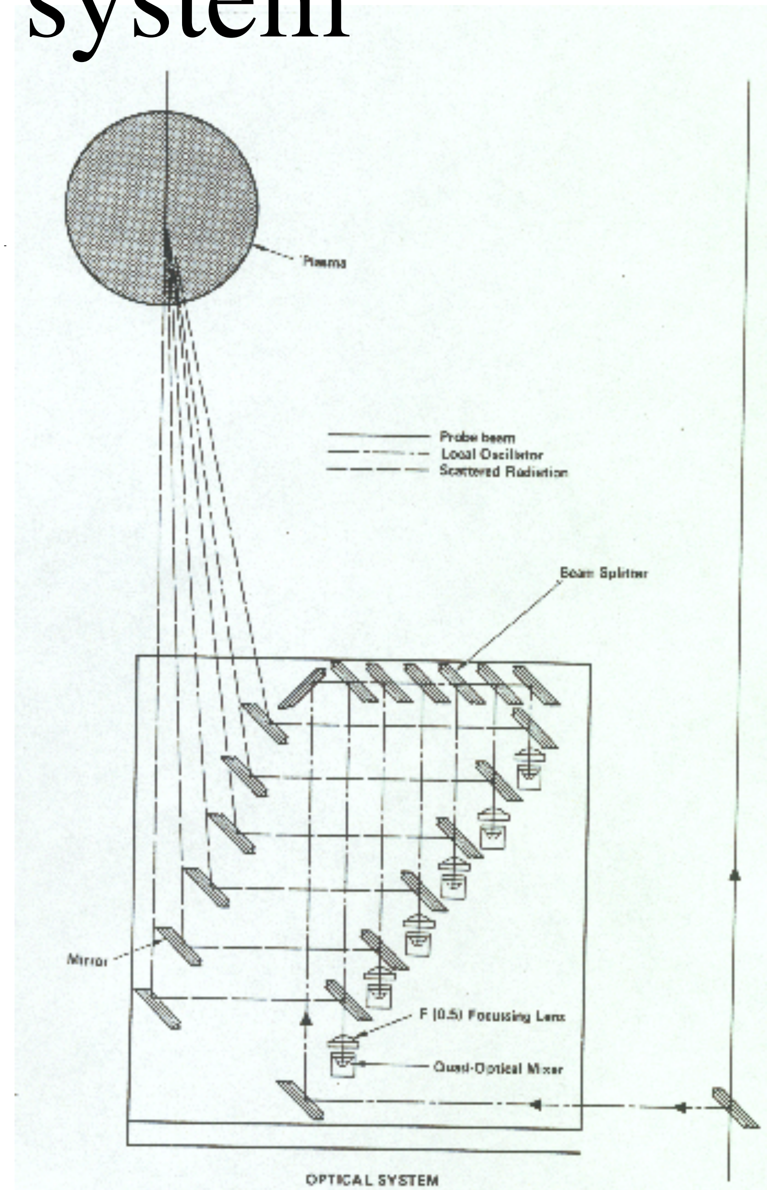


FIG. 19 Multichannel FIR scattering system homodyne receiver schematic.

Scattering

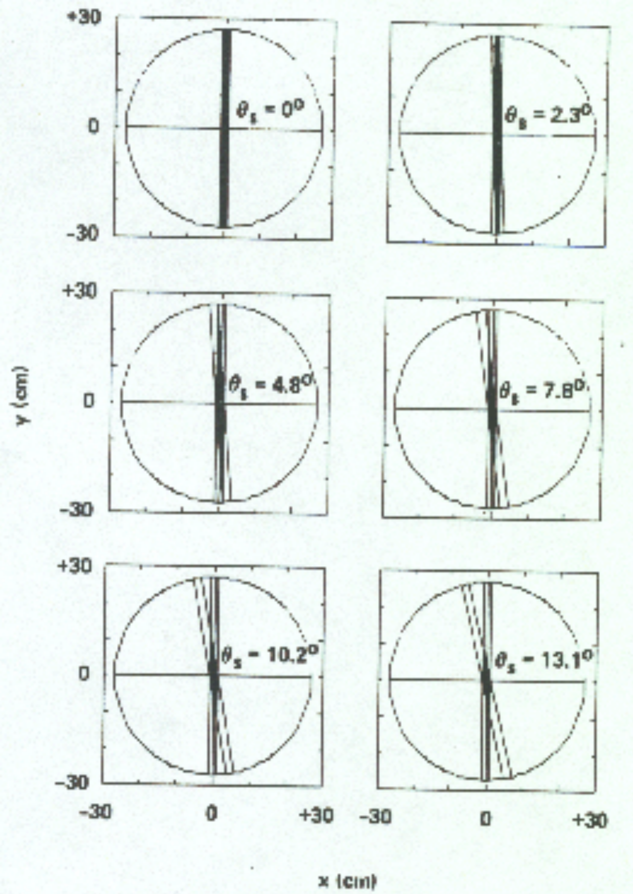


FIG. 20 Scattering volume length versus scattering angle (θ_s) for a source beam at λ_s = 1222 μm.

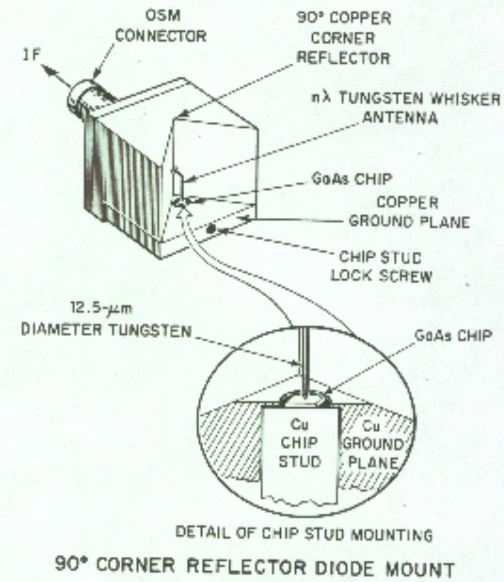


FIG. 31 Schematic of quasi-optical corner reflector Schottky diode mixer (Fetterman et al., 1978).

Design Conflict
Long wavelength to scatter
Short wavelength for resolution

Scattering of radiation by a bound electron: LIF Laser Induced Florescence

The equation of motion for a driven harmonic oscillator (bound electron) with damping is given by

$$\ddot{r} + \gamma \dot{r} + \omega_r^2 r = \frac{e}{m} \hat{e} E_o e^{-i\omega t}$$

The classical scattering cross section

$$\frac{d\sigma}{d\Omega} = r_o^2 \sin^2\theta \frac{\omega_o^4}{\left(\omega_r^2 - \omega_o^2\right)^2 + \gamma^2 \omega_o^2}$$

In the high frequency range $\omega_o >> \omega_r$, the cross section reduces to that of a free electron. The bound electron scatters like a free electron. [X-ray attenuation]

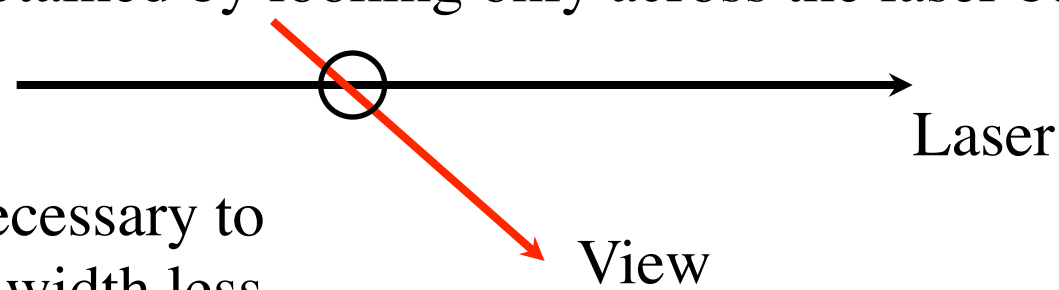
For the case $\omega_o << \omega_r$, $$\frac{d\sigma}{d\Omega} = r_o^2 \sin^2\theta \left(\frac{\omega_o}{\omega_r}\right)^4 \approx \frac{1}{\lambda^4}$$

LIF

In the case of resonance scattering or resonance fluorescence ω_r is about the same as ω_o and then

$$\frac{d\sigma}{d\Omega} = r_o^2 \sin^2\theta \frac{\omega_o^2}{\gamma^2} >> r_o^2$$

Since the strong (quantum-mechanical) resonance is extremely sharp, strong scattering occurs only when the incident (laser) radiation matches the atomic resonance in the rest frame of the atom. The scattering therefore depends on $f(v_\mathbf{k})$, and spatial resolution may be obtained by looking only across the laser beam at a single location.



Note that a laser is necessary to have an incident line width less than the Doppler width of $f(v_\mathbf{k})$.

Crossed-beam technique for spatial resolution