Lecture #5
Motion in Two and Three Dimensions

- If more than one force is applied to an object, it is the *vector sum* of all the forces, $\Sigma\mathbf{F}$, that determines the acceleration of the object through Newton's Second Law: $\Sigma\mathbf{F} = m\mathbf{a}$.

  As long as we are concerned with three orthogonal (i.e., mutually perpendicular) directions, we can deal with the component equations independently:

  $$\Sigma F_x = ma_x, \qquad \Sigma F_y = ma_y, \qquad \text{and} \qquad \Sigma F_z = ma_z.$$

  Often it is simpler to deal with these component equations.

- For motion with a constant acceleration $\mathbf{a}$, the position $\mathbf{r}$ and velocity $\mathbf{v}$ of the object as a function of time are given by

  $$\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2 \qquad \text{and} \qquad \mathbf{v}(t) = \mathbf{v}_0 + \mathbf{a}t$$

  where $\mathbf{r}_0$ and $\mathbf{v}_0$ are the object's position and velocity at time $t = 0$, respectively. These vector equations can also be treated as three independent component equations.

- For projectile motion in the absence of any forces other than gravity (e.g., no air resistance), the projectile experiences a constant acceleration given by $\mathbf{a} = \mathbf{g} = -g\mathbf{j}$, where $\mathbf{j}$ is the unit vector pointing vertically upward. That is, $a_x = 0$, $a_x = -g$, and $a_z = 0$.

- *Tension* is a type of force that is transmitted through an object to another object. For example, if a weight is suspended from the ceiling by a string, then the ceiling exerts a force equal to the object's weight on the string, and the string exterts the same force on the weight (as does gravity). If one thinks of the string as several connected segments, then each segment of the string exerts the same force on its nearest neighbor. That is, each element of the string is under the same tension.

- When a force acts on an object, it is always possible to resolve the force into perpendicular components. It is sometimes useful to consider the component of force that is perpendicular to the surface of the object. This is called the *normal force*. (The component of force that is perpendicular to the normal force is tangential to the surface.)

- We sometimes ignore *friction* in order to simplify problems. Nonetheless, friction is important. Forces of friction are resistive in nature. They oppose the motion. On the atomic level, they are electromagnetic in nature and act between atoms and molecules on the surface of the objects in contact. From a macroscopic point of view, friction is due to the roughness and irregularities of the surface.

- A force of *static friction* f_s opposes the external force on a body until the body begins to move. If the external force on a body at rest increases, the value of f_s increases, reaching its maximum at the instant when motion begins. After the body is in motion, a force of *kinetic friction* f_k acts on the body. From experiment we know that f_s and f_k are proportional to the *normal* force N on the body.

  Specifically,

  1. the direction of the static friction is tangential to the surface and opposite to the tangential component of the external force. Its magnitude is $f_s \leq \mu_s N$, where (the dimensionless) μ_s is called the *coefficient of static friction* and N is the magnitude of the normal force; the equality holds when the body is on the verge of moving (i.e., $f_s = f_{s,\text{maximum}} = \mu_s N$).
  2. the direction of the kinetic friction is opposite to the external force and its magnitude is $f_k = \mu_k N$, where (the dimensionless) μ_k is called the *coefficient of kinetic friction.*
  3. μ_k and μ_s depend on the nature of the surfaces, but usually $\mu_k < \mu_s$.