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Backward Stimulated Raman Scattering of Laser Radiation in Plasma Channels

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Motivation

- Plasma-channel waveguiding can facilitate achieving practically interesting amplification rate in a Raman amplifier (RA) of short laser pulses.
- Elimination of Raman side-scatter instability in a plasma channel is indispensable for regular evolution of a short laser pulse in the RA.
- Increasing bandwidth of the BSRS process is necessary for weakening the requirement of exact frequency matching between the pump beam and amplified signal in the RA.
- Physical properties of sufficiently deep *single-mode* plasma channel meet the above requirements and facilitate further progress in the RA designing.

Outline

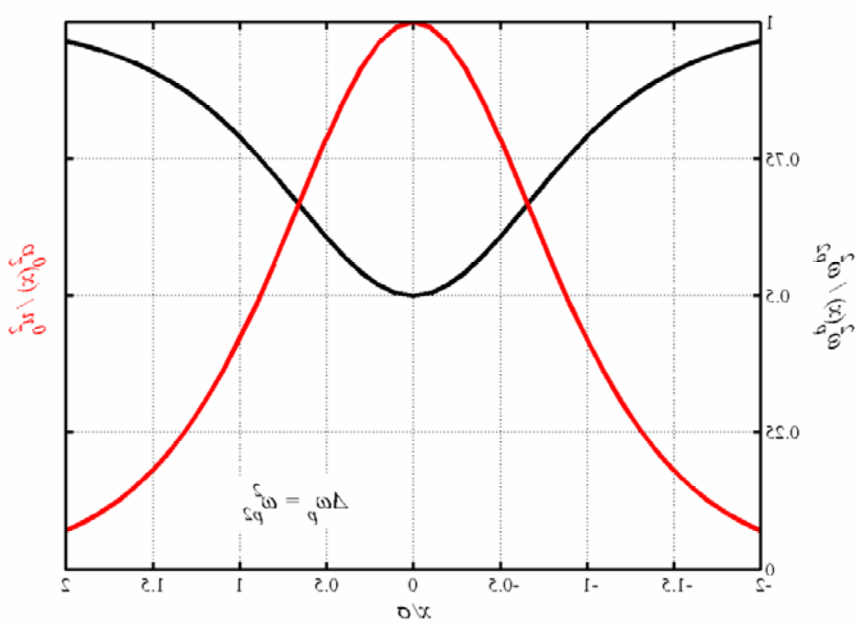
1. Ground state of plasma and electromagnetic field is specified, on which the BSRS instability develops in a plasma channel.
2. Basic set of coupled-mode equations is presented. General dispersion relation for BSRS in a single-mode plasma channel is proposed.
3. Regime of strong localization and its sub-regimes are specified. Spectral features of strongly localized BSRS are established. Effect of the continuum modes of radiation on the growth rate of instability and transversal distribution of field in a channel is investigated.
4. Linear Raman amplification of single-mode EM wave packet in a plasma channel is explored by means of a solution of the initial-value problem for the coupled mode equations.

Linear Theory of BSRS in a Single-mode Plasma Channel:
Starting Points and Basic Assumptions

- Plane shallow channel in a rarefied plasma is considered, which supports a single bound mode of EM radiation. This eliminates the effect of Raman side-scatter.
- Pump EM wave is chosen in the form of a bound channel eigenmode.
- In the process of BSRS a continuum of non-localized radiation modes is excited, and transversal shear of the plasma density couples the continuum modes to the bound mode of scattered light. Coupling between different modes of continuum is neglected.
- Fully electrostatic plasma response to the driving ponderomotive beatwave is assumed, which is valid for the short-wavelength scattering plasma mode.

Ground State: Single-mode Channel

Single-mode plane plasma channel:



Transversal shear of plasma frequency:

$$\omega_p^2(x) = \omega_{p2}^2 - (\Delta\omega_p^2/2)\,\psi_0^2(x)$$

$$\psi_0(x) = \cosh^{-1}(x/\sigma)$$

$$\Delta\omega_p\sigma = 2c$$

Pump EM wave:
Single transversally bounded mode

$$\mathbf{a}_0(x,z,t) = \mathrm{Re}\{\,\mathbf{e}_0 a_0(x) e^{ik_0 z - i\omega_0 t}\}$$

$$a_0(x) = u_0 \psi_0(x)$$

EM Pump Wave: Single Bound Eigenmode of Plasma Channel

Eigenmode problem for the EM pump field in a channel:

$$L_0 a_0(x) \equiv \left(-\frac{\partial^2}{\partial x^2} + \frac{\omega_p^2(x)}{c^2} \right) a_0(x) = \lambda_0 a_0(x)$$

Laser envelope:
Single bound solution of the eigenmode problem

$$a_0(x) = u_0 \psi_0(x)$$

Dispersion relation of a single bound mode of a channel:

$$\lambda_0 \equiv \frac{\omega_0^2}{c^2} - k_0^2 = \frac{\omega_{p2}^2}{c^2} - \frac{1}{\sigma^2}$$

Basic Equations

Scattered EM field and scattering perturbations of electron density

$$\begin{aligned}\mathbf{a}-\mathbf{a}_0 &= \mathrm{Re}\{\mathbf{e}_0 a_s(x,z,t)\exp[-i(\omega_0-\omega_{p1})t-i(k_0-k_{p1})z]\}\\ n_e-n_0(x) &= \mathrm{Re}\{\delta n_e(x,z,t)\ \exp[-i\omega_{p1}t+i(2k_0-k_{p1})z]\}\end{aligned}$$

Basic set of coupled-mode equations

$$\begin{aligned}\left\{\frac{\partial}{\partial\bar{t}}-\frac{\partial}{\partial\bar{z}}+\frac{i}{2\bar{\omega}_0}\left[\hat{L}_0-\bar{\lambda}_0\right]\right\}a_s &= -i\frac{u_0\psi_0}{4\bar{\omega}_0}\nu\\ \left\{\frac{\partial}{\partial\bar{t}}-i(\Delta\bar{\omega}_p/2)^2(1-\psi_0^2)\right\}\nu &= iu_0\psi_0\bar{\omega}_0^2\bar{\omega}_p^2 a_s\end{aligned}$$

Notations:

$$\omega_{p1}=\omega_p(x=0),\quad \bar{t}=\omega_{p1}t,\quad \hat{L}_0=(\omega_{p1}/c)^2L_0,\quad \bar{\lambda}_0=\lambda_0(c/\omega_{p1})^2,\quad \nu=\bar{\omega}_p^2\,\delta n_e^*/n_0(x).$$

Derivation of the Dispersion Equation

1. Fourier-Laplace transform of the amplitudes: $a_s, v \propto \exp(i\bar{\omega}\bar{t} - i\bar{k}\bar{z})$

2. Scattered radiation: Superposition of the single bound capillary mode and continuum of non-bound modes

$$a_s(\bar{x},\bar{k},\bar{\omega}) = \underbrace{P_1^1(y)}_{\text{bound mode}} + \int_{-\infty}^{+\infty} \underbrace{P_1^{iq}(y)\, a_s(q,\bar{k},\bar{\omega})\, dq}_{\text{continuum of non-bound modes}}$$

The mode amplitudes are expressed through associated Legendre functions of $y = \tanh(x/\sigma)$:

$$P_1^1(y) = \sqrt{1-y^2}, \quad P_1^{iq}(y) = \frac{1}{\Gamma(1\mp iq)} \frac{y \mp iq}{1 \mp iq} \left(\frac{1+y}{1-y}\right)^{\pm iq/2}$$

3. Radial shear of plasma density couples the bound and non-bound modes.

(a) Coupling of continuum modes to the bound modes [bound $\leftrightarrow a_s(q)$] is taken into account.

(b) Coupling between different modes of continuum [$a_s(q) \leftrightarrow a_s(q')$] is neglected

General Dispersion Relation

$$p^2 + \frac{G}{2\overline{\omega}}\tilde{Q}(\overline{\omega}) = \overline{\sigma}^2 \left(\frac{G}{2\overline{\omega}}\right)^2 \int_{-\infty}^{+\infty} \frac{F(\overline{\omega}, iq)F(\overline{\omega}, -iq)}{1+(\overline{\sigma}p)^2+q^2} \frac{q\,dq}{\sinh(\pi q)}$$

- Kernel $F(\overline{\omega}, iq)$ describes the coupling of the continuum modes to the bound mode of the channel:

$$F(\overline{\omega}, iq) = \int_{-1}^{1} \frac{1+C^2y^2}{1-B^2y^2}\sqrt{1-y^2}\,P_1^{iq}(y)\,dy$$

- Function $\tilde{Q}(\overline{\omega})$ describes the plasma *electrostatic* response to the ponderomotive beatwave of pump and fundamental bound mode of scattered radiation:

$$\tilde{Q}(\overline{\omega}) = \frac{2}{B^2}\left(1 - \frac{2C^2}{3} + \frac{C^2}{B^2}\right) - \frac{(B^2+C^2)(1-B^2)}{B^5}\ln\left(\frac{1+B}{1-B}\right)$$

- Function $p^2 = 2\omega_0(\overline{\omega} + \overline{k})$ contains information about propagation of the scattered radiation.

Notations: $G = u_0^2\overline{\omega}_0^2/2,\quad C^2 = \Delta\overline{\omega}_p^2/2,\quad B^2 = \Delta\overline{\omega}_p^2/(4\overline{\omega})$

BSRS in the Regime of Strong Localization of Scattering Plasma Wave

The basic physical features of the regime of strong localization:

• The temporal increment of BSRS in homogeneous plasma is smaller than a channel depth:

$$\eta \equiv \Delta\overline{\omega}_p^2 / (2\gamma_{\text{hom}}) > 1, \quad \gamma_{\text{hom}} = (u_0/2)\sqrt{\overline{\omega}_0}$$

• Scattering plasma wave is localized stronger than a driving ponderomotive beatwave:

$$|\delta x| \sim \sigma/\eta < \sigma$$

Parameter areas for BSRS in the regime of strong localization:

• Multi-mode regime:

$$\underbrace{\frac{\Delta\overline{\omega}_p^2}{\sqrt{2\pi}(2\overline{\omega}_0)^{5/4}}}_{\text{multi-mode condition}} < u_0 < \underbrace{\frac{\Delta\overline{\omega}_p^2}{2\sqrt{\overline{\omega}_0}}}_{\text{condition of plasma wave localization}}$$

• Single-mode regime:

$$u_0 < \frac{\Delta\overline{\omega}_p^2}{\sqrt{2\pi}(2\overline{\omega}_0)^{5/4}}$$

Dispersion Equation of Strongly Localized BSRS

- *Multi-mode* regime

$$p^2 + i\frac{G}{2\bar{\varpi}}\frac{\pi}{B} \approx -\left(\frac{G}{2\bar{\varpi}}\right)^2\left(\frac{\pi}{Bp}\right)^2\left(\sqrt{1+(\bar{\sigma}p)^2}-1\right)$$

- *Single-mode* regime

$$p^2 + i\frac{G}{2\bar{\varpi}}\frac{\pi}{B} \approx 0$$

Physical meaning of the single-mode regime of strongly localized BSRS:

Only single localized mode of scattered radiation is involved in BSRS in a plasma channel, whereas the continuum modes are not excited. This makes the single-mode BSRS a channel counterpart of a three-wave BSRS process in a homogeneous plasma.

Spectral Features of the Single-mode Strongly Localized BSRS

- Maximum increment

$$\gamma_{\max} = \frac{\sqrt{3}}{2}\sqrt[3]{\frac{\pi^2}{2\eta}\gamma_{\text{hom}}} < \gamma_{\text{hom}}$$

- Bandwidth limit from the blue side

$$\overline{\omega}_{\text{cutoff}} = -\sqrt[3]{2}\gamma_{\max}/\sqrt{3}$$

- Red-shift of the spectral maximum

$$\Delta\overline{\omega}_{\text{red}} = \gamma_{\max}/\sqrt{3}$$

- Group velocity of scattered radiation

$$v_g \equiv c\frac{\partial \operatorname{Re}\overline{\omega}}{\partial \overline{k}}\bigg|_{\overline{\omega}=\Delta\overline{\omega}_{\text{red}}} = -\frac{2c}{3}$$

Important notes:

- Spectrum of the single-mode strongly localized BSRS reveals *no bandwidth limit from the red side*.
- *Group velocity* of scattered radiation in a channel *is higher than in a homogeneous plasma*, where $v_{g\,\text{hom}} = -c/2$

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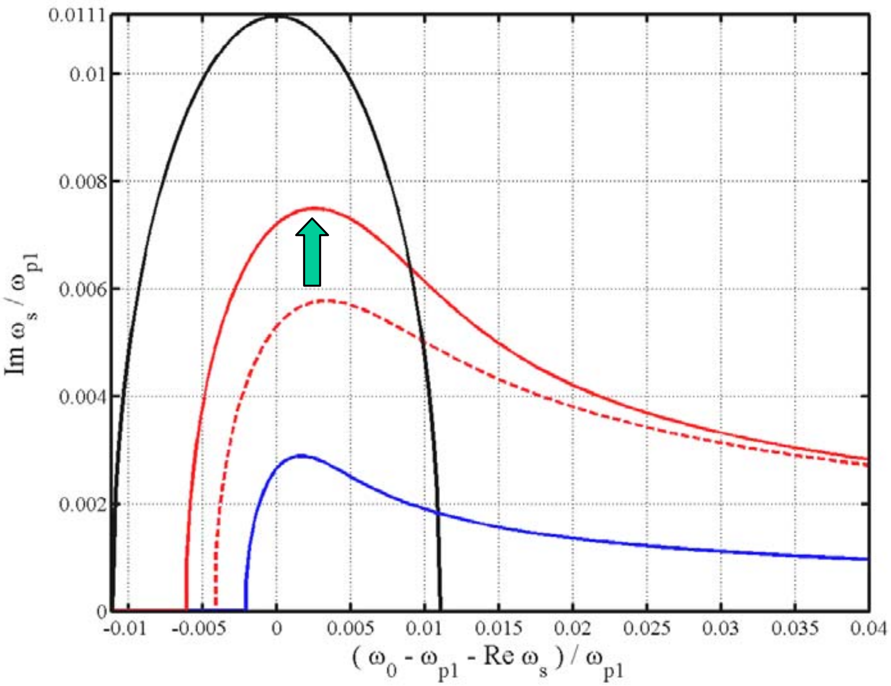
Spectrum of Strongly Localized BSRS



Basic spectral features:

1. Overall red-shift of the spectrum.
2. Significant reduction of the growth rate.
3. Broad bandwidth of the process in comparison with BSRS in a homogeneous plasma.
4. Contribution from the continuum modes enhances the process of strongly localized BSRS.

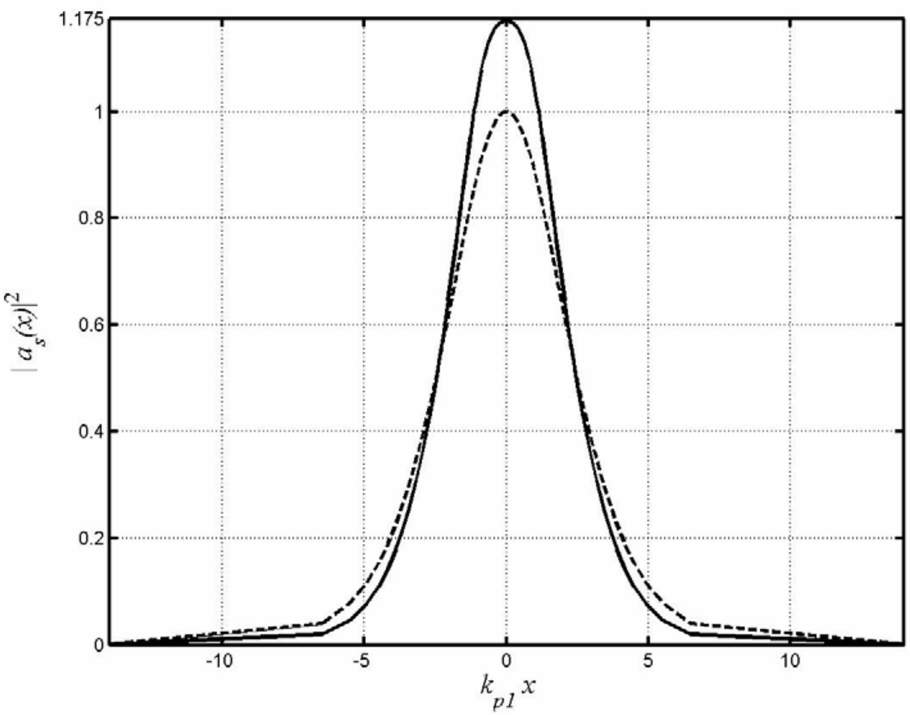
- Parameters of laser: $\varpi_0 = 10, \quad u_0 = 0.007$
- Regimes of BSRS: Blue line – single-mode strongly localized $[n_e(x=0)/n_e(\infty) = 1/3]$.

  Red lines – multi-mode strongly localized $[n_e(x=0)/n_e(\infty) = 4/5]$,

  dashed line – continuum modes contribution is taken off.

  Black line – reference spectrum of BSRS in a homogeneous plasma.

Effect of Continuum Modes on Transverse Distribution of Scattered Radiation in the Multi-mode Regime of Strongly Localized BSRS



Contribution from the continuum modes enhances the localization of waves within a channel:

Transversal distribution of field
Is squeezed and field amplitude is
Increased near the channel axis.

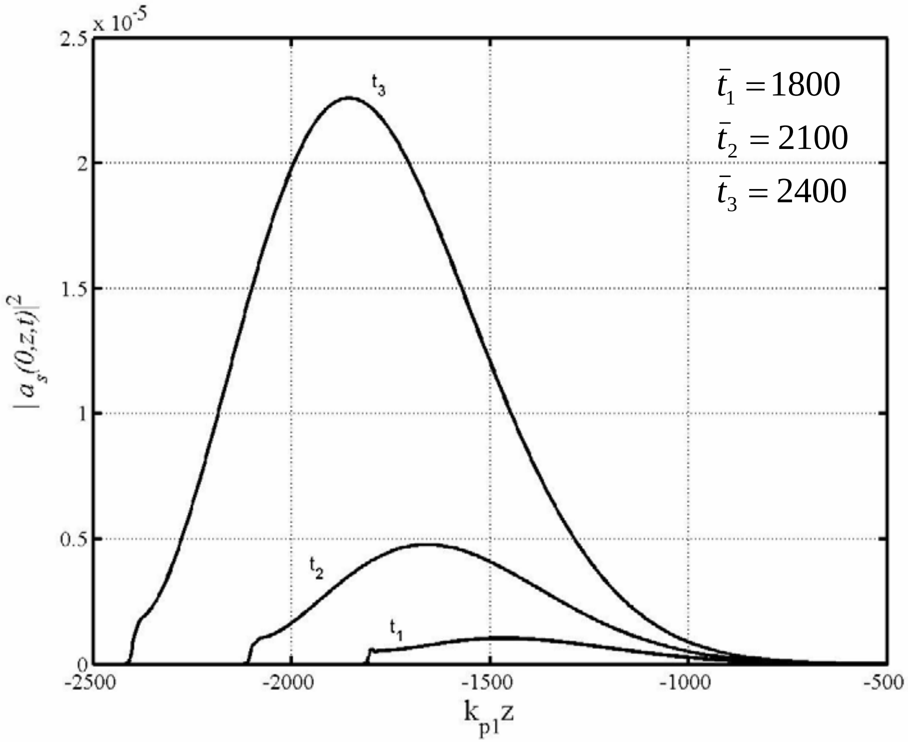
$$\left\langle \bar{x}^2 \right\rangle = \frac{\int_{-\infty}^{+\infty} \bar{x}^2 \left| a_s(\bar{x}) \right|^2 d\bar{x}}{\int_{-\infty}^{+\infty} \left| a_s(\bar{x}) \right|^2 d\bar{x}} \quad \Rightarrow \quad \sqrt{\frac{\left\langle \bar{x}^2_{\text{b.+c.}} \right\rangle}{\left\langle \bar{x}^2_{\text{b.}} \right\rangle}} \approx 0.77$$

Linear Evolution of EM Wave Packet in the Single-mode Regime of Strongly Localized BSRS

- Initial-value problem: Single-mode EM wave packet in unperturbed plasma channel

$$a_s(\bar{x},\bar{z},\bar{t}=0)=a_{s0}(\bar{z})\psi_0(\bar{x})$$
$$\bar{v}(\bar{x},\bar{z},\bar{t}=0)=0$$

- Exact solution of the initial value problem:



$$a_s(\bar{x},\bar{z},\bar{t})=\psi_0(\bar{x})\left\{\underbrace{a_{s0}(\bar{z}+\bar{t})}_{\text{seed perturbation}}+\underbrace{i\pi^2\frac{\gamma_{\text{hom}}^3}{2\eta}\int_0^{t}{}_1F_3\left(\frac{1}{2};1,\frac{3}{2},\frac{3}{2};i\left(\frac{\pi}{2}\right)^2\frac{\gamma_{\text{hom}}^3}{2\eta}\theta_1^2(\bar{t}-\theta_1)\right)a_{s0}(\bar{z}+\theta_1)\theta_1^2\,d\theta_1}_{\text{scattered radiation - amplified wave packet}}\right\}$$

Basic Features of Linear Raman Amplification of a Short Laser Pulse in the Single-mode Regime of Strong Localization

- Amplitude of the amplified pulse grows with time much slower than in a case of a homogeneous plasma.
- Bandwidth of the amplification process greatly exceeds the bandwidth of the ordinary (weakly coupled) BSRS in a homogeneous plasma.
- Group velocity of the amplified pulse is higher than in the case of Raman amplification in a homogeneous plasma.

Summary

- Linear theory of BSRS in a plane single-mode shallow plasma channel is developed.
- The BSRS regime with a **strong localization of scattering electron plasma wave** is identified.
- Transverse shear of the plasma frequency reduces the temporal growth rate and increases the bandwidth of BSRS. Also, the scattered radiation red-shift by more than $\omega_p(x=0)$ is the case in a plasma channel.
- Contribution from the continuum of non-localized radiation modes increases the temporal growth rate of BSRS by enhancing the scattered wave localization in a plasma channel.
- Two regimes of **strongly localized BSRS** are identified, **multi-mode** and **single-mode**.
- Linear Raman amplification (RA) of a single-mode EM wave packet within a plasma channel is investigated. Group velocity of the amplified wave packet is found to be $v_g = -2c/3$, that is higher than in the case of RA in a homogeneous plasma, $v_{g_{\rm hom}} = -c/2$.

Properties of Associated Legendre Functions With a Degree 1 and Imaginary Order *iq*

Differential equation:

$$\left\{\frac{d}{dy}(1-y^2)\frac{d}{dy}+2+\frac{q^2}{1-y^2}\right\}P_1^{\pm iq}(y)=0$$

Orthogonality condition:

$$\int_{-1}^{1}P_1^{iq}(y)P_1^{-iq'}(y)\frac{dy}{1-y^2}=\left(\frac{2}{q}\right)\sinh(\pi q)\delta(q'-q)$$

Expression in terms of a variable x , $y=\tanh(x/\sigma)$:

$$P_1^{\pm iq}(x)=\frac{\tanh(x/\sigma)\mp iq}{(1\mp iq)\Gamma(1\mp iq)}\exp(\pm iqx/\sigma)$$

Cosine decomposition:

$$\cos(qx/\sigma)\equiv\left(\frac{\pi}{4}\right)\frac{(1+q^2)[P_1^{iq}(x)+P_1^{iq}(-x)+P_1^{-iq}(x)+P_1^{-iq}(-x))}{q\,\mathrm{Re}\,\Gamma(1+iq)+\mathrm{Im}\,\Gamma(1-iq)}$$