Plasma Spectroscopy
Inferences from Line Emission

- From line λ, can determine element, ionization state, and energy levels involved
- From line shape, can determine bulk and thermal velocity and often other properties of the ion
- To be useful as a plasma measurement, must know <u>where</u> the ion properties occur
- Optics of viewing instrument define <u>only a chord*</u>
- Atomic physics defines <u>where</u> ion can exist (temperature, density) -- which flux surfaces

* *Profiles not often suitable for inversion, e.g. hollow*

References

- H. R. Griem, Plasma Spectroscopy, McGraw Hill (1964)
- I. Hutchinson, Principles of Plasma Diagnostics, Cambridge (1987)
- H. R. Griem, Principles of Plasma Spectroscopy, Cambridge (1997)
- T. Fujimoto, Plasma Spectroscopy, Clarendon Press, Oxford (2004)

Types of equilibria

- Complete thermodynamic equilibrium
  - All processes are balanced by their inverse
  - Does not exist in laboratory plasmas
- Local thermal equilibrium
  - Distribution of states is nearly the same
  - Collisional rates >> radiative rates
- Coronal equilibrium
  - Radiative rates >> collisional rates

LTE Local Thermal Equilibrium

The distribution of states is nearly the same as if system were in complete thermodynamic equilibrium. In practice the criteria is that collisional processes are much faster than radiative processes.

Radiative rate ~ $A \ \sec^{-1}$

Collisional rate ~ $[nv\sigma] \ \sec^{-1}$

Boltzmann relation: For the population of a quantum state j relative to the ground state population N

$$N_j = \frac{N}{g_{ground}} \frac{g_j e^{-Ej/kT}}{Z(T)}$$

LTE

g_j is the statistical weight of state of energy E_j and
Z(T) is the partition function

$$Z(T) = \sum_j g_j e^{-E_j/kT}$$

The Boltzmann distribution refers to the distribution of states of a given atom or ion. If we extend the development of ideas used in finding the Boltzmann relation, we can look at the density of the next ionization stage as compared to an lower ionization stage. You count the number of electrons in some energy state.

$$e + N^{z} \text{-->} N^{z+1} + e + e$$

LTE II

This yields the Saha equation

$$\frac{N_e N^z}{N^{z-1}} = \frac{2g^z}{g^{z-1}}\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} e^{-I/kT}$$

$$= \frac{2Z^z(T)}{Z^{z-1}(T)}\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} e^{-I/kT}$$

LTE composition is given by the Saha equation + conservation of mass + macroscopic neutrality.
A crude rule of thumb is that for LTE to be valid, you must have

$$N_e > 10^{19} T^{1/2}(eV)\Delta E(eV) m^{-3}$$

where ΔE is the energy difference between the energy levels.
The most demanding criterion is the transition from the ground state to the first excited state.

Often, this is roughly $N_e > 10^{19} T^{3/2}(eV) m^{-3}$

LTE in hydrogen

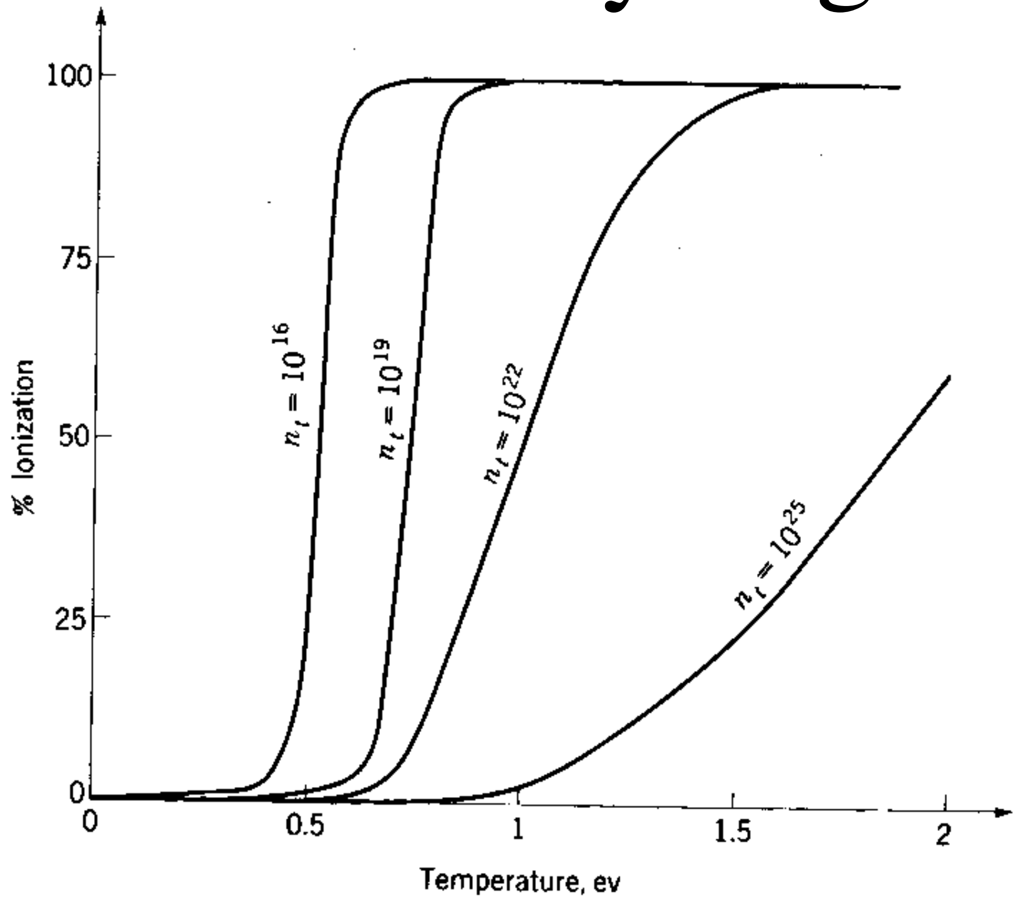

Fig. 1.5 Percent ionization n_e/n_t for hydrogen as a function of temperature for several number densities ($n_t = n_e + n_n$ in particles per cubic meter).

Z^{+n} present where $T_e << E_{ionization}(Z^{+n-1})$

Coronal equilibrium

- Each collisional process is balanced by a (fast) radiative process rather than by a collisional process.
- Assumptions
  - Electrons have a Maxwellian velocity distribution
  - All ions are in ground state -- only a few are in excited states   ($A \gg n v \sigma$)
  - Densities of excited states are determined by rate of collisional excitation from ground state and rate of radiative decay, which is faster than excitation rate

Typically, LTE for inertial confinement; coronal for magnetic confinement

Coronal equilibrium   I

The equations are written as a gain-loss equation

$$\frac{dn}{dt} = gain - loss$$

If we write an equation for Z = 2 state of ionization we can see the form the equations take in terms of ionization and recombination

$$\frac{dn^{z-2}}{dt} = n_e n^{z=1} S(T_e, z) - \alpha(T_e, z) n_e n^{z=2}$$
$$-n_e n^{z=2} S(T_e, z=2) + \alpha(T_e, z=3) n_e n^{z=3}$$

Write one equation for each stage of ionization

Here the ρ dependence in n(ρ) and T_e(ρ) has been suppressed. One must solve the full set of equations at each ρ to obtain the equilibrium at a given position, and transport has been ignored.

Coronal equilibrium II

The ionization rates are given by $S(T_e,z)$ and the recombination rates are given by $\alpha(T_e, z)$.* There will be one equation for each stage of ionization. In steady state we can ignore the time derivative and then we are left with a set of algebraic equations. Solve the algebraic equations and you have the composition of the ionization states as a function of temperature.

* *Recombination is special. Why? In practice, a transport loss rate $1/\tau$ may become an important addition.*

Coronal equilibrium oxygen

Very roughly, an ionization state will be present in coronal equilibrium when $T_e \sim E_{ion}$ for that state.
However, much more careful calculations are required to obtain useful location data.

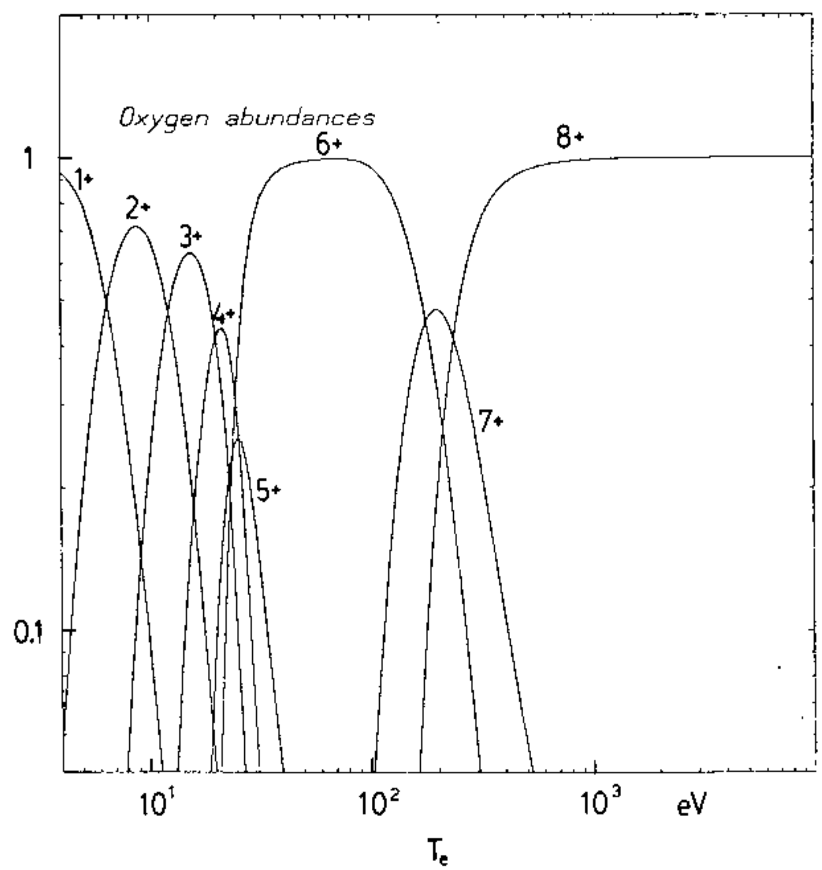

Fig. 6.10. Fractional abundances of different ionization stages of oxygen in coronal equilibrium as a function of electron temperature [after Piotrowicz and Carolan (1983)].

Coronal equilibrium III

Given the distribution of ionization states, use an excitation rate to obtain the populations of the excited states.
Once the populations of excited states are given, then you can compare with experiment or observation.

Since the total $n(Z^{+n})$ is also the density of the ground state, one needs only $n(Z^{+n})$, the excitation rates (from the ground state), and branching ratios to calculate emission rates for each line.

Applications are in high temperature, low density plasmas:
Solar corona
Fusion plasmas
Most plasma thrusters

Collisional radiative model

The collisional radiative model is a set of coupled rate equations taking into account explicitly both collisional and radiative rates. There will be one rate equation for each bound level of the atom or ion stage considered.

Immediately becomes complex – many more unknowns

In one limit [high density] the CR model goes to LTE. In the low density limit, it goes to a coronal equilibrium.

Collisional radiative model for H

- References:
  - Sawada & Fujimoto, J. Appl. Phys. **78** (1995)
  - Fujimoto et al J. Appl. Phys. **66** (1989)
  - Johnson, L, & Hinnov, E, JQSRT
- Motivations
  - Can we obtain T_e, n_e from measurements of Balmer series lines?

CR model for H

Need to write an equation for each level in the hydrogen atom. allowing for transitions between levels due to electron collisions and radiative transitions. Also need to consider transitions from H_2, H_2^+, which lead to upper state populations under some conditions. Radiative recombination can also lead to upper state population.

What is energy cost to create an electron-ion pair beginning with H_2 molecular gas?

Dissociation energy of H_2 molecule is 4.47 eV.

Electron collisional dissociation requires more energy; it produces Franck-Condon H^o with several eV kinetic energy.

Ionization of H^o requires ~20eV, but ionization typically done by tail electrons while bulk excites H^o many times with rapid return to ground state -- significant radiation loss.

Net result is ~100eV required/H^+, but with considerable variation depending on density and electron temperature.

Charge exchange process

H + p --> p + H
The electron jumps from neutral H atom to the proton. This collision does not alter the orbits of H or p, so that the end result is a neutral with the velocity distribution that is characteristic of the velocity distribution of the protons.

Application
Charge exchange measurement of ion temperature in a hot magnetically confined plasma.

Passive Charge Exchange -- Very limited use because H neutrals present only near the edge, but an ion energy loss process

Active Charge Exchange -- Inject a beam of energetic H neutrals to fuel charge exchange in core; not generally useful because new neutrals ionize before escaping

Active Spectroscopy

Charge exchange recombination spectroscopy (CXRS)

$H + ion^{z} \text{-->} p + ion^{z-1*} \text{-->} p + ion^{z-1}$ + photon

Photon emitted will have information about the velocity distribution of ions of charge z and their number. Since the electron is generally captured to a highly excited state, a cascade of photons follows, including several visible lines.

Application: How would you know how many C^{6+} ions (the fully stripped form commonly found) there were in your hot plasma, how hot they were, and whether there was a bulk flow?

Because most modern tokamaks have very powerful (2 MW@) neutral beams for heating, this is a universal technique. (If heating beams are not available, smaller "Diagnostic Neutral Beams" may be used for CXRS.) This is another instance of the "crossed-chord" method to achieve spatial resolution.

Line Broadening

Finite lifetime $\Delta t \Delta \omega \geq 2\pi$

leads to a Lorentzian profile

where γ is width (damping), Δ is shift

$$I(\omega) = \frac{\gamma}{2\pi} \frac{1}{\left(\omega - \Delta - \omega_o v/c\right)^2 + \left(\gamma/2\right)^2}$$

Doppler broadening due to the motion of emitting atoms or ions $\frac{\Delta\lambda}{\lambda} = \frac{\Delta\omega}{\omega} = \frac{v}{c}$

Gives a Gaussian profile for a Maxwellian velocity distribution

$$I(\omega)d\omega = \frac{1}{\sqrt{\pi}} e^{-\left(\frac{(\omega-\omega_0)}{\omega_o v/c}\right)^2} d\omega$$

The half width for the profile is

μ is mass of radiator in units of atomic weight

$$\Delta\lambda_{1/2} = 7.16 \times 10^{-7} \lambda \left(\frac{T[K]}{\mu}\right)^{1/2}$$

Temperature measurements require that the Doppler broadening exceed all other contributions to the line width.

Line Broadening II -- Collisions

Collisional broadening

Collisions reduce the lifetime of a state. The line profile will be a Lorentzian as lifetime broadening, but broadened further by collisions.

Stark Broadening

Due to linear Stark effect from the fields of nearby electrons and ions. Generally consider only the nearest neighbor as a perturber to the radiating species. The nearest neighbor approximation gives a scaling of the field with density as

$$E_o = 2\pi\left(\frac{4}{15}\right)^{2/3} ZeN^{2/3}$$

This gives a half width of line profile that goes as $N^{2/3}$.

Both are generally less than Doppler broadening.

Motional Stark Effect -- $\mathbf{E}=\mathbf{v}\times\mathbf{B}$, Beam emission to measure **B**

Line Broadening III

Fluid Flow Velocities

Fluid (bulk) flows will Doppler-shift the center of lines, but shifts generally much less than Doppler broadening from T.

Sophisticated and powerful techniques (pioneered in astronomy) permit the measurements of very small shifts in the line center as a measurement of $\mathbf{v}_{||}$, the component of $\mathbf{v}$ along the line of sight, even in the presence of large line widths ($v/c \sim 10^{-7}$ possible). CXRS widely used to measure fluid velocities -- only limitation is whether velocity of measured ion is representative of total plasma mass.